ONE-SHOT GAME

STATIC OLIGOPOLY MODELS → STATIC GAMES OF COMPLETE INFORMATION

- COURNOT MODEL (1838)
- BERTRAND MODEL OF PRICE COMPETITION
- STACKELBERG MODEL

COURNOT (1838)

FIRST, LET'S LIST THE ASSUMPTIONS FOR OLIGOPOLISTIC MARKETS.

1. CONSUMERS ARE PRICE TAKERS
2. ALL FIRMS PRODUCE HOMOGENEOUS PRODUCTS
3. THERE IS NO ENTRY INTO THE INDUSTRY
4. FIRMS COLLECTIVELY HAVE "MARKET POWER"
5. EACH FIRM SETS ONLY ITS OUTPUT OR ITS PRICE (NOT OTHER VARIABLES SUCH AS ADVERTISING)

SECOND, SUMMARY OF RESULTS ON OLIGOPOLISTIC MARKETS.

1. THE EQUILIBRIUM PRICE LIES BETWEEN THAT OF MONOPOLY AND PERFECT COMPETITION
2. FIRMS MAXIMIZE PROFITS BASED ON "THEIR BELIEFS ABOUT ACTIONS OF OTHER FIRMS"
3. FIRM'S EXPECTED PROFITS ARE MAXIMIZED WHEN EXPECTED MARGINAL REVENUE EQUALS MARGINAL COST.
4. *** MARGINAL REVENUE FOR A FIRM DEPENDS ON ITS RESIDUAL DEMAND CURVE (= MARKET DEMAND CURVE MINUS OUTPUT SUPPLIED BY OTHER FIRMS)

COURNOT (1838)

1. THERE ARE 2 FIRMS W/ NO ADDITIONAL ENTRY
2. HOMOGENEOUS PRODUCT SUCH THAT $q_1 + q_2 = Q$ WHERE Q IS INDUSTRY OUTPUT, q_i IS THE OUTPUT OF THE i^{th} FIRM.
3. SINGLE PERIOD OF PRODUCTION AND SALES (EX: THINK OF PERISHABLE CROP SUCH AS WATERMELON, MANGOSTEEN)
4. MARKET AND INVERSE MARKET DEMAND IS A LINEAR FUNCTION OF PRICE: $P = A - BQ = A - B(q_1 + q_2)$.

$$\boxed{P = A - Bq_1 - Bq_2}$$

$$\boxed{Q = \frac{A-P}{B} = \frac{A}{B} - \frac{P}{B}.}$$

5. EACH FIRM HAS CONSTANT AND EQUAL MARGINAL COST $= c$. [i.e, W/ CONSTANT MC, YOU KNOW THAT $MC = AC = c$]
6. THE DECISION VARIABLE IS "QUANTITY OF OUTPUT TO PRODUCE AND MARKET"

RECALL THAT

$$\pi_1 = \pi_1(q_1, q_2)$$

$$\pi_2 = \pi_2(q_1, q_2)$$

COURNOT-NASH EQUILIBRIUM: FOR q_1^c AND q_2^{c} TO BE THE NASH EQUILIBRIUM QUANTITIES, THE FOLLOWING TWO CONDITIONS MUST BE TRUE:

$$\pi_1(q_1^c, q_2^c) \geq \pi_1(q_1, q_2^c) \quad \text{FOR ANY } q_1 \quad \text{—①}$$

$$\pi_2(q_1^c, q_2^c) \geq \pi_2(q_1^c, q_2) \quad \text{FOR ANY } q_2 \quad \text{—②}$$

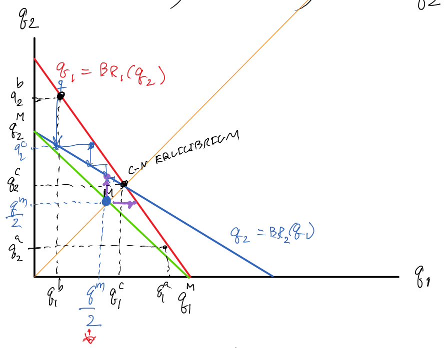
THE COURNOT-NASH EQUILIBRIUM CAN BE FOUND BY USING "BEST RESPONSE FUNCTIONS".

FIRM 1'S BEST RESPONSE FUNCTION GIVES THE PROFIT MAXIMIZING CHOICE OF OUTPUT

FOR FIRM 1 FOR ANY OUTPUT CHOSEN BY FIRM 2 .

$$q_1^c = BR_1(q_2^c)$$

FIRM 2'S BEST RESPONSE FUNCTION, LIKEWISE, IS $q_2^c = BR_2(q_1^c)$



q^M = MONOPOLY OUTPUT IF THE TWO FIRMS COLLUDE.

STABILITY ?

IF THE TWO FIRMS COLLUDE, THE PRODUCTION = q^M $\frac{q^m}{2}$, $\frac{q^m}{2}$

WE WILL BE AT POINT M.

BY DOING SO, $\pi_i\left(\frac{q_i^m}{2}, \frac{q_{-i}^m}{2}\right) > \pi_i(q_i^c, q_{-i}^c)$.

SO, BOTH FIRMS HAS AN INTEREST TO COLLUDE. ARE THEY GOING TO HONOR THE AGREEMENT UNDER THE CARTEL ? ANSWER IS NO ! (CHEAT)

COMPETITION $\longleftrightarrow$ COOPERATION

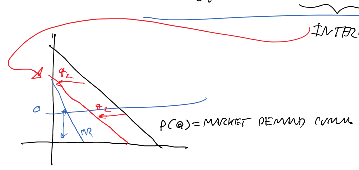
LET'S SEE A NUMERICAL EXAMPLE :

MARKET DEMAND CURVE : $P(Q) = A - BQ$ WHERE $Q = q_1 + q_2$

WHERE A AND B ARE PARAMETERS.

COSTS OF PRODUCTION : $\underbrace{C^i = c \cdot q_i}_{\text{SYMMETRIC}}$ WHERE $i = 1, 2$

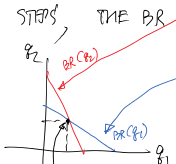
FIRM 1'S RESIDUAL DEMAND CURVE : $\boxed{P(q_1, q_2) = \underbrace{(A - Bq_2)}_{\text{INTERCEPT}} - Bq_1}$



- FIRM 1'S MARGINAL REVENUE CURVE : $MR(q_1, q_2) = (A - Bq_2) - 2Bq_1$.
- EQUATING MR AND MC GIVES $(A - Bq_2) - 2Bq_1 = c$

$$\boxed{q_1 = \frac{A - Bq_2 - c}{2B}}.$$

BY THE SAME STEPS, THE BR FUNCTION FOR FIRM 2 IS



$$\boxed{q_2 = \frac{A - Bq_1 - c}{2B}}.$$

2 EQUATIONS $\rightarrow$ SOLVING FOR q_1, q_2 GIVES

2 UNKNOWN

$$\boxed{q_1^c = q_2^c = \frac{A - c}{3B}}$$

C-N EQUILIBRIUM QUANTITIES

SO $\boxed{\text{AGGREGATE OR MARKET OUTPUT } (Q) = q_1^c + q_2^c = 2 \cdot \left(\frac{A-c}{3B}\right)}$

PRICE (P^c) = A - B·Q = A - $\cancel{B}$ · 2(A-c)

SO $\boxed{\text{AGGREGATE OR MARKET OUTPUT } (Q) = q_1^c + q_2^c = 2 \cdot \left(\frac{A-c}{3B}\right)}$

AND PRICE $(P^c) = A - B \cdot Q = A - \cancel{B} \cdot 2\left(\frac{A-c}{\cancel{3B}}\right)$

$$\boxed{P^c = \frac{A+2c}{3}}$$

FIRM 1's PROFIT $(\pi_1) = P \cdot q_1 - c \cdot q_1$. WHEN $P = P^c$ AND $q_1 = q_1^c$,

$$\boxed{\pi_1^c = \frac{(A-c)^2}{9B}}$$

PROPERTIES OF C-N EQUILIBRIUM

- MARKET POWER AND EFFICIENCY:

RECALL THAT $MR(q_1, q_2) = \boxed{P(q_i, q_j) + \frac{dP(q_i, q_j)}{dQ} \cdot q_1 = MC(q_1)}$

$TR = P(Q) \cdot Q$
$MR = P(Q) + Q \cdot \frac{dP(Q)}{dQ}$

$\frac{dQ}{dP} \cdot \frac{P}{Q} = \mathcal{E}$

LET'S DIVIDE BOTH SIDES BY $P(q_i^c, q_j^c)$

$i, j = 1, 2$
$i \neq j$

$$\frac{P(q_i^c, q_j^c) - MC(q_1)}{P(q_i^c, q_j^c)} = -\frac{dP(q_i^c, q_j^c)}{dQ} \cdot q_i \cdot \frac{1}{P(q_i^c, q_j^c)} \cdot \frac{Q^c}{Q^c}$$

$$\boxed{\frac{P - MC}{P} = \frac{1}{|\mathcal{E}|} \cdot s_i}$$

$s_i = \frac{q_i}{Q^c}$

WHERE $|\mathcal{E}|$ = PRICE ELASTICITY OF DEMAND

s_i = MARKET SHARE OF FIRM i, $\frac{q_i^c}{Q^c}$

OBSERVATION #1 : COURNOT DUOPOLISTS EXERCISE SOME DEGREE OF MARKET POWER, i.e. PRICE WILL BE SET ABOVE MC.

OBSERVATION #2 THE HIGHER THE $|\mathcal{E}|$, THE LOWER DEGREE OF MARKET POWER (= THE LESS OF MARKUP ABILITY)

OBSERVATION #3 COURNOT MARKUP < MONOPOLY MARKUP (WHY?) SINCE s_i WILL BE LESS THAN ONE

OBSERVATION #4 THE GREATER THE NUMBER OF COMPETITORS, THE LOWER EACH FIRM'S MARKET SHARE, AND THE LESS MARKET POWER.

WHAT WE LEARN IS THAT "BARRIERS TO ENTRY" IS THE KEY FOR MARKUP ABILITY FOR A FIRM IN AN INDUSTRY

$\uparrow$ BARRIERS TO ENTRY $\longrightarrow$ $\downarrow$ # OF COMPETITORS $\longrightarrow$ $\uparrow \left(\frac{P - MC}{P}\right)$

SUPPOSE WE HAVE AN OLIGOPOLY W/ N FIRMS IN THE INDUSTRY.

SO FIRM i WILL TAKE INTO A/C OUTPUTS PRODUCED BY $N-1$ FIRMS WHEN IT FINDS A PROFIT MAXIMIZING OUTPUT LEVEL.

$$\boxed{q_{-i} = (q_1, q_2, \ldots, q_{i-1}, q_{i+1}, \ldots, q_N)}$$

THEN, IN EQUILIBRIUM, $\frac{P(q_i^c, q_{-i}^c) - MC_i(q_i^c)}{P(q_i^c, q_{-i}^c)} = \frac{s_i}{|\mathcal{E}|}$

IF WE MULTIPLY BOTH SIDES BY s_i AND SUM BOTH SIDES OVER ALL N FIRMS

$$\sum_{i=1}^{N} s_i \left(\frac{P^c - MC(q_i^c)}{P^c}\right) = \sum_{i=1}^{N} \frac{s_i^2}{|\mathcal{E}|}$$

WHERE $\sum^{N} s_i^2$ = HHI (HERFINDAHL - HIRSCHMAN INDEX)

p^c = COURNOT EQUILIBRIUM PRICE

HHI = SUM OF THE SQUARES OF MARKET SHARES

MEASURES DEGREE OF MARKET CONCENTRATION WHICH CAN VARY FROM 0 (PERFECT COMPETITION) TO 1 (MONOPOLY)

EXAMPLE

CASE 1: $\underbrace{\overbrace{\overset{15\%}{A}, \overset{15\%}{B}, \overset{15\%}{C}, \overset{15\%}{D}, \overset{15\%}{E}, \overset{15\%}{F}}^{90\%}}_{\text{6 LARGEST FIRMS}}$ AND 10% CAPTURED BY 10 EQUALLY SIZED FIRMS.

0.01×10

CASE 2: THE LARGEST FIRM PRODUCES 80% AND THE NEXT FIVE LARGEST FIRMS PRODUCE 2% EACH AND THE REST OF 10 FIRMS PRODUCE 1% EACH

CASE 1 $\Rightarrow$ $HHI = (0.15)^2 + (0.15)^2 + (0.15)^2 + (0.15)^2 + (0.15)^2 + (0.15)^2 + \underbrace{(0.01)^2 + \dots + (0.01)^2}_{\text{10 FIRMS}} = 0.136$ (OR

CASE 2 $HHI = (0.80)^2 + (0.02)^2 + (0.02)^2 + (0.02)^2 + (0.02)^2 + (0.02)^2 +$
$10 \cdot (0.01)^2 = 0.643$ (OR 64.3%)

GUIDE LINE

HHI BELOW 0.01 (OR 100) $\Rightarrow$ HIGHLY COMPETITIVE INDU

HHI BELOW 0.15 (OR 1500) $\Rightarrow$ UNCONCENTRATED INDUSTRY

HHI BET 0.15 TO 0.25 $\Rightarrow$ MODERATE CONCENTRATION

HHI ABOVE 0.25 (ABOVE 2500) $\Rightarrow$ HIGH CONCENTRATI

EX SUPPOSE 4 FIRMS, EACH PRODUCES 25%

$HHI = (0.25)^2 \cdot 4 = 0.25$ OR $(25)^2 \cdot$

$$\boxed{\frac{1}{0.25} = 4}$$

REMARK $\frac{1}{HHI}$ = NUMBER OF FIRMS IN THE INDUSTR (IN CASE THAT ALL FIRMS H EQUAL MKT SHARE.)

COMPARATIVE STATICS

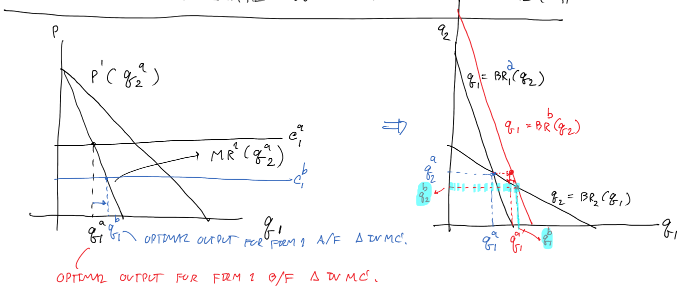
Q: HOW DO CHANGES IN THE EXOGENOUS PARAMETERS MODEL AFFECT THE COURNOT EQUILIBRIUM?

EX) WHAT IF MC CHANGES?

" ——— " MR " ——— " ?

" ——— " # OF FIRMS CHANGES?

EFFECT OF DECREASE IN FIRM i'S MARGINAL COST



IF ALL FIRMS HAVE THE SAME MARKET SHARE, $s_i = \frac{1}{N}$.

SO $$\boxed{\frac{p^c - MC}{p^c} = \frac{1}{|\varepsilon| \cdot N}}$$

13.6%)

STRY

N

4 = 2500

Y
ANE

OF THE

ALL FIRMS HAVE THE SAME MARKET SHARE, $s_i = \frac{1}{N}$.

SO $$\boxed{\frac{P^c - MC}{P^c} = \frac{1}{|\varepsilon| \cdot N}}$$

- WHEN $N \uparrow$, $\frac{P^c - MC}{P^c} \downarrow$
- IN LIMIT, IF $N \to \infty$, $P \to MC$.

EXERCISE $P(Q) = A - BQ$ WHERE $Q = \sum_{i=1}^{N} q_i$.

SUPPOSE ALL FIRMS HAVE IDENTICAL COST FUNCTION

$C_i = c\, q_i \to$ MC IS CONSTANT AND EQUAL TO c.

SOLUTION:

$$MR_i\left(q_i, \sum_{i \neq j} q_j\right) = A - B\sum_{i \neq j} q_j - 2B q_i$$

$$A - B\sum_{i \neq j} q_j - 2B q_i = c \quad \to$$

IN EQ

q_i

$$A - B(N-1) q^c - 2b\, q^c = c$$

$$q^c = \frac{A - c}{(N+1)B}$$

$$Q = N \cdot q^c = N \cdot \frac{A - c}{(N+1)B}$$

$$P^c = \frac{A + Nc}{N+1}$$

$$\pi^c = \left(\frac{A - c}{N+1}\right)^2 \frac{1}{B}$$

$\uparrow N \to$ EACH FIRM'S OUTPUT $(q^c) \downarrow$

$\to$ TOTAL OUTPUT IN THE INDUSTRY (Q)

$\to$ PRICE $(P^c) \downarrow$

$\to$ FIRM'S PROFIT $(\pi^c) \downarrow$

v :

ILIBRIUM

$= q_2 = \dots = q_N$

N EQUATIONS

N UNKNOWNS

$\frac{\partial q^c}{\partial N} < 0$

$\uparrow \quad \frac{\partial Q}{\partial N} > 0$

$\rightarrow \frac{\partial P^c}{\partial N} < 0$

$\rightarrow \frac{\partial \pi^c}{\partial N} < 0$