BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

FN211: Lecture Note 4
Working with Probability and Introduction to Multivariate Distribution

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Outline

- Working with Probability
- Introduction to Multivariate Distribution

Working with Probability

Working with probabilities

- We use the **multiplication rule** to assess the joint probability of *A* and *B* occurring.

$$P(AB) = P(A|B)P(B)$$

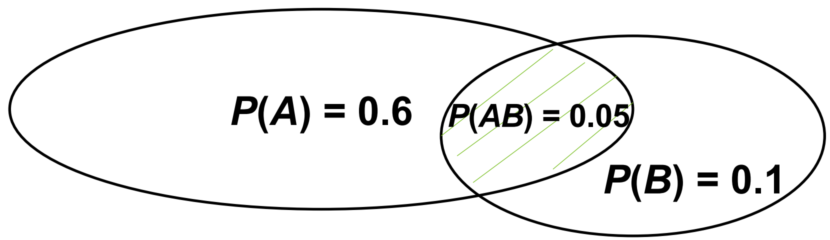
  - Note that from this equation, we can calculate the conditional probability of *A* given *B*, as long as the probability of *B* isn't zero.

- We use the **addition rule** to assess the probability that *A* or *B* <u>or both</u> occur:

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$



- If the probability of relaxed import restrictions is 0.60 and the probability of a trade war is 0.10, then the probability of relaxed trade restrictions or a trade war is 0.65 when the joint probability of a trade war and relaxed trade restrictions is 0.05.

Multiplication rule

Example: Recall the probability of relaxed trade restrictions has been estimated at 60%.

If the probability of reduced sales, given that the trade restrictions are relaxed, is 80%, then the probability of relaxed trade restrictions and reduced sales is

$$P(AB) = P(A|B)P(B)$$

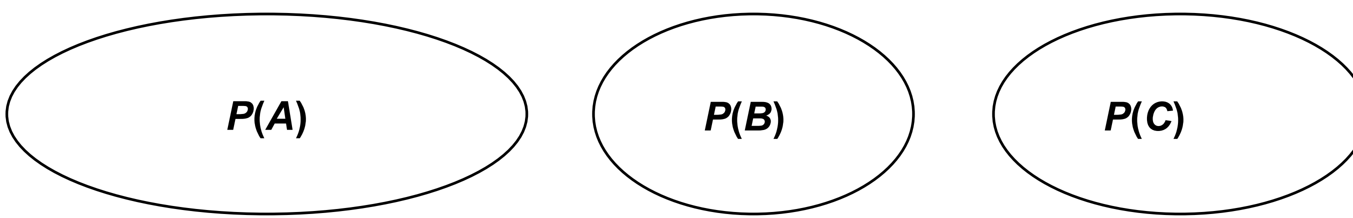
Dependent and Independent events

- When events are independent, the occurrence of one does not affect the probability of the other. In other words, $P(A|B = 0)$ and $P(B|A) = 0$.
  - To determine the joint probability, we use the multiplication rule:

$$P(AB) = P(A)P(B)$$

or, more generally,

$$P(ABC\ ...) = P(A)P(B)P(C)\ ....$$



Joint probability of independent events

Example:

- Recall our analyst's estimated probability of relaxed import restrictions (60%).
- If A is the probability of a heavy winter snowfall (40%) and

  B is the independent probability of relaxed restrictions:
- Then the joint probability of relaxed import restrictions and a heavy winter snowfall is:

Total probability rule

- Recall that we have a 60% (40%) chance of trade restrictions being relaxed (maintained).
  - $P(RR) = 0.6$, where RR is relaxed restrictions.
  - $P(NR) = 0.4$, where NR is no relaxation.
- Our analysts believe that the stock price will decrease if trade restrictions are relaxed with a probability of 30% and that they will decrease if trade restrictions are not relaxed with a probability of 15%.
  - $P(DS|RR) =$
  - $P(DS|NR) =$
- What is the probability of a decrease in stock prices?

$$P(DS) = P(DS|RR)P(RR) + P(DS|NR)P(NR)$$

Expected value

Random Variable

- The expected value of a random variable is the probability-weighted average of the possible outcomes for that variable.

$$E(X) = \sum_{i=1}^{n} X_i P(X_i)$$

- We anticipate that there is a 15% chance that next year's return on holding Cleveland Corp will be 4%, a 60% chance it will be 6%, and a 25% chance it will be 8%. What is the expected return on Cleveland Corp stock?

Variance

- The variance of a random value is the sum of the squared deviations from the expected value weighted by their associated probabilities.

$$\sigma^2(X) = \sum_{i=1}^{n} [X_i - E(X)]^2 P(X_i) = E\{[X - E(X)]^2\}$$

  - This value is a measure of the dispersion of possible values.
  - Because it has units that are squared, it is not easy to interpret. Accordingly, we use its positive square root, standard deviation, more often because it also measures dispersion but has the same units as expected value.
- The standard deviation of returns for Cleveland Corp. is then:

Conditional Expectations

- The total probability rule applies to expected values just as it does any mutually exclusive and exhaustive set of possible outcomes across a set of states.

$$E(X) = \sum_{i=1}^{n} E(X|S_i)P(S_i)$$

- This equation allows us to calculate the expected value of a random variable (X) as a function of the probabilities of future possible states, $P(S)$, and the conditional value of the expected value of X in those states, $E(X|S)$.

Conditional Expected value

Example:

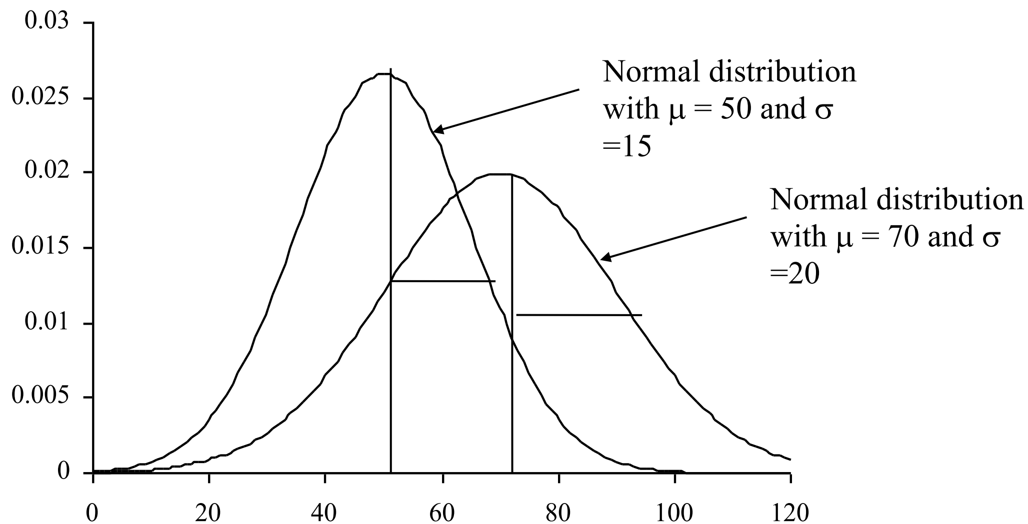
Example: Recall that we have a 60% chance of relaxed trade restrictions and, therefore, a 40% of maintaining them. If we expect the Cleveland Corp. stock to return 6% if trade restrictions are maintained and lose 11% if they are relaxed, what is the expected change in return for Cleveland Corp.?

Introduction to Multivariate Distribution

Background: Normal distribution

Given $X \sim N\left(\mu, \sigma^2\right)$: Then, PDF is characterized by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad , -\infty < x < \infty$$



Probability and Random Variable

Example Joint Cumulative Distribution Function:

Air Conditioner Maintenance

- A company that services air conditioner units in residences and office blocks is interested in how to schedule its technicians in the most efficient manner
- The random variable X, taking the values 1,2,3 and 4, is the **service time** in hours
- The random variable Y, taking the values 1,2 and 3, is **the number of air conditioner units**

Probability and Random Variable

Example Joint Distribution Function:

Air Conditioner Maintenance

Y= number of units	X=service time			
	1	2	3	4
1	0.12	0.08	0.07	0.05
2	0.08	0.15	0.21	0.13
3	0.01	0.01	0.02	0.07

- Joint p.m.f
- Joint cumulative distribution function

Probability and Random Variable

Marginal probability distribution

= Obtained by **summing or integrating the joint probability distribution** over the values of the other random variable

- Discrete $$P(X = i\ ) = p_{i+} = \sum_j p_{ij}$$

- Continuous

  Joint pdf: $f(x, y)$

  Marginal pdf's of X and Y: $$f_X(x) = \int f(x, y)dy$$ $$f_Y(y) = \int f(x, y)dx$$

Probability and Random Variable

Example Marginal probability distribution:

Air Conditioner Maintenance

Y= number of units	X=service time			
	1	2	3	4
1	0.12	0.08	0.07	0.05
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Marginal p.m.f of X = 1

Marginal p.m.f of Y = 1

Probability and Random Variable

Conditional Probability Distributions

- The conditional probability distribution is a probability distribution.
- The probabilistic properties of the random variable X **under the knowledge provided by the value of Y**

- Discrete: $$p_{i|j} = P(X = i \mid Y = j) = \frac{P(X = i, Y = j)}{P(Y = j)} = \frac{p_{ij}}{p_{+j}}$$

- Continuous: $$f_{X|Y=y}(x) = \frac{f(x, y)}{f_Y(y)}$$

Probability and Random Variable

Example Conditional Probability Distributions:

Air Conditioner Maintenance

Y= number of units	X=service time			
	1	2	3	4
1	0.12	0.08	0.07	0.05
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Conditional distribution of

X =1 given Y = 3

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Question?