THE SOCIAL DISCOUNT RATE

EE465/EE463 Project Evaluation

Semester 2/2014

Based on Chapter 10 of *Cost-Benefit Analysis: Concepts and Practice*, Boardman et al., 2011.

Topics

- Social discount rate: Introduction
- Does the choice of discount rate matter?
- The theory behind the appropriate SDR
- Alternatives in deriving SDR from market rates
- The shadow price of capital

Social Discount Rate (1)

- When evaluating government policies or projects, analysts must decide on the appropriate weights to apply to policy impacts that occur in different years.
- Given these weights, denoted by w_t, and estimates of the real annual net social benefits, NB_t, the estimated net present value (NPV) of a project is given by

$$NPV = \sum_{t=0}^{n} w_t NB_t$$

- Selection of the appropriate social discount rate (SDR) is equivalent to deciding on the appropriate *set of weights* for the above problem.
- These weights are sometimes referred to as *social discount factors*.

Social Discount Rate (2)

- Discounting reflects the idea that a given amount of real resources in the future is worth less today than the same amount is worth now because:

  1) Via investment, one can transform resources that are currently available into a greater amount in the future.

  2) People prefer to consume a given amount of resources now, rather than in the future.

- Thus, it is generally accepted that the social discount weights *decline over time*:

$$0 \le w_n \le w_{n-1} \le \ldots \le w_1 \le w_0 = 1.$$

Social Discount Rate (3)

- Three unresolved issues:
  1) Whether market interest rates can be used to determine the weights.
  2) Whether to include unborn future generations in determining the weights and, if so, what weight they should have.
  3) Whether society values a unit of investment the same as a unit of consumption.
- Different assumptions about these issues lead to different approaches towards determining the SDR, which, in turn, lead to different discount weights

Does the Choice of Discount rate Matter?

Example: Annual Net Benefits and NPV for 3 Alternative Projects

Year	Project A	Project B	Project C
0	−80,000	−80,000	−80,000
1	25,000	80,000	0
2	25,000	10,000	0
3	25,000	10,000	0
4	25,000	10,000	0
5	25,000	10,000	140,000
NPV ($i = 2\%$)	37,836	35,762	46,802
NPV ($i = 10\%$)	14,770	21,544	6,929

In general,

- Low SDRs favor projects with the highest total benefits,
- High SDRs favor projects where the benefits are front-end loaded.

THE THEORY BEHIND THE APPROPRIATE SOCIAL DISCOUNT RATE (SDR)

An Individual's Marginal Rate of Time Preference (MRTP)

- An individual's MRTP is the proportion of additional consumption that an individual requires in order to be willing to postpone (a small amount of) consumption for one year.

Example:

- Your rich grandma offers to give you either $1000 this year or $1200 next year.
- Suppose you are indifferent between the two options.

  ➔ Your MRTP is __________

Equality of Discount Rates in Perfect Markets

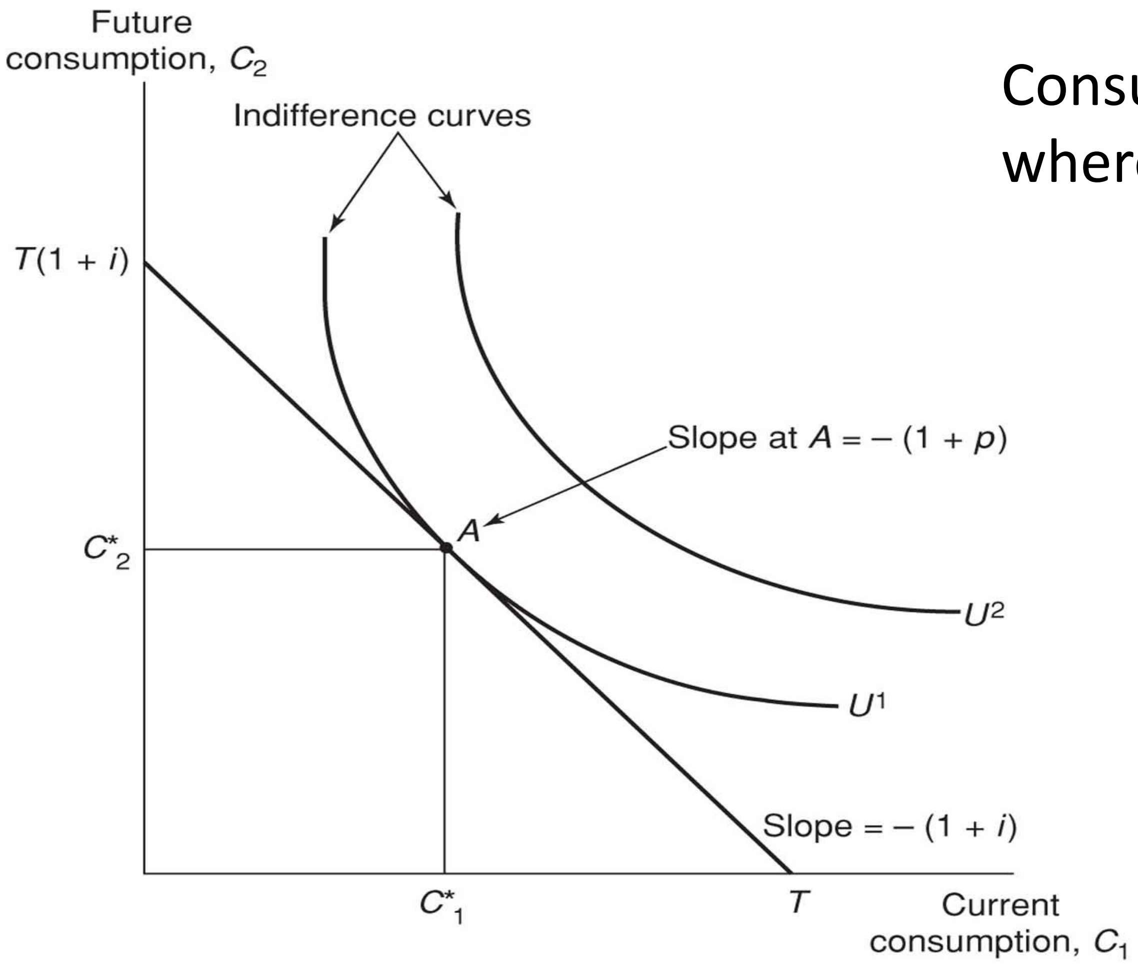
- In a perfectly competitive capital market, an individual's MRTP (p) equals the market interest rate (i).
- This result is obtained from solving consumer's U-max problem:

$$\max_{C_1,C_2} U(C_1, C_2)$$

$$\text{subject to } C_1 + \frac{C_2}{1+i} = T$$

➔ Rewrite BC: $C_2 =$ ________________

MRTP in a Two-Period Model



Consumption is maximized at A where: MRS = slope of BC

MRS = -(1+p) &
slope of BC = -(1+i).
➔

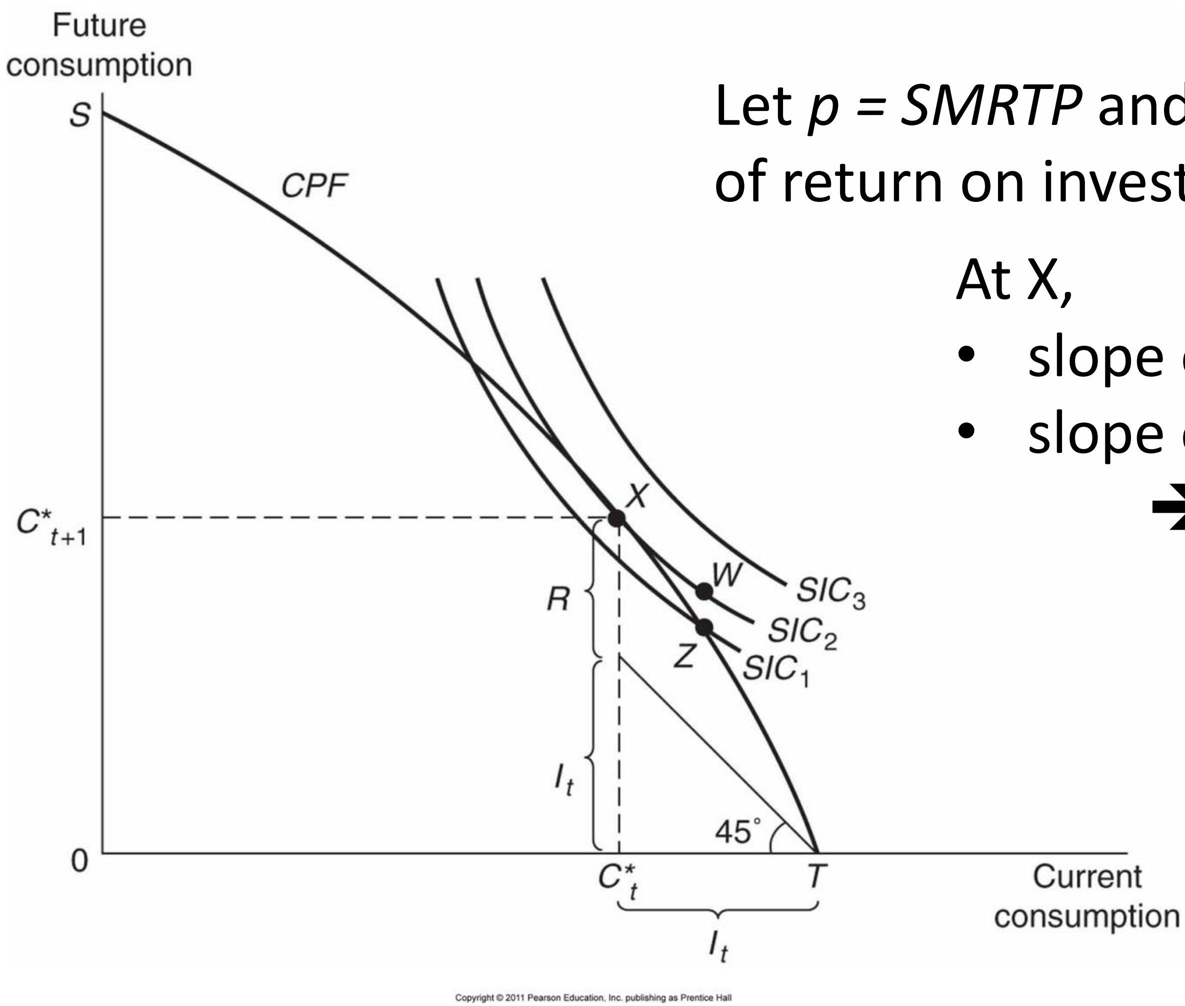
➔

Here, the market interest rate can be used as the social discount rate.

A Simple Two-Period Model with Production

- Consider a more general, multi-individual, two-period model with production of a hypothetical country.
- Assume: no tax, no transaction cost, no market failures, etc.
- Trade-off between current and future consumptions are represented by consumption possibility frontier (CPF), which is a concave curve.
- Society's preference is represented by social indifference curve (SIC), and its slope is the *social marginal rate of time preference (SMRTP)*.
- SMRTP is the extra amount of future consumption that society requires as a compensation for giving up one unit of current consumption.

The Optimal Levels of Consumption and Investment in a Two-Period Model



Let *p* = *SMRTP* and r_x be the marginal rate of return on investment, at point X.

At X,

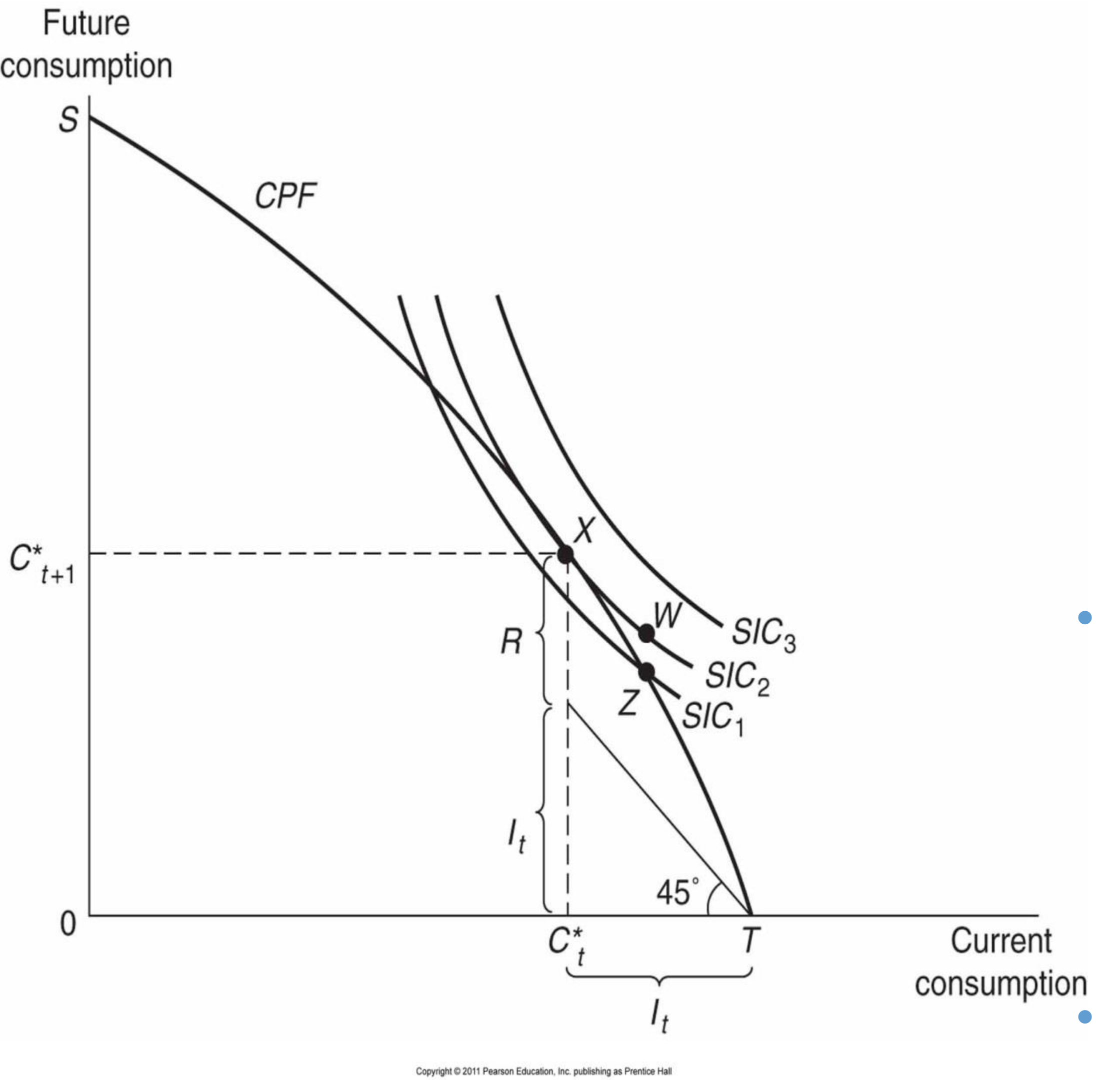
- slope of CPF = ______
- slope of SIC =______

➔

The First-Best Solution in the Two-Period Model

- From previous slide, society achieves its first-best solution when the SMRTP will equal the marginal rate of return on investment (i.e. $p_x = r_x$).
- These rates would also equal the economy-wide market interest rate, i (= r).
- This implies that all individuals will have the same MRTP because if their *MRTP > i* they would borrow at *i* and consume more in the current period until their *MRTP = i*; if *MRTP < i* they would invest until their *MRTP = i*.
- Since everyone's MRTP equals *i*, it should be the unanimous choice for the social discount rate.

Problems with Real Economies: The Second Best



- An actual economy (with taxes, risk and transaction costs) would not operate at the optimal point X, but at a point such as Z, which is a **second-best outcome**. Here, society would *underinvest* and $r_x > p_x$.
- Also, different people have different preference, face different tax rates, numerous values exist for both MRTPs and 1+r.
- No obvious choice for SDR.

DERIVING THE SDR FROM THE MARKET

Alternative SDRs

- Assume for now that the social discount weights are constant over time.
- Four potential social discounting rates that are derived from rates observable in markets.
  1. Using the marginal rate of return on private investment (r_z)
  2. Using the Marginal Social Rate of Time Preference (p_z) method
  3. Using the government's borrowing rate (i)
  4. Using the weighted average approach

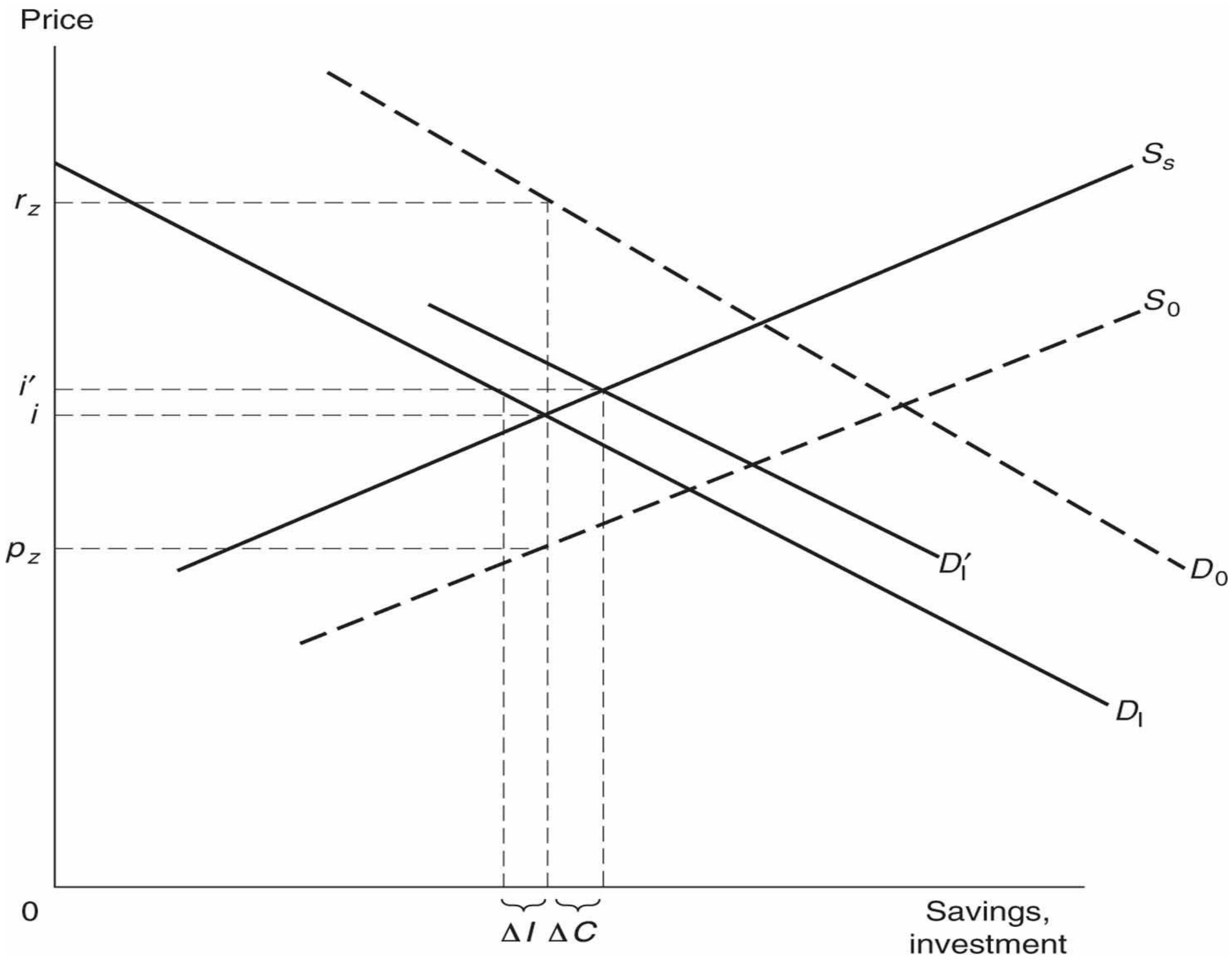
I. Marginal Rate of Return on Private Investment (r_z)

- Argument: Society should receive an equal rate of return in the public sector to what it would have received had the resources remained in the private sector.
- Arnold Harberger's paper:
  - Assume that a government project would be financed entirely by borrowing in a closed domestic financial market, which results in higher market rate of interest.
  - 2 effects:

    1. Private-sector investment falls (ΔI).

    2. Private savings increase ~ Private consumption falls (ΔC).
  - i.e. the project would "crowd-out" both investment and consumption

The Effects of Taxes and Government Borrowing on Savings and Investment



Marginal Rate of Return on Private Investment (cont'd)

- Harberger suggests that the social discount rate should be obtained by weighting r_z and p_z by the relative contributions that I and C would make toward funding the project.
- Hence, SDR should be computed using the weighted average cost of capital:

$$WSOC = ar_z + bp_z$$

where $a = \Delta I/(\Delta I + \Delta C)$ and b$= \Delta b/(\Delta I + \Delta C)$

- Moreover, Harberger claims that savings are not very responsive to changes in interest rates, and hence $\Delta C \approx 0$. Thus, r_z is a good approximation of the SDR.
- Numerical value of r_z– the best proxy of r_z is the real, before-tax rate of return on corporate bonds.

II. Marginal Social Rate of Time Preference (p_z)

- Argument: SDR should be thought of as the rate at which individuals in society are willing to postpone a small amount of current consumption in exchange for additional future consumption (and vice versa).
- In principle, p_z represents this rate.
- Numerical Values of p_z - In practice, the best return that most people can earn in exchange for postponing consumption is the real after-tax return on savings.
- Example: Suppose the nominal, pre-tax interest rate on government bonds and adjusting for taxes on savings and inflation yields estimates of p_z between 0.00 and 0.04.
    - This method suggests using a real SDR = ___ with sensitivity analysis at 0% and 4%.

III. Government's Borrowing Rate (i)

- Argument: the government should discount projects using its long-term borrowing rate, *i*, it reflects the actual cost of financing a project.
- <u>Numerical Values of i</u> – the real rate of return on 10-year Treasury bonds (Adjusted by inflation)
- Example:
- The average monthly yield on 10-year U.S. Treasury bonds for the period between April 1953 and December 2001 was 6.71% and CPI during that period is 3.89%.
  - A value for i of _____, with a plausible range for sensitivity analysis of 1.7% to 3.7%.

IV. Weighted Average Approach (WSOC)

- Argument: SDR should be calculated in terms of the social opportunity cost of the different sources weighted according to the relative contribution of each source.
- If *a* is the proportion of t_a project's resources that displaces private domestic investment, *b* is the proportion financed by borrowing from foreigners, and 1-*a*-*b* is the proportion that displaces domestic consumption, then SDR is the weighted social opportunity cost of capital (WSOC) :

$$WSOC = ar_z + bi + (1 - a - b)p_z$$

where i = the government's real, long-term borrowing rate.

Note that: As $p_z < i < r_z$, it follows that __________________.

SHADOW PRICE OF CAPITAL

Shadow Price of Capital (SPC)

Argument:

- Due to market distortions, the rate at which individuals are willing to trade present for future consumption, p_Z, differs from the rate of return on private investment, r_Z.
- Thus, flows of investment should be treated differently from flows of consumption; i.e. investment flows should be weighted by some parameters.
- The **shadow price of capital (SPC)**, denoted by θ, is a parameter that converts investment gains or losses into *consumption equivalents*.
- In SPC discounting method, these consumption equivalents, like consumption flows themselves, are then discounted at p_Z.

Shadow Price of Capital Method (1)

The shadow price of capital method requires that discounting be done in four steps:

1) Costs and benefits in each period are divided into those that affect consumption and those that affect investment.
2) Flows into and out of investment are multiplied by the SPC to convert them into consumption equivalents.
3) Changes in consumption are added to changes in consumption equivalents.
4) Resulting amounts are discounted at $p_{z.}$

Shadow Price of Capital Method (2)

- A general expression for the shadow price of capital is:

$$\theta = \frac{(r_z+\delta)(1-f)}{p_z-r_z f+\delta(1-f)}$$

where r_z is the net return on capital after depreciation,

δ is the depreciation rate of the capital invested,

f is the fraction of the gross return on capital reinvested,

and p_z is the marginal social rate of time preference.

- Note: In the absence of reinvestment and depreciation, SPC becomes: $\theta =$ ….

Example: Shadow Price of Capital (2)

Example: Let r_z=2% , p_z=1.5%. ➔ θ =______.

- Consider a project that is financed by a bond issue of $3 million.
- Assume the bond yields real interest rate of 4% per year through five equal annual installments of $673,854.
- The estimated benefits from the project are $700,000 per year for five years.
- Assume the entire amount borrowed would displace private-sector investment (Harberger's crowd-out assumption).

Example: Shadow Price of Capital (2)

Year	Principal	Annual Installment	Interests	Loan Repayment	Loan Balance
1	3,000,000	673,854	120,000	553,854	2,446,146
2	2,446,146	673,854	97,846	576,008	1,870,138
3	1,870,138	673,854	74,806	599,048	1,271,089
4	1,271,089	673,854	50,844	623,010	648,080
5	648,080	673,854	25,923	648,080	0

Assume also:

- the full amount of each loan repayment will be reinvested.
- changes in consumption in each period include the project benefits, loan repayment, and interest on the loan.

Example: Shadow Price of Capital (2)

ΔI_0	-3,000,000
ΔI_1	553,854
ΔI_2	576,008
ΔI_3	599,048
ΔI_4	623,010
ΔI_5	648,080

ΔC_0	=	0
ΔC_1	700,000 – 673,854 +120,000 =	146,146
ΔC_2	700,000 – 673,854 +97,846 =	123,992
ΔC_3	700,000 – 673,854 +74,806 =	100,952
ΔC_4	700,000 – 673,854 +50,844 =	76,146
ΔC_5	700,000 – 673,854 +25,923 =	52,146

- With SPC, NPV =
- Without SPC, NPV =