Quiz 2: Solution

1. Find $\dfrac{dy}{dx}$ for $\cos(x^k y) = mx + y$, when k and m are some fixed constants.

   Solution:

   Since y is defined in terms of x implicitly, we will use **implicit differentiation** to find $\dfrac{dy}{dx}$.

$$\begin{aligned}
\frac{d}{dx}\cos(x^k y) &= \frac{d}{dx}(mx + y) \\
-\sin(x^k y)\frac{d(x^k y)}{dx} &= m + \frac{dy}{dx} \\
-\sin(x^k y)\left[x^k \frac{dy}{dx} + y\frac{d}{dx}x^k\right] &= m + \frac{dy}{dx} \\
-\sin(x^k y)\left[x^k \frac{dy}{dx} + kx^{k-1} y\right] &= m + \frac{dy}{dx} \\
-x^k \sin(x^k y)\frac{dy}{dx} - kx^{k-1} y \sin(x^k y) &= m + \frac{dy}{dx} \\
\left[-x^k \sin(x^k y) - 1\right]\frac{dy}{dx} &= m + kx^{k-1} y \sin(x^k y) \\
\frac{dy}{dx} &= \frac{m + kx^{k-1} y \sin(x^k y)}{-x^k \sin(x^k y) - 1}
\end{aligned}$$

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