MA332 Linear algebra
Exercise I

1. Suppose $\underline{A}$ is 3 by 5, $\underline{B}$ is 5 by 3, $\underline{C}$ is 5 by 1, and $\underline{D}$ is 3 by 1. Which of these matrix operations are allowed, and what are the shape of the results?

$$\underline{BA},\ \underline{A}(\underline{B}+\underline{C}),\ \underline{ABD},\ \underline{AC}+\underline{BD},\ \underline{ABABD}$$

2. Compute $\underline{AB}$ where $\underline{A}=\begin{bmatrix}2 & 0 & -3\\ -1 & 6 & 5\end{bmatrix}$ and $\underline{B}=\begin{bmatrix}0 & 3\\ -4 & 7\\ 2 & -5\end{bmatrix}$. Show how you obtain the result using all three methods you study in the class.

3. Find all solutions to the following system of linear equations:

$$\begin{aligned} x+y-3z &= 3\\ -2x-y &= -4\\ 4x+2y+3z &= 7 \end{aligned}$$

4. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system

a) $\left[\begin{array}{cc|c}2 & 3 & h\\ 4 & 6 & 7\end{array}\right]$ b) $\left[\begin{array}{cc|c}1 & -3 & 2\\ 5 & h & -7\end{array}\right]$

5. For matrix

$$A=\begin{bmatrix}0 & 1 & 4 & 0\\ 0 & 2 & 8 & 0\end{bmatrix}$$

determine the echelon form U, the basic variables, the free variables and the general solution to $\underline{Ax}=0$. Then apply, elimination to $\underline{Ax}=\underline{b}$, with b_1 and b_2 on the right side; find the conditions for $\underline{Ax}=\underline{b}$ to be consistent and find the general solution. What is the rank of $\underline{A}$.

6. a) Find all solutions to

$$\underline{U}\,\underline{x}=\begin{bmatrix}1 & 2 & 3 & 4\\ 0 & 0 & 1 & 2\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

b) if the right side is changed from (0,0,0) to (a,b,0), what are the solutions?

7. a) Show that $\underline{A}=\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}$ has inverse $\underline{C}=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$

b) Show that $\underline{A}=\begin{bmatrix}2 & -3\\ 0 & 0\end{bmatrix}$ has no inverse.

8. Suppose

$$Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \text{has no solution}$$

but

$$Ax = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \text{ has infinitely many solutions.}$$

a) Find all possible information about r, m and n. (r is the rank and A is of the size $m \times n$)

b) Find an example of such a matrix A with r, m and n all as small as possible.

9. Find an LU-factorisation for $\underline{A} = \begin{bmatrix} 5 & -5 & 10 & 0 & 5 \\ -3 & 3 & 2 & 2 & 1 \\ -2 & 2 & 0 & -1 & 0 \\ 1 & -1 & 10 & 2 & 5 \end{bmatrix}$ and verify your result.

10. If $\underline{A} = \begin{bmatrix} 0 & 0 & -2 \\ 2 & 4 & 2 \\ 1 & -1 & 4 \end{bmatrix}$, find a permutation matrix P such that PA has an LU-factorisation and then find the factorisation.

10. a) Find the inverse of the matrix $\underline{A} = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 7 & 1 \\ 1 & 3 & 0 \end{bmatrix}$ using Gauss-Jordan method.

b) show that the matrix $\underline{A} = \begin{bmatrix} 1 & 3 & -2 \\ 1 & 2 & 0 \\ 2 & 8 & -8 \end{bmatrix}$ is not invertible.

11. a) Apply row elimination to A and find the pivots and the upper triangular U. Factor this matrix into L times U.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}$$

b) How do L and U and the pivots confirm that A is invertible?

c) If you change the entry "20" to what number (??) the A will become singular.

d) What permutation matrix P will multiply A so that the rows of PA are in reverse order (row 1,2,3,4 of A become rows 4,3,2,1 of PA)? What matrix multiplication would put the columns in reverse order?