EE435: Assignment 2

1. Reduce form model

$$\ln S_t = \beta_{10} + \beta_{11}\ln P_{0t} + \beta_{12}\ln P_{X2t} + \beta_{13}\ln P_{X3t} + \beta_{14}\ln P_{X4t} + \varepsilon_{1t} \quad (1)$$

$$\ln D_t = \beta_{20} + \beta_{21}\ln P_{0t} + \beta_{22}\ln GDP_t + \varepsilon_{2t} \quad (2)$$

Assume the market always clear:

$\ln S_t = \ln D_t$ — ③

① = ② ,

$$\beta_{10} + \beta_{11}\ln P_{0t} + \beta_{12}\ln P_{X2t} + \beta_{13}\ln P_{X3t} + \beta_{14}\ln P_{X4t} + \varepsilon_{1t} = \beta_{20} + \beta_{21}\ln P_{0t} + \beta_{22}\ln GDP_t + \varepsilon_{2t}$$

$$(\beta_{11} - \beta_{21})\ln P_{0t} = \beta_{20} + \beta_{22}\ln GDP_t + \varepsilon_{2t} - \beta_{10} - \beta_{12}\ln P_{X2t} - \beta_{13}\ln P_{X3t} - \beta_{14}\ln P_{X4t} - \varepsilon_{1t}$$

$$\ln P_{0t} = \frac{\beta_{20}}{\beta_{11}-\beta_{21}} + \frac{\beta_{22}}{\beta_{11}-\beta_{21}}\ln GDP_t + \frac{\varepsilon_{2t}}{\beta_{11}-\beta_{21}} - \frac{\beta_{10}}{\beta_{11}-\beta_{21}} - \frac{\beta_{12}}{\beta_{11}-\beta_{21}}\ln P_{X2t} - \frac{\beta_{13}}{\beta_{11}-\beta_{21}}\ln P_{X3t} - \frac{\beta_{14}}{\beta_{11}-\beta_{21}}\ln P_{X4t} - \frac{\varepsilon_{1t}}{\beta_{11}-\beta_{21}}$$ — ④

④ in ②,

$$\ln D_t = \beta_{20} + \beta_{21}\left(\frac{\beta_{20}}{\beta_{11}-\beta_{21}} + \frac{\beta_{22}}{\beta_{11}-\beta_{21}}\ln GDP_t + \frac{\varepsilon_{2t}}{\beta_{11}-\beta_{21}} - \frac{\beta_{10}}{\beta_{11}-\beta_{21}} - \frac{\beta_{12}}{\beta_{11}-\beta_{21}}\ln P_{X2t} - \frac{\beta_{13}}{\beta_{11}-\beta_{21}}\ln P_{X3t} - \frac{\beta_{14}}{\beta_{11}-\beta_{21}}\ln P_{X4t} - \frac{\varepsilon_{1t}}{\beta_{11}-\beta_{21}}\right) + \beta_{22}\ln GDP_t + \varepsilon_{2t}$$ — ⑤

Let:

$$\pi_{20} = \beta_{20} + \beta_{21}\left(\frac{\beta_{20}-\beta_{10}}{\beta_{11}-\beta_{21}}\right),\ \pi_{21} = \beta_{22} + \beta_{21}\left(\frac{\beta_{22}}{\beta_{11}-\beta_{21}}\right),\ \pi_{22} = \left(-\frac{\beta_{21}\beta_{12}}{\beta_{11}-\beta_{21}}\right),\ \pi_{23} = \left(-\frac{\beta_{21}\beta_{13}}{\beta_{11}-\beta_{21}}\right),\ \pi_{24} = \left(-\frac{\beta_{21}\beta_{14}}{\beta_{11}-\beta_{21}}\right),\ w_{2t} = \beta_{21}\left(\frac{\varepsilon_{2t}-\varepsilon_{1t}}{\beta_{11}-\beta_{21}}\right) + \varepsilon_{2t}$$

⑤ in ①,

$$\ln S_t = \beta_{10} + \beta_{11}\left(\frac{\beta_{20}}{\beta_{11}-\beta_{21}} + \frac{\beta_{22}}{\beta_{11}-\beta_{21}}\ln GDP_t + \frac{\varepsilon_{2t}}{\beta_{11}-\beta_{21}} - \frac{\beta_{10}}{\beta_{11}-\beta_{21}} - \frac{\beta_{12}}{\beta_{11}-\beta_{21}}\ln P_{X2t} - \frac{\beta_{13}}{\beta_{11}-\beta_{21}}\ln P_{X3t} - \frac{\beta_{14}}{\beta_{11}-\beta_{21}}\ln P_{X4t} - \frac{\varepsilon_{1t}}{\beta_{11}-\beta_{21}}\right) + \beta_{12}\ln P_{X2t} + \beta_{13}\ln P_{X3t} + \beta_{14}\ln P_{X4t} + \varepsilon_{1t}$$

Let:

$$\pi_{10} = \beta_{10} + \beta_{11}\left(\frac{\beta_{20}-\beta_{10}}{\beta_{11}-\beta_{21}}\right),\ \pi_{11} = \frac{\beta_{11}\beta_{22}}{\beta_{11}-\beta_{21}},\ \pi_{12} = -\frac{\beta_{11}\beta_{12}}{\beta_{11}-\beta_{12}} + \beta_{12},\ \pi_{13} = -\frac{\beta_{11}\beta_{13}}{\beta_{11}-\beta_{21}} + \beta_{13},\ \pi_{14} = -\frac{\beta_{11}\beta_{14}}{\beta_{11}-\beta_{21}} + \beta_{14},\ w_{1t} = \beta_{11}\left(\frac{\varepsilon_{2t}-\varepsilon_{1t}}{\beta_{11}-\beta_{21}}\right) + \varepsilon_{1t}$$ — ⑥

1. Reduce form model:

$$\ln S_t = \pi_{10} + \pi_{11}\ln GDP_t + \pi_{12}\ln P_{X2t} + \pi_{13}\ln P_{X3t} + \pi_{14}\ln P_{X4t} + w_{1t}$$

$$\ln P_t = \pi_{20} + \pi_{21}\ln GDP_t + \pi_{22}\ln P_{X2t} + \pi_{23}\ln P_{X3t} + \pi_{24}\ln P_{X4t} + w_{2t}$$

2.Estimate reduce form model

```
. g lnst=ln(st)

. g lndt=ln(dt)

. g pd=pm+t

. g lnpx2=ln(px2)

. g lnpx3=ln(px3)

. g lnpx4=ln(px4)

. g lngdp=ln(gdp)

. reg lnst lngdp lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
				F(4, 17)	=	37.32
Model	4.64569724	4	1.16142431	Prob > F	=	0.0000
Residual	.529104674	17	.031123804	R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lngdp	.3438812	.1913463	1.80	0.090	-.0598242	.7475865
lnpx2	-.4503744	.1515961	-2.97	0.009	-.7702142	-.1305347
lnpx3	-.9242052	.2783356	-3.32	0.004	-1.511442	-.3369685
lnpx4	-.3883793	.4222332	-0.92	0.371	-1.279214	.5024549
_cons	24.65741	5.309757	4.64	0.000	13.4548	35.86002

```
. predict lnsthat, xb

. reg lndt lngdp lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
				F(4, 17)	=	26.43
Model	3.4026552	4	.850663799	Prob > F	=	0.0000
Residual	.54721789	17	.032189288	R-squared	=	0.8615
				Adj R-squared	=	0.8289
Total	3.94987309	21	.188089195	Root MSE	=	.17941

lndt	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lngdp	.1265855	.194594	0.65	0.524	-.2839719	.5371429
lnpx2	-.4887365	.1541691	-3.17	0.006	-.8140049	-.1634682
lnpx3	-.7243134	.2830597	-2.56	0.020	-1.321517	-.1271097
lnpx4	-.577921	.4293997	-1.35	0.196	-1.483875	.3280333
_cons	27.18614	5.399879	5.03	0.000	15.79339	38.57889

```
g lnpd=ln(pd)

reg lnpd lngdp lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
				F(4, 17)	=	6.76
Model	.17707359	4	.044268398	Prob > F	=	0.0019
Residual	.111247189	17	.006543952	R-squared	=	0.6142
				Adj R-squared	=	0.5234
Total	.288320779	21	.013729561	Root MSE	=	.08089

lnpd	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lngdp	.1632779	.0877392	1.86	0.080	-.0218357	.3483914
lnpx2	.1318015	.0695123	1.90	0.075	-.0148567	.2784596
lnpx3	.0939842	.127627	0.74	0.472	-.1752851	.3632535
lnpx4	.4939641	.1936093	2.55	0.021	.0854842	.9024439
_cons	2.87652	2.434717	1.18	0.254	-2.260283	8.013322

```
predict lnpdhat, xb
```

3.Estimate structural form

```
. predict lndthat, xb

. reg lnst lnpdhat lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
				F(4, 17)	=	37.32
Model	4.64569773	4	1.16142443	Prob > F	=	0.0000
Residual	.529104183	17	.031123775	R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879	4.578612
lnpx2	-.727963	.1840856	-3.95	0.001	-1.11635	-.3395762
lnpx3	-1.122146	.2824139	-3.97	0.001	-1.717988	-.5263052
lnpx4	-1.428722	.4751381	-3.01	0.008	-2.431176	-.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274	36.63092

```
. reg lndt lnpdhat lngdp
```

Source	SS	df	MS	Number of obs	=	22
				F(2, 19)	=	44.99
Model	3.26129847	2	1.63064924	Prob > F	=	0.0000
Residual	.688574614	19	.036240769	R-squared	=	0.8257
				Adj R-squared	=	0.8073
Total	3.94987309	21	.188089195	Root MSE	=	.19037

lndt	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675	-1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194	.802766
_cons	35.93498	7.189835	5.00	0.000	20.88648	50.98347

4.OLS

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), ols

Multivariate regression
```

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnst						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344	.1766516
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

2SLS

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 2sls inst(lnpx2 lnpx3 lnpx4 lngdp)

Two-stage least-squares regression
```

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	10.67	0.0000
lndt	22	2	.1454858	0.8982	77.04	0.0000

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnst						
lnpd	2.10611	2.191774	0.96	0.343	-2.339013	6.551233
lnpx2	-.7279628	.3442892	-2.11	0.041	-1.426214	-.0297119
lnpx3	-1.122146	.528189	-2.12	0.041	-2.193363	-.0509293
lnpx4	-1.428722	.8886357	-1.61	0.117	-3.230959	.3735147
_cons	18.59914	15.98447	1.16	0.252	-13.81886	51.01715
lndt						
lnpd	-2.574157	.4354519	-5.91	0.000	-3.457295	-1.69102
lngdp	.5212921	.1027745	5.07	0.000	.3128558	.7297283
_cons	35.93499	5.494663	6.54	0.000	24.7913	47.07868

```
Endogenous variables:  lnst lnpd lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
```

3SLS

```
.  reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls inst(lnpx2 lnpx3 lnpx4 lngdp)

Three-stage least-squares regression
```

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.2963642	0.6266	57.47	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnst						
lnpd	2.171576	1.926095	1.13	0.260	-1.603501	5.946652
lnpx2	-.7990055	.2985983	-2.68	0.007	-1.384247	-.2137635
lnpx3	-1.329743	.4560002	-2.92	0.004	-2.223487	-.4359989
lnpx4	-1.171403	.775654	-1.51	0.131	-2.691657	.348851
_cons	17.84948	14.04122	1.27	0.204	-9.670808	45.36976
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

```
Endogenous variables:  lnst lnpd lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
```

I3SLS

```
. .  reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls ireg3 inst(lnpx2 lnpx3 lnpx4 lngdp)

Iteration 1:   tolerance =   .1059484
Iteration 2:   tolerance =  .04569793
Iteration 3:   tolerance =  .01846611
Iteration 4:   tolerance =  .00725496
Iteration 5:   tolerance =  .00281814
Iteration 6:   tolerance =  .00108981
Iteration 7:   tolerance =  .00042072
Iteration 8:   tolerance =  .00016231
Iteration 9:   tolerance =   .0000626
Iteration 10:  tolerance =  .00002414
Iteration 11:  tolerance =  9.310e-06
Iteration 12:  tolerance =  3.590e-06
Iteration 13:  tolerance =  1.384e-06
Iteration 14:  tolerance =  5.339e-07

Three-stage least-squares regression, iterated
```

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnst						
lnpd	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

```
Endogenous variables:  lnst lnpd lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
```

Concerning the asymptotic property, 3SLS is more asymptotically efficient than other models.

5. β_{11} is the price elasticity of quantity supply.

   β_{21} is the price elasticity of quantity demand.