Understanding Interest Rates

Chapter 4 of Mishkin

Debt Instruments

- A ***debt instrument*** is a tool an entity can utilize to raise capital.
- It is a documented, binding obligation that provides funds to an entity in return for a promise from the entity to repay a lender or investor in accordance with terms of a contract.
- Credit cards, credit lines, loans, and bonds can all be types of debt instruments.
- Different debt instruments have different streams of cash payments to the holder (known as ***cash flows***) with different timing.

Credit Market Instruments

1. ***Simple loan***: The loan that must be repaid at the ***maturity date***, along with an additional payment for the interest.
2. ***Fixed-payment loan***: The loan that must be repaid by a fixed amount in every period.
3. ***Coupon bond***: Pays the owner a fixed interest payment (or ***coupon***) every year until the maturity date, when the ***par value*** (or ***face value***) is repaid.
4. ***Discount bond***: Has discounted price and repays the par value at its maturity date (***zero-coupon bond***).

Measuring Interest Rates

- An ***interest rate*** is the cost of borrowing for the rental of funds.
- In economics, the ***yield to maturity*** is the most accurate measure of interest rates. To measure these interest rates, we need to know the concept of ***present value***.
- The concept of ***present value*** (or ***present discounted value***) is based on the commonsense notion that a dollar paid to you one year from now is less valuable to you than a dollar paid to you today.
- This is true because you can deposit a dollar today in a saving account that earns interest and have more than a dollar in one year.

Present Value

- Let's look at a simple loan example: Jane borrow you \$100 for one year, with \$10 interest payment.
- In this case, we can calculate the ***simple interest rate*** as:

$$i = \frac{\$10}{\$100} = 0.10 = 10\%$$

- At the end of the year, you would have \$110, which can be calculated from

$$\$100 \times (1 + 0.10) = \$110.$$

- If you then lend out this \$110 to Jane for another year, at the end of the second year you would have

$$\$110 \times (1 + 0.10) = \$121,$$

or, equivalently,

$$[\$100 \times (1 + 0.10)] \times (1 + 0.10)$$
$$= \$100 \times (1 + 0.10)^2$$
$$= \$121.$$

Present Value

- Continuing with the loan again, at the end of the third year you would have

$$\$121 \times (1 + 0.10)$$
$$= [\$100 \times (1 + 0.10)^2] \times (1 + 0.10)$$
$$= \$100 \times (1 + 0.10)^3$$
$$= \$133.1$$

- Generalizing, we can see that

$$PV \times (1 + i)^n = FV,$$

or we can write

$$PV = \frac{FV}{(1 + i)^n},$$

where

PV is the present value
FV is the future value
i is the interest rate
n is number of years.

Measuring Interest Rates

- For a ***simple loan***, we apply the concept of present value directly to calculate the yield-to-maturity:

$$PV \times (1 + i)^n = FV$$

$$(1 + i)^n = \frac{FV}{PV}$$

$$1 + i = \left[\frac{FV}{PV}\right]^{1/n}$$

$$i = \left[\frac{FV}{PV}\right]^{1/n} - 1$$

Measuring Interest Rates

- For a ***discounted bond***, we apply the concept of present value directly to calculate the yield-to-maturity:

$$i = \left[\frac{F}{P}\right]^{1/n} - 1,$$

where F = face value

P = bond price

n = years to maturity date.

Measuring Interest Rates

- For a ***fixed-payment loan***, we need a financial calculator or a computer program, e.g. Ms Excel, to calculate for yield-to-maturity.
- We want to find the interest rate *i* that solves:

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \cdots + \frac{FP}{(1+i)^n}$$

where LV = loan value

FP = fixed payment

n = years to maturity date.

Now, consider a coupon bond

Note:

- We have bonds in both primary and secondary markets.
- ***Coupon rate**** may not be the same as ***yield to maturity***, which may be different from ***market interest rate***.
- ***Face value**** may not be the same as ***bond price***.
- ***Maturity date**** for a bond is fixed, which makes ***time to maturity*** (The time remaining until a financial contract expires) varied as time pass by.

Measuring Interest Rates

- For a ***coupon bond***, we need a computer program to calculate yield-to-maturity.
- We want to find the interest rate *i* that solves:

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

where P = price of the bond

F = face value

C = yearly payment (Coupon)

$= coupon\ rate \times face\ value$

n = years to maturity date.

Relationship between bond price and interest rate

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Three Interesting Facts in Table 1

1. When bond is at par, yield equals coupon rate.
2. Price and yield are negatively related.
3. Yield greater than coupon rate when bond price is below par value.

Calculation

- Suppose the market price is at $1,200.
- We want to calculate *i* such that:

$$1{,}200 = \frac{100}{(1+i)} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + \frac{100}{(1+i)^4} + \frac{100}{(1+i)^5} + \frac{100}{(1+i)^6}$$

$$+ \frac{100}{(1+i)^7} + \frac{100}{(1+i)^8} + \frac{100}{(1+i)^9} + \frac{100}{(1+r)^{10}} + \frac{1{,}000}{(1+r)^{10}}$$

- It is challenging to do this with pencil and paper!

Calculation

- We know that if *i* goes up, the value of each term goes down.
- We also know that:

$$1{,}000 = \frac{100}{(1+0.1)} + \frac{100}{(1+0.1)^2} + \frac{100}{(1+0.1)^3} + \frac{100}{(1+0.1)^4} + \frac{100}{(1+0.1)^5} + \frac{100}{(1+0.1)^6} + \frac{100}{(1+0.1)^7} + \frac{100}{(1+0.1)^8} + \frac{100}{(1+0.1)^9} + \frac{1{,}100}{(1+0.1)^{10}}$$

- Then, we can conclude that *i* must be smaller than 10%.
- We can try the process trial and error.
- Or we can switch to some computer programs.

Some interesting facts

- With compound interest at 10 percent, you can double your saving in 7-8 years. Let’s try saving 100,000 baht for 15 years.
- We may need at least 3,000,000 baht before we retire?
- If we can find high (enough) return, finding 3,000,000 baht at the age of 60 is not that hard.

Interest Rate Vs Returns

- Recall that our interest rate is the ***yield-to-maturity***.
- If you do not hold an asset to its maturity date, ***rate of return*** on the asset may not be equal to the interest rate.

$$RETURN = i + g$$

where g is the capital gains (or losses).

Interest Rate Vs Returns (2)

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

Calculation

- Suppose the market interest rate is 20%.
- This means people can find alternative investment with 20% rate of return.
- Here, *i* must be 20% to induce people to buy this bond:

$$P = \frac{100}{(1+0.2)} + \frac{100}{(1+0.2)^2} + \frac{100}{(1+0.2)^3} + \frac{100}{(1+0.2)^4} + \frac{100}{(1+0.2)^5} + \frac{100}{(1+0.2)^6}$$

$$+ \frac{100}{(1+0.2)^7} + \frac{100}{(1+0.2)^8} + \frac{1{,}100}{(1+0.2)^9}$$

$$P = 596.90$$

Interest Rate Vs Returns

Key Findings from Table 2

1. Bond whose return = yield is bond that maturity = holding period
2. For bonds with maturity > holding period, $i\uparrow$ P$\downarrow$ implying capital loss
3. Longer is maturity, greater is % price change associated with interest rate change
4. Longer is maturity, more return changes with change in interest rate
5. Bond with high initial interest rate can still have negative return if $i\uparrow$

Interest Rate Vs Returns

Conclusion from Table 2 Analysis

1. Prices and returns more volatile for long-term bonds because have higher ***interest-rate risk***
2. No interest-rate risk for any bond whose maturity equals holding period

Nominal Vs Real Interest Rates

Real Interest Rate is the interest rate that is adjusted for expected change in price level:

$$i^r = i - \pi^e$$

where i^r is the real interest rate, and π^e is the expected inflation.

Nominal Vs Real Interest Rates

Real Interest Rate can be thought of as interest rate in term of goods and services (real term).

- It more accurately reflects true cost of borrowing.
- When real rate is low, greater incentives to borrow and less to lend