Exericse Solution: Solving Inequality (Part I)

1. Let a, b, and c be real numbers. Show that if $a \geq b$ and $c < 0$, then $a^2 + b^2 + 5a - 5b - 4c > 0$.

   Solution:

   (i) Since $a^2 \geq 0$, $b^2 \geq 0$, then $a^2 + b^2 \geq 0$.

   (ii) Since $a \geq b$, then $a - b \geq 0$ and $5(a-b) \geq 0$ or $5a - 5b \geq 0$.

   (iii) Since $c < 0$, then $-4c > 0$.

   From (i) and (ii),

   $$a^2 + b^2 + (5b - 5b) \geq 0$$

   and by adding $-4c$ both sides and use (iii), we have

   $$\begin{aligned} a^2 + b^2 + (5a - 5b) - 4c &\geq -4c \\ &> 0. \end{aligned}$$

   That is, $a^2 + b^2 + 5a - 5b - 4c > 0$. ■

2. Let x, y be real numbers. Suppose that $x < 0$ and $y > 3$. Determine whether each of the following inequalities is true or not. Explain your answer.

   (a) $3x - xy > 0$.

   (b) $\frac{3}{x} - \frac{y}{x} + x^2 + y^3 > 18$

   Solution:

   (a) $3x - xy > 0$ is true. First note that, since $y > 3$, then

   $$3 - y < 0 \tag{1}$$

   and by multiplying throughout the inequality with a negative number x ($x < 0$), the inequality sign is changed:

   $$x(3 - y) > x \cdot 0$$

   or

   $$3x - xy > 0.$$

   ■

   (b) $\frac{3}{x} - \frac{y}{x} + x^2 + y^3 > 18$ is true. Notice again that since $y > 3$, then $3 - y < 0$. Since $x < 0$, then $\frac{1}{x} < 0$ and by multiplying $\frac{1}{x}$ throughout the inequality, we have

   $$\frac{1}{x}(3 - y) > \frac{1}{x} \cdot 0 \qquad \text{or} \qquad \frac{3}{x} - \frac{y}{x} > 0.$$

Since $x < 0$, then $\boxed{x^2 > 0}$ and since $y > 3$, then $\boxed{y^3 > 27}$. That is,

$$\begin{aligned}
\frac{3}{x} - \frac{y}{x} &> 0 \\
\frac{3}{x} - \frac{y}{x} + x^2 &> x^2 \\
&> 0 \\
\frac{3}{x} - \frac{y}{x} + x^2 + y^3 &> y^3 \\
&> 27 \\
&> 18
\end{aligned}$$

That is, $\frac{3}{x} - \frac{y}{x} + x^2 + y^3 > 18$.

3. Find the solution set for each of following inequalities.

(a)

$$\frac{2x}{1-x} \geq \frac{1-x}{2x}$$

Solution:

$$\begin{aligned}
\frac{2x}{1-x} &\geq \frac{1-x}{2x} \\
\frac{2x}{1-x} - \frac{1-x}{2x} &\geq 0
\end{aligned}$$

Notice that

$$\frac{2x}{1-x} - \frac{1-x}{2x} = \frac{(2x)^2 - (1-x)^2}{2x(1-x)} = \frac{4x^2 - (1-2x+x^2)}{2x(1-x)} = \frac{3x^2+2x-1}{2x(1-x)} = \frac{(3x-1)(x+1)}{2x(1-x)}.$$

Therefore the solution set of the given inequality is the same as the one of

$$\frac{(3x-1)(x+1)}{2x(1-x)} \geq 0$$

which can be solved by looking at the intervals constructed from $x = -1, 0, \frac{1}{3}, 1$.

	$x \in (-\infty, -1)$	$x \in (-1, 0)$	$x \in (0, 1/3)$	$x \in (1/3, 1)$	$x \in (1, \infty)$
$(3x-1)$	$-$	$-$	$-$	$+$	$+$
$(x+1)$	$-$	$+$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$	$+$
$(1-x)$	$+$	$+$	$+$	$+$	$-$
$\frac{(3x-1)(x+1)}{2x(1-x)}$	$-$	$\boxed{+}$	$-$	$\boxed{+}$	$-$

To have non-zero denominator, we must have $x \neq 0$, $x \neq 1$. Since we have "greater than or equal" sign, then the solution set is

$$[-1, 0) \cup [1/3, 1).$$

■

(b)

$$\frac{16x^4 - 81}{6x^2 + x - 12} < 0$$

Solution: Notice that

$$16x^4 - 81 = (4x^2)^2 - 9^2 = (4x^2 - 9)(4x^2 + 9) = (2x - 3)(2x + 3)(4x^2 + 9)$$

and

$$6x^2 + x - 12 = (2x + 3)(3x - 4).$$

That is,

$$\frac{16x^4 - 81}{6x^2 + x - 12} = \frac{(2x - 3)(2x + 3)(4x^2 + 9)}{(2x + 3)(3x - 4)} = \frac{(2x - 3)(4x^2 + 9)}{(3x - 4)} < 0$$

with $(2x + 3) \neq 0$ or $x \neq -\frac{3}{2}$. Since $4x^2 + 9 > 0$ for all $x \in \mathbb{R}$, then we only need to look at the sign of $\frac{(2x-3)}{(3x-4)} < 0$. Consider the intervals divided by $x = 3/2$, $x = 4/3$.

	$x \in (-\infty, 3/2)$	$x \in (3/2, 4/3)$	$x \in (4/3, \infty)$
$(2x - 3)$	$-$	$+$	$+$
$(3x - 4)$	$-$	$-$	$+$
$\frac{(2x-3)}{(3x-4)}$	$+$	$\boxed{-}$	$+$

Therefore, the solution set is $(3/2, 4/3)$. ■

(c)

$$\frac{x^4 - 2x^2 - 8}{2x + 1} \geq 0$$

Solution: Notice that

$$x^4 - 2x^2 - 8 = (x^2 - 4)(x^2 + 2) = (x - 2)(x + 2)(x^2 + 2).$$

and $\frac{x^4-2x^2-8}{2x+1} \geq 0$ is equivalent to

$$\frac{(x - 2)(x + 2)(x^2 + 2)}{2x + 1} \geq 0.$$

Since $x^2 + 2 > 0$ for all $x \in \mathbb{R}$, we will consider $\frac{(x-2)(x+2)}{2x+1} \geq 0$. The rational function $\frac{(x-2)(x+2)}{2x+1}$ has zeros at $x = -2, 2$ and an undefined point at $x = -1/2$.

	$x \in (-\infty, -2)$	$x \in (-2, -1/2)$	$x \in (-1/2, 2)$	$x \in (2, \infty)$
$x + 2$	$-$	$+$	$+$	$+$
$2x + 1$	$-$	$-$	$+$	$+$
$x - 2$	$-$	$-$	$-$	$+$
$\frac{(x-2)(x+2)}{2x+1}$	$-$	$\boxed{+}$	$-$	$\boxed{+}$

To have non-zero denominator, we must have $x \neq -1/2$. Since we have "greater than or equal" sign, then the solution set is $[-2, -1/2) \cup [2, \infty)$. ■

(d)

$$\frac{x^4+x^3+2x^2+2x}{x-1} \le 0$$

Solution: Notice that

$$x^4+x^3+2x^2+2x = x^2(x^2+2)+x(x^2+2) = (x^2+x)(x^2+2) = x(x+1)(x^2+2).$$

That is, $\frac{x^4+x^3+2x^2+2x}{x-1} \le 0$ is equivalent to

$$\frac{x(x+1)(x^2+2)}{x-1} \le 0.$$

Note that $x^2+2>0$ for any real number x so we need to only consider $\frac{x(x+1)}{x-1} \le 0$. To obtain the subintervals, consider $x=-1,0,1$

	$x \in (-\infty,-1)$	$x \in (-1,0)$	$x \in (0,1)$	$x \in (1,\infty)$
$x+1$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
$\frac{x(x+1)}{x-1}$	$\boxed{-}$	$+$	$\boxed{-}$	$+$

To have non-zero denominator, we must have $x \ne 1$. Since we have "less than or equal" sign, then the solution set is $(-\infty,-1] \cup [0,1)$. ■

(e)

$$\left(\frac{x}{x-3}-2\right)\left(\frac{e^x}{\cos(x)+2}\right) \ge 0$$

Solution: Notice that $e^x>0$ for all real numbers x and

$$\begin{aligned} -1 &< \cos(x) < 1 \\ -1+2 &< \cos(x)+2 < 1+2 \\ 1 &< \cos(x)+2 < 3 \end{aligned}$$

That is, $\cos(x)+2>0$ and $\left(\frac{e^x}{\cos(x)+2}\right)>0$ for any real number x. So, the solution set of the given inequality can be found from solving $\left(\frac{x}{x-3}-2\right) \ge 0$. Note that

$$\frac{x}{x-3}-2 = \frac{x-2x+6}{x-3} = \frac{-x+6}{x-3}.$$

That is, we obtain the solution set by solving $\frac{-x+6}{x-3} \ge 0$ or $\frac{x-6}{x-3} \le 0$. Consider the intervals divided by $x=3$, $x=6$.

	$x \in (-\infty,3)$	$x \in (3,6)$	$x \in (6,\infty)$
$x-3$	$-$	$+$	$+$
$x-6$	$-$	$-$	$+$
$\frac{x-6}{x-3}$	$+$	$\boxed{-}$	$+$

To have non-zero denominator, we must have $x \ne 3$. Since we have "less than or equal" sign, then the solution set is $(3,6]$. ■

4. Let x and y be real numbers with $|x| < \frac{1}{2}$. Show that $|xy - x| < \frac{|y|+1}{2}$.

Solution: From $|x| < \frac{1}{2}$,

$$|xy - x| = |x(y-1)| = |x||y-1| < \frac{1}{2}|y-1| \leq \frac{1}{2}\left(|y|+1\right).$$

■

5. Let x be a real number with $|x| \leq 3$. Determine if the following inequality is true or not.

$$|x^2 - 4| \geq 5|x+2|$$

Explain your answer.

Solution: False. First note that, for $|x| \leq 3$. $|x-2| \leq |x| + 2 \leq 3 + 2 = 5$. That is,

$$|x-2| \leq 5.$$

From $x^2 - 4 = (x+2)(x-2)$ and from $|x-2| \leq 5$,

$$|x^2 - 4| = |(x-2)(x+2)| = |(x-2)||(x+2)| \leq 5|x+2|.$$

That is, $|x^2 - 4| \leq 5|x+2|$. Therefore, the given inequality is false. ■

6. Let x and y be real numbers with $x > 1$ and $y < -2$. Determine if the following inequality is true or not.

$$1 + \frac{|y|}{x} > \frac{4-y}{2|x|}$$

Explain your answer.

Solution: True. Since $x > 1$ and $y < -2$, we have $x > 0$ and $y < 0$. This implies $x = |x|$ and $y = -|y|$. I.e., the above inequality is equivalent to

$$1 - \frac{y}{x} > \frac{4-y}{2x}.$$

Since $x > 0$, we have

$$2x\left(1 - \frac{y}{x}\right) > 4 - y \quad \Leftrightarrow \quad 2x - 2y > 4 - y \quad \Leftrightarrow \quad 2x - 2 > 2 + y.$$

That is, the given inequality is true if and only if $\boxed{2x - 2 > 2 + y}$ is true. Notice that, since $x > 1$,

$$2x - 2 = 2(x-1) > 0$$

and since $y < -2$,

$$2 + y < 0.$$

That is, $2x - 2$ is positive and $2 + y$ is negative. Hence, it is true that

$$2x - 2 > 2 + y.$$

and therefore the given inequality is true. ■

7. Find the solution set for each of following inequalities.

(a) $1 < |x-1| \leq 6$

Solution: There are two possible ways to solve this problem.

Method I By definition,

$$|x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$$

We consider 2 cases for $|x-1|$: $x \geq 1$ and $x < 1$.

Case I: $x \in [1, \infty)$: For $x \geq 1$, $1 < |x-1| \leq 6$ becomes

$$\begin{aligned} 1 &< x-1 \leq 6 \\ 1+1 &< x \leq 6+1 \\ 2 &< x \leq 7 \end{aligned}$$

or $x \in (2, 7]$. That is, the solution set for this case is $[1, \infty) \cap (2, 7] = (2, 7]$.

Case II: $x \in (-\infty, 1)$: For $x < 1$, $1 < |x-1| \leq 6$ becomes

$$\begin{aligned} 1 &< -x+1 \leq 6 \\ 1-1 &< -x \leq 6-1 \\ 0 &> x \geq -5 \end{aligned}$$

or $x \in [-5, 0)$. That is, the solution set for this case is $(-\infty, 1) \cap [-5, 0) = [-5, 0)$.

From cases I and II, the solution set is $[-5, 0) \cup (2, 7]$. ∎

Method II We can consider the given statement $1 < |x-1| \leq 6$ as two inequalities: $1 < |x-1|$ and $|x-1| \leq 6$

(i) For $1 < |x-1|$, we have either $x-1 > 1$ or $x-1 < -1$.
That is, $x-1 > 1 \Rightarrow x > 2$ or $x-1 < -1 \Rightarrow x < 0$.
I.e. $x \in (-\infty, 0) \cup (2, \infty)$.

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(ii) For $|x-1| \leq 6$, we have $-6 \leq x-1 \leq 6$.
That is,

$$\begin{aligned} -6 &\leq x-1 \leq 6 \\ -6+1 &\leq x \leq 6+1 \\ -5 &\leq x \geq 7 \end{aligned}$$

or $x \in (-5, 7)$.

Since both (i) and (ii) have to be true, the solution set is

$$\{(-\infty,0)\cup(2,\infty)\}\cap(-5,7)=[-5,0)\cup(2,7].$$

■

(b)

$$\frac{(|x|+5)(x-1)}{|x-2|+x^2+x+1}\geq 0$$

Solution: First notice that, $x^2+x+1>0$ for all $x\in\mathbb{R}$ (because for $a=1,b=1,c=1$, $b^2-4ac=1-4=-3<0$ and $a=1>0$). Also notice that $|x-2|\geq 0$ for all $x\in\mathbb{R}$. That is, the denominator

$$|x-2|+x^2+x+1>0$$

for all $x\in\mathbb{R}$. Therefore, in order to have $\frac{(|x|+5)(x-1)}{|x-2|+x^2+x+1}\geq 0$, we only need to consider when $(|x|+5)(x-1)\geq 0$, which can be separated into 2 subcases by using $|x|=\begin{cases}-x, & x<0\\ x, & x\geq 0\end{cases}$.

Case I When $x<0$,

$$(|x|+5)(x-1)=(-x+5)(x-1)=-(x-5)(x-1).$$

So we find x such that $-(x-5)(x-1)\geq 0$ or $(x-5)(x-1)\leq 0$. By setting $(x-5)(x-1)=0$, we consider $x=5$ and $x=1$ to construct 3 subintervales.

	$x\in(-\infty,1)$	$x\in(1,5)$	$x\in(5,\infty)$
$x-1$	$-$	$+$	$+$
$x-5$	$-$	$-$	$+$
$(x-5)(x-1)$	$+$	$\boxed{-}$	$+$

So, $x\in[1,5]$ I.e. the solution set of $(|x|+5)(x-1)\geq 0$ for this case is $(-\infty,0)\cap[1,5]=\emptyset$.
Remark: It is also possible to argue that, since

$$x<0\Rightarrow x-1<0$$

and

$$|x|\geq 0\ \Rightarrow\ |x|+5>0\ \Rightarrow\ (|x|+5)>0$$

for any $x\in\mathbb{R}$, then the product $(|x|+5)(x-1)$ must always be negative and it is impossible to have $(|x|+5)(x-1)\geq 0$ and the solution set for this case is $\emptyset$.

Case II When $x\geq 0$,

$$(|x|+5)(x-1)=(x+5)(x-1)=(x+5)(x-1).$$

So we find x such that $(x+5)(x-1)\geq 0$. By setting $(x+5)(x-1)=0$, we consider $x=-5$ and $x=1$ to construct 3 subintervales.

	$x\in(-\infty,-5)$	$x\in(-5,1)$	$x\in(1,\infty)$
$x+5$	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
$(x+5)(x-1)$	$\boxed{+}$	$-$	$\boxed{+}$

That is, $x \in (-\infty, -5] \cup [1, \infty)$. I.e. the solution set of $(|x|+5)(x-1) \geq 0$ for this case is $[0, \infty) \cap \{(-\infty, -5] \cup [1, \infty)\} = [1, \infty)$.

From cases I and II, the solution set is $\emptyset \cup [1, \infty) = [1, \infty)$. ■

(c)

$$2x + \frac{1}{|x|} \geq 1$$

Solution: $[-\frac{1}{2}, 0) \cup (0, \infty)$. From $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$, we will consider 2 cases.

Case I: $x \in (-\infty, 0)$ When $x < 0$, $|x| = -x$ and the given inequality becomes

$$\begin{aligned} 2x - \frac{1}{x} &\geq 1 \\ 2x^2 - x - 1 &\geq 0 \\ (2x+1)(x-1) &\geq 0. \end{aligned}$$

By setting $(2x+1)(x-1) = 0$, we consider $x = -1/2$ and $x = 1$ to construct 3 subintervales.

	$x \in (-\infty, -1/2)$	$x \in (-1/2, 1)$	$x \in (1, \infty)$
$2x+1$	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
$(2x+1)(x-1)$	$\boxed{+}$	$-$	$\boxed{+}$

That is, $x \in (-\infty, -1/2] \cup [1, \infty)$. I.e. the solution set for this case is $(-\infty, 0) \cap \{(-\infty, -1/2] \cup [1, \infty)\} = [-1/2, 0)$.

Case II: $x \in [0, \infty)$ When $x \geq 0$, we first note that we must have $x \neq 0$ because x is the denominator. For $x > 0$, $|x| = x$ and the given inequality becomes

$$\begin{aligned} 2x + \frac{1}{x} &\geq 1 \\ 2x^2 - x + 1 &\geq 0. \end{aligned}$$

For $a = 2, b = 1, c = 1$, $b^2 - 4ac = 1 - 8 < 0$, so we cannot factor $2x^2 - x + 1$. Since $a > 0$, then $2x^2 - x + 1 > 0$ for all $x \in \mathbb{R}$. Recall $x \neq 0$. That is, the solution set for this case is $[0, \infty) \cap \mathbb{R} - \{0\} = (0, \infty)$.

From cases I and II, the solution set is $[-1/2, 0) \cup (0, \infty)$. ■

(d)

$$\frac{|2-x|}{x^3 + 3x^2 + 11x + 18} \leq 0$$

Solution: First notice that

$$x^3+3x^2+11x+18 = \underbrace{x^3 + 2x^2} + \underbrace{x^2 + 11x + 18} = x^2(x+2)+(x+9)(x+2) = (x^2+x+9)(x+2).$$

For $a = 1, b = 1, c = 9$, $b^2 - 4ac = 1 - 36 < 0$, so we cannot factor $x^2 + x + 9$. Since $a > 0$, then $x^2 + x + 9 > 0$ for all $x \in \mathbb{R}$. That is,

$$\frac{|2-x|}{x^3 + 3x^2 + 11x + 18} = \frac{|2-x|}{(x^2 + x + 9)(x + 2)}$$

and since $x^2 + x + 9 > 0$ for all $x \in \mathbb{R}$, the solution set of $\frac{|2-x|}{x^3+3x^2+11x+18} \leq 0$ is the same as the one for

$$\frac{|2-x|}{x+2} \leq 0.$$

Two approaches for solving this will be presented here (short and long approaches).

(i) **Short Approach** Since $|2 - x| \geq 0$ (by property of absolute value) for all $x \in \mathbb{R}$, then we can only have $\frac{|2-x|}{x+2} \leq 0$ when $x + 2 < 0$. That is, $x < -2$ or the solution set for x is $(-\infty, -2)$. ■

(i) **Long Approach** From $|2 - x| = \begin{cases} -(2-x), & 2 - x < 0 \Leftrightarrow x > 2 \\ 2 - x, & 2 - x \geq 0 \Leftrightarrow x \leq 2 \end{cases}$, we will consider 2 cases.

Case I: $x \in (2, \infty)$ We have $|2 - x| = x - 2$ and $\frac{|2-x|}{x+2} = \frac{x-2}{x+2}$ So we want to solve

$$\frac{x-2}{x+2} \leq 0$$

Consider the intervals divided by $x = -2$, $x = 2$.

	$x \in (-\infty, -2)$	$x \in (-2, 2)$	$x \in (2, \infty)$
$x + 2$	$-$	$+$	$+$
$x - 2$	$-$	$-$	$+$
$\frac{x+2}{x-2}$	$+$	$\boxed{-}$	$+$

To have non-zero denominator, we must have $x \neq -2$. Since we have "less than or equal" sign, then $x \in (-2, 2]$. Hence, the solution set is $(2, \infty) \cap (-2, 2] = \emptyset$.

Case II: $x \in (-\infty, 2]$ We have $|2 - x| = 2 - x$ So we want to solve

$$\frac{2-x}{x+2} \leq 0 \quad \text{or} \quad \frac{x-2}{x+2} \geq 0$$

Consider the intervals divided by $x = -2$, $x = 2$.

	$x \in (-\infty, -2)$	$x \in (-2, 2)$	$x \in (2, \infty)$
$x + 2$	$-$	$+$	$+$
$x - 2$	$-$	$-$	$+$
$\frac{x+2}{x-2}$	$\boxed{+}$	$-$	$\boxed{+}$

To have non-zero denominator, we must have $x \neq -2$. Since we have "less than or equal" sign, then $x \in (-\infty, -2) \cup (2, \infty)$. Hence, the solution set is $(-\infty, 2] \cap \{(-\infty, -2) \cup (2, \infty)\} = (-\infty, -2)$

From cases I and II, $\emptyset \cup (-\infty, -2) = (-\infty, -2)$ ■

(e)

$$\frac{x^2+1-|x-1|}{5-|x+3|} \le 0$$

Solution:

From $|x-1| = \begin{cases} -(x-1), & x-1<0 \;\Leftrightarrow x<1 \\ x-1, & x-1\ge 0 \;\Leftrightarrow x\ge 1 \end{cases}$, and $|x+3| = \begin{cases} -(x+3), & x+3<0 \;\Leftrightarrow x<-3 \\ x+3, & x+3\ge 0 \;\Leftrightarrow x\ge -3 \end{cases}$,

we will consider 3 cases:

Case I: $x \in (-\infty,-3)$, Case II:$x \in [-3,1)$, Case III: $x \in [1,\infty)$.

Case I: $x \in (-\infty,-3)$, $|x-1| = -(x-1)$, $|x+3| = -(x+3)$ and

$$\frac{x^2+1-|x-1|}{5-|x+3|} = \frac{x^2+1+x-1}{5+x+3} = \frac{x^2+x}{x+8} = \frac{x(x+1)}{x+8}.$$

So we will solve $\frac{x(x+1)}{x+8} \le 0$. To obtain the subintervals, consider $x=-8,-1,0$

	$x\in(-\infty,-8)$	$x\in(-8,-1)$	$x\in(-1,0)$	$x\in(0,\infty)$
$x+8$	$-$	$+$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$
x	$-$	$-$	$-$	$+$
$\frac{x(x+1)}{x+8}$	$\boxed{-}$	$+$	$\boxed{-}$	$+$

To have non-zero denominator, we must have $x \ne -8$. Since we have "less than or equal" sign, then the solution set is $(-\infty,-8)\cup[-1,0]$. I.e., the solution set is $(-\infty,-3)\cap\{(-\infty,-8)\cup[-1,0]\} = \boxed{(-\infty,-8)}$.

Case II: $x \in [-3,1)$, $|x-1| = -(x-1)$, $|x+3| = x+3$ and

$$\frac{x^2+1-|x-1|}{5-|x+3|} = \frac{x^2+1+x-1}{5-x-3} = \frac{x^2+x}{-x+2} = -\frac{x(x+1)}{x-2}.$$

So we will solve $-\frac{x(x+1)}{x-2} \le 0$ or $\frac{x(x+1)}{x-2} \ge 0$. To obtain the subintervals, consider $x=-1,0,2$

	$x\in(-\infty,-1)$	$x\in(-1,0)$	$x\in(-1,2)$	$x\in(2,\infty)$
$x+1$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
$\frac{x(x+1)}{x-2}$	$-$	$\boxed{+}$	$-$	$\boxed{+}$

To have non-zero denominator, we must have $x \ne 2$. Since we have "less than or equal" sign, then $x \in [-1,0]\cup(2,\infty)$. I.e., the solution set is
$[-3,1)\cap\{x\in[-1,0]\cup(2,\infty)\} = \boxed{[-1,0]}$.

Case III: $x \in [1,\infty)$, $|x-1| = x-1$, $|x+3| = x+3$ and

$$\frac{x^2+1-|x-1|}{5-|x+3|} = \frac{x^2+1-x+1}{5-x-3} = \frac{x^2-x+1}{-x+2} = -\frac{x^2-x+1}{x-2}.$$

So we want to solve

$$\frac{x^2-x+1}{x-2} \ge 0.$$

For $a = 1, b = -1, c = 1$, $b^2 - 4ac = 1 - 4 < 0$, so we cannot factor $x^2 - x + 1$. Since $a > 0$, then $x^2 - x + 1 > 0$ for all $x \in \mathbb{R}$. So, in order to have $\frac{x^2-x+1}{x-2} \geq 0$, we can consider instead $x - 2 > 0$ or $x \in (2, \infty)$. That is, the solution set for this case is $[1, \infty) \cap (2, \infty) = \boxed{(2, \infty)}$.

From cases I, II, III, the solution set is $(-\infty, -8) \cup [-1, 0] \cup (2, \infty)$. ■