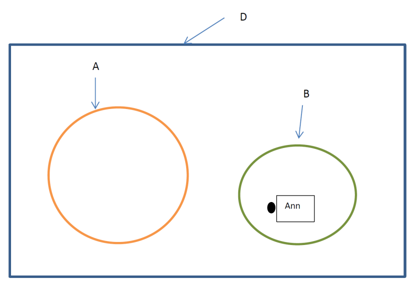
Solution: Quiz 2

1. Let the domain D be the set of students who took TU152. Suppose Ann is in the set D. Use the **diagram** to determine whether the following argument is valid or invalid. Explain your answer.
"Everyone who passed the class TU152 did not miss the final exam."
"Ann missed the final exam."
$\therefore$ *"Ann did not pass the class TU152."*

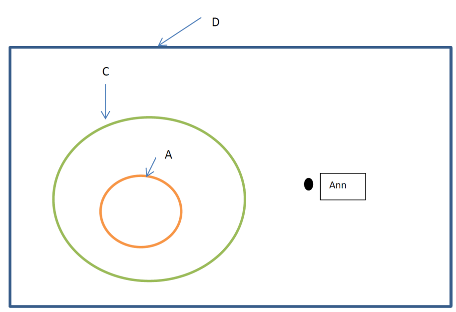
Solution: To use the diagram, let A be the set of students who passed the class TU152, and let B be the set of students who missed the final exam.
Then
- the first premise "Everyone who passed the class TU152 did not miss the final exam." implies " Passing the class TU152 implies not missing on the exam" or "$A \subseteq B^c$." or "$A \cap B = \emptyset$" where B^c is the compliment of set B.
- the second premise " Ann missed the final exam.." implies that Ann $\in B$,
- the conclusion implies that "Ann is in not the set A."

From the diagram, since we know that "Ann" is an element in B, which does not intersect with A , it is always true that "Ann' is not an element in A ." That is, the conclusion is always true. Therefore the argument is **valid**.



Remarks: The following is an alternative solution. let A be the set of students who passed the class TU152, and
let C be the set of students who **did not** miss the final exam.
Then, (i) the first premise implies "$A \subseteq C$." (ii) the second premise implies Ann $\notin C$, (iii) the conclusion implies "Ann is not in the set C."

From the diagram, since we know that "Ann" is outside the set C, which contains A , then "Ann" cannot be inside A and it is always true that "Ann" is not an element in A. That is, the conclusion is always true and the argument is **valid**.



2. Let $B = \left\{1, \frac{1}{2}, \frac{1}{4}\right\}$. Let the set of all positive real numbers $\mathbb{R}^+$ be the domain for the predicate $P(y)$ which is defined as

$$P(y): \ \sim \{\forall x \in B, \ 2xy \ > \ 1\}.$$

Find the truth set of the predicate $P(y)$.

Solution: First, we can simplify $P(y)$ by finding the negation of $\{\forall x \in B, \ 2xy \ > \ 1\}$:

$$P(y): \sim \{\forall x \in B, \ 2xy \ > \ 1\} \ = \ [\exists x \in B, \ 2xy \leq 1] \, .$$

To find the truth set T_p of $P(y)$, we have to consider all elements in $\mathbb{R}^+$ that make $P(y)$ true.

Consider each element of $B = \{1, \frac{1}{2}, \frac{1}{4}\}$.
Note that $y \in \mathbb{R}^+$.
(i) For $x = 1$, $2xy = 2(1)y \leq 1$ is true when $y \leq \frac{1}{2}$ or $y \in (-\infty, 1/2] \cap \mathbb{R}^+ = (0, 1/2]$.
(ii) For $x = \frac{1}{2}$, $2xy = 2(1/2)y \leq 1$ is true when $y \leq 1$ or $y \in (-\infty, 1] \cap \mathbb{R}^+ = (0, 1]$.
(iii) For $x = \frac{1}{4}$, $2xy = 2(1/4)y \leq 1$ is true when $y \leq 2$ or $y \in (-\infty, 2] \cap \mathbb{R}^+ = (0, 2]$.
Note that the truth set T_p of $P(y)$ is

$$T_p = \{y \in \mathbb{R}^+, P(y) \text{ is true.}\}$$

$P(y) = [\exists x \in B, \ 2xy \leq 1]$ is true when there is at least one $x \in B$ make $2xy \leq 1$ true which occurs when

$$y \in (0, 1/2] \cup (0, 1] \cup (0, 2] = (0, 2].$$

That is the truth set is

$$T_p(y) = (0, 2] = \{x \in \mathbb{R}, 0 < x \leq 2\}.$$

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