Assignment 1 EE320 (Section Aj. Kittichai)

Due on Oct., 27th 2020

Instruction

1) Question 0 is required for all groups.
2) Odd-numbered group must attempt all odd-numbered questions.
3) To submit your homework, write your filename as follow **hw1_Group_0x**. One point will be deducted if you don't follow the format of suggested filename.

Question 0: (required for all)

0.1) Given that $Z = \frac{x^3 - y^3}{x^2 y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

0.2) Given that $Z = \frac{x - y}{x + y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

0.3) If z = $2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

0.5) Given that $ln\,(x + y + z) + xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

Question 1: Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2.$$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

1.1) What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative

1.2) Is the product X considered an inferior product?

1.3) What is the level of quantity demanded if $P_x = 10, P_y = 25 \ and \ I = 10$?

1.4) Calculate the own-price elasticity of demand, and evaluate the value when $P_x = 10, P_y = 25 \ and \ I = 10$.

1.5) Calculate the cross-price elasticity of demand when$P_x = 10, P_y = 25 \ and \ I = 10$.

1.6) Calculate income elasticity of demand when $P_x = 10, P_y = 25 \ and \ I = 10$. Is the product a necessary or luxurious product?

Question 2 Given the production function $Q = f(K,L) = A[K^n + L^n]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

2.1) To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

2.2) Under the assumption used in (2.1), show that the production function satisfies the law of diminishing returns.

2.3) Calculate the marginal rate of technical substitution (MRTS) of labor (L) for capital (K).

2.4) how that MRTS is a decreasing function in L. That is, as labor increases, the value of MRTS decreases.

Suppose that $\mathrm{K(t)} = \frac{1}{2}t^2 + 2t + 3$ and $\mathrm{L(t)} = e^t + 3$, where $t \geq 0$ is the number of periods from now. Consider the following problem

2.5) Show that Q is increasing over time.

2.6) Compute $\frac{\frac{dQ}{Q}}{dt}$ when t = 0, i.e. growth of output in the initial period.

Question 3: Suppose that the preference set of a household can be given by

$$U(x,y) = x^{1/2} + y^{1/2},$$

where x is the amount of consumption on good-x, and y is the amount of consumption on good-y. Consider the following problems.

3.1) Calculate the marginal utility of good x and good y, respectively.

3.2) Does the utility function satisfy with the law of diminishing marginal utility?

3.3) Does the marginal utility curve of good x shift up when the consumer consumes more units of good-y?

3.4) What is the level of the household utility when the consumer consumes 1 unit of good-x and 2 units of good-y?

3.5) Following (3.4), use the total differential to calculate the change in the level of utility under which the consumer increases the consumption on good x by 3 units and reduces the consumption on good y by 1 unit.

3.6) Derive the MRS and show that MRS is decreasing in x.

Question 4: Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using L units of labor. If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$. Solve for L^* and calculate $\frac{\partial L^*}{\partial w}$ *and* $\frac{\partial L^*}{\partial P}$.

0.1) Given that $Z = \frac{x^3 - y^3}{x^2y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

$$\frac{\partial z}{\partial x} = \frac{x^2y^2(3x^2) - [(x^3-y^3)(2xy^2)]}{(x^2y^2)^2}$$

$$\frac{\partial z}{\partial y} = \frac{x^2y^2(-3y^2) - [(x^3-y^3)(2yx^2)]}{(x^2y^2)^2}$$

$$= x\left[\frac{3x^4y^2 - 2x^4y^2 + 2xy^5}{(x^2y^2)^2}\right] + y\left[\frac{-3x^2y^4 - 2yx^5 + 2x^2y^4}{(x^2y^2)^2}\right]$$

$$= \frac{3x^5y^2 - 2x^5y^2 + 2x^2y^5 - 3x^2y^5 - 2y^2x^5 + 2x^2y^5}{(x^2y^2)^2}$$

$$= \frac{x^2y^5 - x^5y^2}{(x^2y^2)^2} = \frac{(\cancel{x^2y^2})[y^3 - x^3]}{(x^2y^2)^{\cancel{2}}}$$

$$= \frac{y^3 - x^3}{x^2y^2} = -\left[\frac{x^3-y^3}{x^2y^2}\right] = -z$$

0.2) Given that $Z = \frac{x-y}{x+y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

total differential

$$dz = \frac{\partial z}{\partial x}.dx + \frac{\partial z}{\partial y}.dy$$

$$= \frac{x+y(1) - (x-y)(1)}{(x+y)^2}dx + \frac{(x+y)(-1) - (x-y)(1)}{(x+y)^2}dy$$

$$dz = \frac{2y}{(x+y)^2}dx + \frac{-2x}{(x+y)^2}dy$$

when $x=1, y=1$

$$dz = \frac{1}{2}dx - \frac{1}{2}dy$$

If $x, y > 0$

$$\frac{\partial z}{\partial x} = \frac{2y}{(x+y)^2} > 0 \quad \therefore x\uparrow \to z\uparrow$$

$$\frac{\partial z}{\partial y} = \frac{-2x}{(x+y)^2} < 0 \quad \therefore y\uparrow \to z\downarrow$$

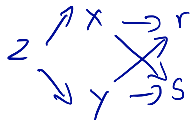
If $dx = 2$, $dy = -2$

$$dz = \frac{1}{\cancel{2}}.\cancel{2}^1 - \frac{1}{\cancel{2}}.(-\cancel{2}^1)$$

$$dz = 1 + 1 = 2$$

$$dz = 2$$

0.3) If $z = 2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0



a) $\frac{\partial z}{\partial s} = \left(\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s}\right) + \left(\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}\right)$

$= (4xy+3y)(2r) + (2x^2+3x+2y)(-4)$

$\frac{\partial z}{\partial r} = \left(\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r}\right) + \left(\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}\right)$

$= (4xy+3y)(2r+2s) + (2x^2+3x+2y)(2)$

b) $r = 1 \quad s = 0$

$x = (1)^2 + 2(1)(0) = 1 \qquad y = 2(1) - 4(0) = 2$

$\frac{\partial z}{\partial s} = (8+6)(2) + (2+3+4)(-4)$

$= 28 - 36$

$= -8$ when increase s 1 unit, z will decrease 8 units.

$\frac{\partial z}{\partial r} = (8+6)(2+0) + (2+3+4)(2)$

$= 28 + 18$

$= 46$ when increase r 1 unit, z will increase 46 units.

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

$2x^2 + 3y^2 + 2z^2 = 16$

$z^2 = \frac{16}{2} - \frac{2x^2}{2} - \frac{3y^2}{2}$

$z = \pm\sqrt{8 - x^2 - \frac{3}{2}y^2}$

$\frac{\partial z}{\partial y} = \frac{1}{2}\left(8 - x^2 - \frac{3}{2}y^2\right)^{-\frac{1}{2}}(-3y)$

$= \frac{1}{2}\left(\frac{1}{\pm\sqrt{8 - x^2 - \frac{3}{2}y^2}}\right)(-3y)$

$= \frac{1}{2}\cdot\frac{1}{z}\cdot(-3y)$

$= -\frac{3y}{2z}$

$\left.\frac{\partial z}{\partial y}\right|_{x=1,\, y=2,\, z=-1} = -\frac{3(2)}{2(-1)} = 3$

when y increase 1 unit, z will increase 3 unit

0.5) Given that $ln\ (x + y + z) + xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

$$F(x,z) = 0 = \ln(x+y+z) + xyz - ze^{x+y+z}$$

$$= -\frac{F_x}{F_z} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{\frac{1}{x+y+z} + yz - ze^{x+y+z}}{\frac{1}{x+y+z} + xy - (ze^{x+y+z} + e^{x+y+z})}$$

$$= -\frac{\frac{1}{1} + 0 - 0}{\frac{1}{1} + 0 - e^1}$$

$$= \frac{1}{e}$$

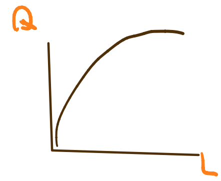
Question 2 Given the production function $Q = f(K,L) = A[K^{n} + L^{n}]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

2.1) To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

decreasing return to scale

$= \alpha + \beta < 1$

2.2) Under the assumption used in (2.1), show that the production function satisfies the law of diminishing returns.



Law of diminishing → concave function

F.O.C $MPL = \frac{\partial Q}{\partial L} = nAL^{n-1} > 0$ increasing function

S.O.C $= \frac{\partial^2 Q}{\partial^2 L} = n-1(nAL^{n-2})$

$= n^2 A L^{n-2} - nAL^{n-2}$

$n^2 < n$ due to $n < 1$ and that makes less than. So, $n^2AL^{n-2} - nAL^{n-2}$ will be less than 0

∴ Production function is concave function

↓

law of diminishing return

2.3) Calculate the marginal rate of technical substitution (MRTS) of labor (L) for capital (K).

$$MRTS_{L,K} = \frac{MPL}{MPK} \qquad MRTS_{K,L} = \frac{MPK}{MPL}$$

$$MPL = \frac{\partial Q}{\partial L} = nAL^{n-1} \qquad MPK = \frac{\partial Q}{\partial K} = nAK^{n-1}$$

$$MRTS_{L,K} = \frac{\cancel{nA}L^{n-1}}{\cancel{nA}K^{n-1}} = \frac{L^{n-1}}{K^{n-1}}$$

$$MRTS_{K,L} = \frac{\cancel{nA}K^{n-1}}{\cancel{nA}L^{n-1}} = \frac{K^{n-1}}{L^{n-1}}$$

2.4) how that MRTS is a decreasing function in L. That is, as labor increases, the value of MRTS decreases.

MRTS decreasing function

from 2.3) that $MRTS_{L,K} = \frac{L^{n-1}}{K^{n-1}}$

and from 2.1) If it is decreasing return to scale

$\therefore n<1$

That is we can rewrite $MRTS_{L,K}$ to be $\frac{K^{n-1}}{L^{n-1}}$

because $n-1$ is negative $= \frac{K^{-(n+1)}}{L^{-(n+1)}}$

Therefore, as labor increase L^{n-1} will increase too and that make value of $MRTS_{L,K}$ decreases.

Suppose that $\mathrm{K(t)} = \frac{1}{2}t^2 + 2t + 3$ and $\mathrm{L(t)} = e^t + 3$, where $t \geq 0$ is the number of periods from now. Consider the following problem

Question 2 Given the production function $Q = f(K, L) = A[K^n + L^n]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

2.5) Show that Q is increasing over time.

$$Q = A\left[\left(\tfrac{1}{2}t^2+2t+3\right)^n + \left(e^t+3\right)^n\right]$$

$$= A\left(\tfrac{1}{2}t^2+2t+3\right)^n + A\left(e^t+3\right)^n$$

$$\frac{\partial Q}{\partial t} = nA\left(\tfrac{1}{2}t^2+2t+3\right)^{n-1}(t+2) + nA\left(e^t+3\right)^n\left(e^t\right)$$

According to $0<n<1$, $A>0$, $t \geq 0$

So, all variables are greater or equal to zero.

$$\therefore nA\left(\tfrac{1}{2}t^2+2t+3\right)^{n-1}(t+2) + nA\left(e^t+3\right)^n\left(e^t\right) > 0$$

$\Rightarrow$ increasing function.

2.6) Compute $\frac{\frac{dQ}{Q}}{dt}$ when t = 0, i.e. growth of output in the initial period.

$$\frac{dQ}{dt} = nA\left(\tfrac{1}{2}t^2+2t+3\right)^{n-1}(t+2) + nA\left(e^t+3\right)^n\left(e^t\right)$$

when $t=0$

$$\Rightarrow nA\left(\tfrac{1}{2}(0)^2+0+3\right)^{n-1}(0+2) + nA\left(e^0+3\right)^n\left(e^0\right)$$

$$= nA(3)^{n-1}(2) + nA(4)^n(1)$$

$$= 2nA(3)^{n-1} + nA(4^n)$$

Question 4: Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using L units of labor. If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$. Solve for L^* and calculate $\frac{\partial L^*}{\partial w}$ *and* $\frac{\partial L^*}{\partial P}$.

$Q = L^{1/2}$

$Q^2 = L$

$\Pi = TR - TC$

$= (Q \cdot P) - w(Q^2)$

$= L^{\frac{1}{2}}P - wL$

$\frac{d\Pi}{dL} = \frac{1}{2}PL^{-\frac{1}{2}} - w = 0$

$w = \frac{1}{2}PL^{-\frac{1}{2}}$

$2w = PL^{-\frac{1}{2}}$

$L^{-\frac{1}{2}} = \frac{2w}{P}$

$L^* = \left(\frac{P}{2w}\right)^2 = \frac{P^2}{4w^2} = \frac{1}{4}P^2w^{-2}$

$\frac{\partial L^*}{\partial w} = -\frac{1}{2}P^2w^{-3}$

$\frac{\partial L^*}{\partial P} = \frac{1}{2}Pw^{-2}$