Oligopoly
Friday, April 5, 2019 11:13 AM

Now, we can apply our knowledge on game theory to study about a market structure so called "oligopoly"

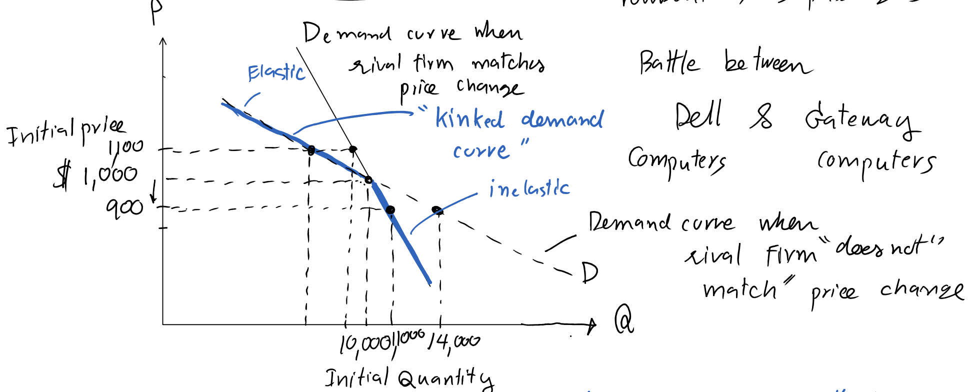
Oligopoly ⇒ market w/ a few firms that dominate the market

⇒ a few giant firms together w/ many small firms
↓
can be considered as "oligopoly" as well.

⇒ key features — ① Mutual Interdependence
② Repeated interaction

Mutual Interdependence : Action chosen by ① affects ②'s outcomes (Ex: output, price, profits), vice versa.

Battle between Dell & Gateway
Computers computers



This diagram shows "mutual interdependence" between Dell and Gateway.

Next, we will begin w/ 3 models of oligopoly competition:

Ⅰ Cournot Model of quantity competition (1838)

$\boxed{q_1, q_2} \rightarrow (\pi_1, \pi_2)$

- simultaneous move game
  output (q_i) is firm's choice variable.

Ⅱ Stackelberg Model of quantity competition (1934)

$q_1 \rightarrow q_2 \rightarrow (\pi_1, \pi_2)$

- sequential move game
  output (q_i) is firm's choice variable.

Ⅲ Bertrand Model of price competition (1883)

$\boxed{P_1, P_2} \rightarrow \boxed{\pi_1, \pi_2}$

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \longrightarrow \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}$$

simultaneous move game

p_i is firm's choice variable

Note: we study case of homogeneous products first.

(I) Cournot (1838)

- 2 players: 1 and 2
- Each chooses its own output, q_i simultaneously $(i = 1, 2)$
- Profits will be revealed after that.

Ex: consider a demand function $\boxed{P = 30 - Q}$

where $Q = q_1 + q_2$

$Q \downarrow$ Total output

For simplicity, assume that $MC = 0$.

Consider Firm 1: It attempts to maximize its profit by choosing output, q_1.

- $\pi = TR - TC$
- $TR_1 = p \cdot q_1$

$$= (30 - Q) q_1$$

$$= (30 - q_1 - q_2) q_1$$

$$TR_1 = 30 q_1 - q_1^2 - q_1 q_2 \quad \text{— (1)}$$

$$MR_1 = \frac{dTR_1}{dq_1} = 30 - 2q_1 - q_2 \quad \text{— (2)}$$

$$MC_1 = 0 \quad \text{— (3)}$$

π_1 is maximized at q_1 where $MR_1 = MC_1$:

$$30 - 2q_1 - q_2 = 0$$

$$2q_1 = 30 - q_2$$

$$q_1 = \frac{30 - q_2}{2}$$

$$\boxed{q_1^* = 15 - \frac{1}{2} q_2}$$

	$MR_1 = 30 - 2q_1 - q_2$	q_1^*
$q_2 = 0$	$MR_1 = 30 - 2q_1$	15
$q_2 = 5$	$MR_1 = 25 - 2q_1$	12.5
$q_2 = 10$	$MR_1 = 20 - 2q_1$	10
$q_2 = 15$	$MR_1 = 15 - 2q_1$	7.5
$q_2 = 30$	$MR_1 = -2q_1$	0

↳ Firm 1's Reaction Function

or

Firm 1's Best Response Function

OR

in short, BR_1 :

$$q_1 = BR(q_2)$$

Consider Firm 2 :
$$TR_2 = p \cdot q_2$$
$$= (30 - Q)\, q_2$$
$$= (30 - q_1 - q_2)\, q_2$$
$$TR_2 = 30q_2 - q_1 q_2 - q_2^2$$

Π_2 is maximized at q_2 where $MR = MC$:
$$30 - q_1 - 2q_2 = 0$$
$$2q_2 = 30 - q_1$$
$$q_2 = \frac{30 - q_1}{2}$$
$$\boxed{q_2^* = 15 - \frac{1}{2} q_1}$$

Firm 2's Reaction Function

or Firm 2's Best Response Function or BR_2

In equilibrium,
$$\boxed{\begin{aligned} q_1^* &= 15 - \tfrac{1}{2} q_2 \quad (1) \\ q_2^* &= 15 - \tfrac{1}{2} q_1 \quad (2) \end{aligned}}$$

For Firm 1, $q_1^* = 15 - \frac{1}{2}\left[15 - \frac{1}{2} q_1^*\right]$

For Firm1

$$q_1^* = 15 - \frac{1}{2}\left[15 - \frac{1}{2}q_1^*\right]$$

$$q_1^* = 15 - \frac{15}{2} - \frac{1}{4}q_1^*$$

$$\frac{3}{4}q_1^* = \frac{15}{2}$$

$$q_1^* = \frac{\cancel{15}^{5}}{\cancel{2}_1} \times \frac{\cancel{4}^{2}}{\cancel{3}_1} = 10 \text{ units}$$

For firm2 $\rightarrow$ by the same process, $q_2^* = 10$ units.

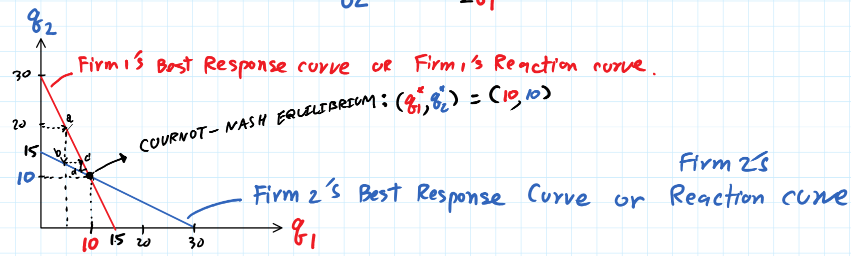
$\boxed{\text{So in Cournot-Nash Equilibrium } (q_1^*, q_2^*) = (10, 10).}$

(No one has incentive to alter its output in equilibrium.

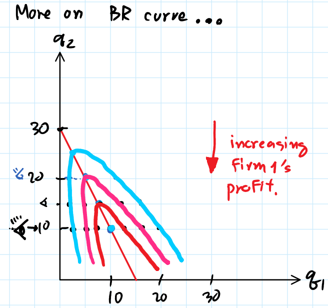
Graphically...

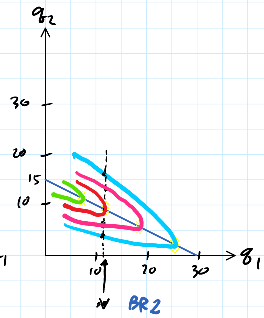
Firm 1's BR Function : $q_1 = 15 - \frac{1}{2} q_2$

Firm 2's BR Function : $q_2 = 15 - \frac{1}{2} q_1$

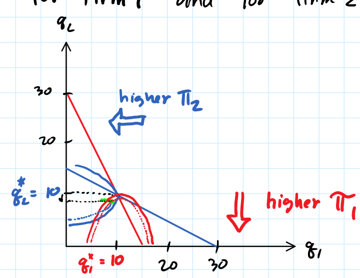


More on BR curve...





- we introduce "Iso-profit curves" for Firm 1 and for Firm 2.



- Firms have clear incentive to "collude" by making an agreement to produce **less than** the cournot quantity (10, 10). (In picture, both will jump to a higher Iso profit curve!)

\# If both firms form "a cartel", what would happen?

Consider $P = 30 - Q$.

$$TR = P \cdot Q = (30 - Q)Q = 30Q - Q^2$$

$$MR = \frac{dTR}{dQ} = 30 - 2Q.$$

$$MC = 0 \quad (\text{assumed})$$

- Set Q where MR = MC :

$$30 - 2Q = 0$$

$$\boxed{Q^* = 15} \quad \boxed{\begin{aligned} q_1^* &= \tfrac{15}{2} = 7.5 \\ q_2^* &= \tfrac{15}{2} = 7.5 \end{aligned}}$$

JOINT OPTIMAL OUTPUT A CARTEL SHOULD PRODUCE

- Find P :

$$\boxed{P^* = 30 - Q = 30 - 15 = 15}$$

- Find Joint Profit of the cartel

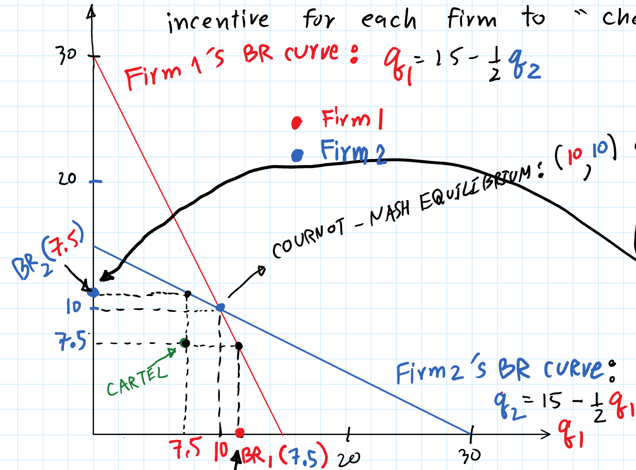
$$\boxed{\pi = P \cdot Q = 15 \cdot 15 = 225} \quad \boxed{\begin{aligned} \pi_1^* &= \tfrac{225}{2} = 112.5 \\ \pi_2^* &= \tfrac{225}{2} = 112.5 \end{aligned}}$$

$$\boxed{\pi = P \cdot Q = 15 \cdot 15 = 225}$$

$$\pi_1^* = \frac{225}{2} = 112.5$$

$$\pi_2^* = \frac{225}{2} = 112.5$$

Next concern is that, to ensure the above outcome, both must not "cheat". Unfortunately, there is an incentive for each firm to "cheat" or "being dishonest" (produce more than agreed amount of output)



Cartel's output by each firm is (7.5, 7.5)

If Firm 1 keeps promise to produce at $q_1 = 7.5$, Firm could earn higher profit if it produces more than the agreed output of 7.5.

If Firm 2 keeps promise to produce at $q_2 = 7.5$, firm 1 could earn higher profit if it produces more than the agreed output of 7.5.

q_1 The same argument holds for Firm 1.

Therefore, cartel is not stable.

However, keep in mind that this analysis is for one-shot game. In reality, both firms meet at several time. So, mechanism to punish a cheater will be introduced.

II Stackelberg model of quantity competition (1934)

$$\boxed{q_1} \longrightarrow \boxed{q_2} \longrightarrow \boxed{\begin{matrix}\pi_1 \\ \pi_2\end{matrix}}$$

- 2 players (2 firms) : 1 vs. 2
- Homogeneous products
- Timing of events $\Rightarrow$
  1. Firm 1 chooses q_1
  2. q_1 will be observed by firm 2 and then firm 2 chooses its q_2.
  3. Payoffs are realized.

Q: What is the best response that the follower (Firm 2) can make to the choice of q_1 that firm 2 has observed

A: Firm 2 would choose $q_2 = BR_2(q_1)$

Firm 1 perfectly knows the above reaction of firm 2, i.e., firm 2's reaction to any q_1 chosen by firm 1.

Then, as a result, Firm 1's (leader) profit function is:

$$\pi_1^s(q_1) = P\big(\underbrace{q_1 + BR_2(q_1)}_{\text{Total output}}\big) \cdot q_1 - c_1(q_1).$$

Q: Will the leader make profit at least as large as its Cournot-Nash equilibrium profit?

A: Yes! Let's see...

Ex: Recall that BR_1: $q_1^* = 15 - \frac{1}{2}q_2$

BR_2: $q_2^* = 15 - \frac{1}{2}q_1$

STEP 1: Find firm 1's optimal output, q_1^*

Recall that $\boxed{P = 30 - Q}$, $q_1 + q_2$

$$TR_1 = (30 - q_1 - q_2)\cdot q_1$$
$$= 30q_1 - q_1^2 - q_1 q_2$$
$$= 30q_1 - q_1^2 - q_1\left(15 - \tfrac{1}{2}q_1\right)$$

$$MR_1 = \frac{dTR_1}{dq_1} = 30 - 2q_1 - 15 + q_1 = 15 - q_1.$$

Firm 1's profit is maximized by chosing q_1 where $MR_1 = MC_1$

$15 - q_1 = 0$ (MC = 0 is assumed)

$$\boxed{q_1^* = 15}$$

Then, for firm 2 (the follower) will take $q_1^* = 15$ into a/c and then choose its optimal output, $q_2^*(q_1^*)$:

$$q_2^* \quad 15 - \tfrac{1}{2}q_1^* = 15 - \tfrac{1}{2}(15) = 15 - 7.5 = 7.5$$

$$\boxed{q_2^* = 7.5}$$

Find p: $P = 30 - Q = 30 - (q_1^* + q_2^*) = 30 - (15 + 7.5)$
$= 30 - 22.5$
$= 7.5$

$$\boxed{P^* = 7.5}$$

Find π_1^* and π_2^*:

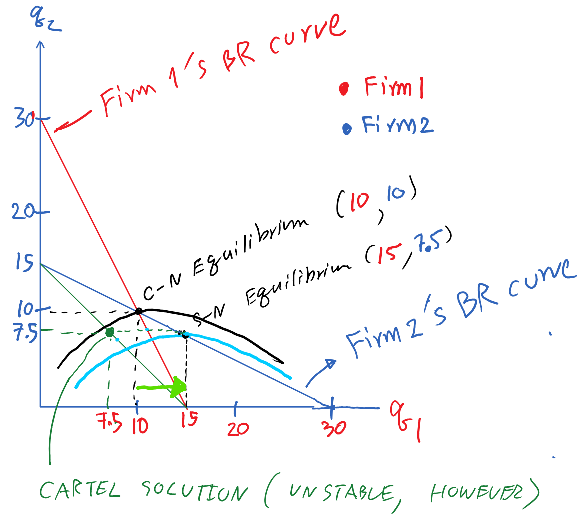
$$\boxed{\begin{aligned}\pi_1^* &= p\cdot q_1^* = 7.5\cdot 15 = 112.5 \\ \pi_2^* &= p\cdot q_2^* = 7.5\cdot 7.5 = 56.25\end{aligned}}$$

Notice that firm 1 earns higher profits compared to firm 2!

As $\pi_1^* = 112.5 > \pi_2^* = 56.25$, then "advantage of being a first mover" exists. (First-Mover Advantage)

q_2 curve

exists.
(First - Mover Advantage)



- $P = 30 - Q$

$$q_1 = 15 - \frac{1}{2} q_2$$

$$q_2 = 15 - \frac{1}{2} q_1$$

Firm 1 (Leader) earns <u>higher</u> profit in Stackelberg setup <u>than</u> in Cournot setup

[In the diagram, Firm 1 jumps from Black Isoprofit curve to **Blue** Isoprofit curve]

	q_1^*	q_2^*	Q^* $(= q_1^* + q_2^*)$	P	π_1^*	π_2^*	π_{TOTAL}^* $(= \pi_1^* + \pi_2^*)$
COURNOT	10	10	20	10	100	100	200
STACKELBERG	15	7.5	22.5	7.5	112.5	56.25	168.75
CARTEL (OR COLLUSION)	7.5	7.5	15	15	112.5	112.5	225

- $q^*_{1,\,STACKELBERG}$ (15) $>$ $q^*_{1,\,COURNOT}$ (10)

  $q^*_{1,\,STACKELBERG}$ (15) $>$ $q^*_{2,\,STACKELBERG}$ (7.5) $\rightarrow$ $\pi^*_{1,\,STACKELBERG} > \pi^*_{2,\,STACKELBERG}$ (1^{st}-mover advantage)

- ON TOTAL OUTPUT : Q_{CARTEL} (15) $<$ $Q_{COURNOT}$ (20) $<$ $Q_{STACKELBERG}$ (22.5)

- PRICE : P_{CARTEL} (15) $>$ $P_{COURNOT}$ (10) $>$ $P_{STACKELBERG}$ (7.5)

- Cartel generates the highest joint profits. (225)

- Stackelberg generates the lowest joint profits (168.75)

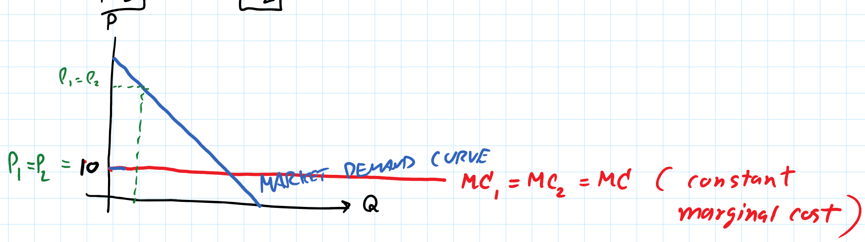
Wednesday, April 10, 2019 12:50 PM

III Bertrand Model of Price Competition (1853)

First, Bertrand argued that usually firms compete on price rather than on quantity. Moreover, it is easier to adjust price than quantity.

Simple setup
1. 2 players : 1 vs. 2
2. homogeneous products
3. Price as a strategic variable.
4. One-shot simultaneous move game.
5. Timing of the events
   - Firm 1 and Firm 2 chooses its price simultaneously.
   - Payoffs are revealed.

$\boxed{p_1, p_2} \rightarrow \boxed{\pi_1, \pi_2}$



Q: Is there a Nash equilibrium when Firm 1 and Firm 2 set their price simultaneously?

A:

$$BR_1 = \begin{cases} q_1 = 0 & \text{if } p_1 > p_2 \\ q_1 = q_2 = \frac{1}{2} & \text{if } p_1 = p_2 \\ q_1 = Q & \text{if } p_1 < p_2 \end{cases} \qquad BR_2 = \begin{cases} q_2 = 0 & \text{if } p_2 > p_1 \\ q_2 = \frac{1}{2} = q_1 & \text{if } p_2 = p_1 \\ q_2 = Q & \text{if } p_2 < p_1 \end{cases}$$

In equilibrium

$P_1 = MC$

$P_2 = MC$!!!

driven by process of undercutting price (price war)

(Bertrand paradox!)

How could a market with 2 sellers end up w/ $P = MC$ (competitive outcome of perfect competition)?