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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i-\bar{X})(Y_i-\bar{Y})$	$(X_i - \bar{X})^2$	$\hat{Y}_i$	$\hat{u}_i$
1	2.8	63	-14.625	-0.4125	6.03	213.89	2.7313	0.0687
2	3.4	72	-5.625	0.1875	-1.05	31.64	3.0274	0.3726
3	3	78	0.375	-0.2125	-0.08	0.14	3.2248	-0.2248
4	3.5	81	3.375	0.2875	0.97	11.39	3.3235	0.1765
5	3.6	87	9.375	0.3875	3.63	87.89	3.5209	0.0791
6	3.0	75	-2.625	-0.2125	0.56	6.89	3.1261	-0.1261
7	2.7	75	-2.625	-0.5125	1.35	6.89	3.1261	-0.4261
8	3.7	90	12 375	0.4875	6.03	153.19	3.6196	0.0804

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$\hat{\beta}_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

$\bar{x} = 77.625$

$\bar{y} = 3.2125$

$$\boxed{\begin{aligned} \hat{\beta}_1 &= 0.0329 \\ \hat{\beta}_0 &= 0.6586 \ \# \end{aligned}}$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \rightarrow 0.6586 + 0.0329 x_i$

$\hat{u}_i = y_i - \hat{y}_i \rightarrow y_i - \hat{y}_i$

$\hat{\beta}_1 = 0.0329$

$\hat{\beta}_0 = 0.6586$

Y_i	X_i	$\hat{y}_i$	$\hat{u}_i$
2.8	63	2.7313	0.0687
3.4	72	3.0274	0.3726
3	78	3.2248	-0.2248
3.5	81	3.3235	0.1765
3.6	87	3.5209	0.0791
3.0	75	3.1261	-0.1261
2.7	75	3.1261	-0.4261
3.7	90	3.6196	0.0804

$+$

$= 0$

1.3 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$\hat{\beta}_1 = \beta_1 + \left(\sum_{i=1}^{n} u_i k_i\right)$$

$$Var(\hat{\beta}_1) = Var(\beta_1) + Var\left(\sum_{i=1}^{n} u_i k_i\right)$$

2. Data is listed in the table

X_i	Y_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i-\bar{X})(Y_i-\bar{Y})$	$(X_i-\bar{X})^2$	$\hat{Y}_i$	$\hat{u}_i$
10	0	-10	-9.1	91	100	0.145	-0.145
12	2	-8	-7.1	56.8	64	1.936	0.064
14	5	-6	-4.1	24.6	36	3.727	1.273
16	6	-4	-3.1	12.4	16	5.518	0.482
18	7	-2	-2.1	4.2	4	7.309	-0.309
22	10	2	0.9	1.8	4	10.891	-0.891
24	10	4	0.9	3.6	16	12.682	-2.682
26	15	6	5.9	35.4	36	14.473	0.527
28	16	8	6.9	55.2	64	16.264	-0.264
30	20	10	10.9	109	100	18.055	1.945

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \longrightarrow \hat{\beta}_0 = 0.8955$

$\hat{\beta}_1 = \dfrac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2} \longrightarrow \hat{\beta}_1 = -8.81$

$\bar{x} = 20$

$\bar{y} = 9.1$

2.2 Find the value of $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \rightarrow 0.8955 - 8.81 X_1$

$\hat{\beta}_0 = 0.8955$

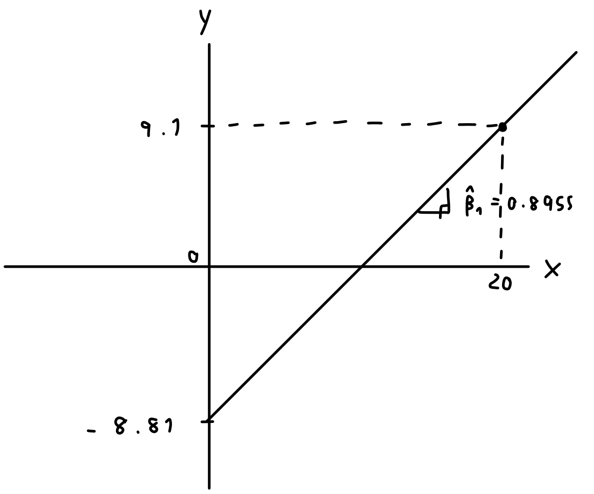
$\hat{u}_1 = y_i - \hat{y}_i$

$\hat{\beta}_1 = -8.81$

X_i	Y_i	$\hat{y}_i$	$\hat{u}_i$
10	0	0.145	-0.145
12	2	1.936	0.064
14	5	3.727	1.273
16	6	$\boxed{5.518}$	0.482
18	7	7.309	-0.309
22	10	10.891	-0.891
24	10	12.682	-2.682
26	15	14.473	0.527
28	16	16.264	-0.264
30	20	18.055	1.945

$\oplus = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$
$= 0.8955 - 8.81 X_1$

2.4 If X_i =16, what is the predicted Y?

$X_i = 16$, $\hat{y} = 5.518$

2.5 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \text{ ,}$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x - \bar{x})^2}$$

suppose $\quad X_i = x_i - \bar{x} \qquad K_i = \frac{x_i}{\sum_i X_i^2}$

$$\beta_1 = \sum_{i=1}^{n} (y_i - \bar{y})\, k_i \qquad \text{OR} \qquad \hat{\beta}_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (x - \bar{x})^2}$$

$$= \sum_{i=1}^{n} (\hat{\beta}_0 - \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x})\, k_i$$

$$= \sum_{i=1}^{n} \beta_1 (x_i - \bar{x})\, k_i + \Sigma_i\, u_i\, k_i$$

$$= \beta_1 \sum_{i=1}^{n} X_i\, k_i + \Sigma_i\, u_i\, k_i$$

$$= \beta_1 \sum_{i=1}^{n} X_i \frac{X_i}{\Sigma_i X_i^2} + \Sigma_i\, u_i\, k_i$$

$$= \beta_1 \frac{\cancel{\Sigma_i X_i^2}}{\cancel{\Sigma_i X_i^2}} + \Sigma_i\, u_i\, k_i$$

$$E(\hat{\beta}_1) = E[\beta_1 + \Sigma_i\, u_i k_i]$$

SLR 4: $E(u_i \mid x_1) = 0$

$$E(\beta_1) = \beta_1 + \Sigma_i\, k_i\, E(u_i)$$

$\qquad\qquad\qquad\qquad 0$

$\therefore \quad E(\hat{\beta}_1) = \beta_1$