

EE 325 Section 3 Take home quiz 2 February 27th, 2020 ☺

Data on X_i and Y_i are given in the table

i	X_i	Y_i
1	4	6
2	3	5
3	2	4
4	8	10
5	1	3

Suppose you decide to fit the following model

$$Y_i = \beta_1 + \beta_2 X_i + u_i \quad \text{where} \quad u_i \sim \text{NIID}(0, \sigma^2)$$

$$\sum X_i = 18, \sum Y_i = 28$$

$$\bar{X} = 3.6, \bar{Y} = 5.6$$

$$\sum (X_i - \bar{X})^2 = 29.2$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 29.2$$

$$\sum X_i Y_i = 130, \sum X_i \sum Y_i = 504, \left(\sum X_i \right)^2 = 94$$

Compute estimators of β_1 and β_2 ($\hat{\beta}_1$ and $\hat{\beta}_2$).

Interpret the regression.

$y_i - \bar{y}$		$\bar{X} = \frac{\sum x_i}{N} = \frac{19}{5} = 3.6$	$\bar{Y} = \frac{\sum y_i}{N} = \frac{27}{5} = 5.6$			29.2
u_i	i	X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$
0.4	1	4	6	0.4	0.4	0.16
-0.6	2	3	5	-0.6	-0.6	0.36
-1.6	3	2	4	-1.6	-1.6	2.56
4.4	4	8	10	4.4	4.4	19.36
-2.6	5	1	3	-2.6	-2.6	6.76

$$y_i = \beta_1 + \beta_2 x_i + u_i$$

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i + \hat{u}_i$$

$$\hat{y}_i = 5.6 + \cancel{0} + \cancel{\hat{u}_i}$$

$$u_i \sim \text{NIID}(0, \sigma)$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 5.6 - 3.6 \hat{\beta}_2 = 5.6 - 3.6 = 2$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{27.2}{29.2} = 1$$

$$u_i \sim \text{NIID}(0, \sigma^2)$$

$$\hat{y}_i = 5.6, \hat{\beta}_1 = 2, \hat{\beta}_2 = 1$$

$\therefore$ The value of the estimator of β_1 & β_2 ($\hat{\beta}_1$ & $\hat{\beta}_2$) are 2 and 1, respectively.

Interpret $\hat{\beta}_2$, from $\hat{\beta}_2 = 1$ this means if x increase by 1 unit, on average, y will increase 1 unit.

Interpret $\hat{\beta}_1$, suppose $x = 0$, on average, y will equal to 2.