

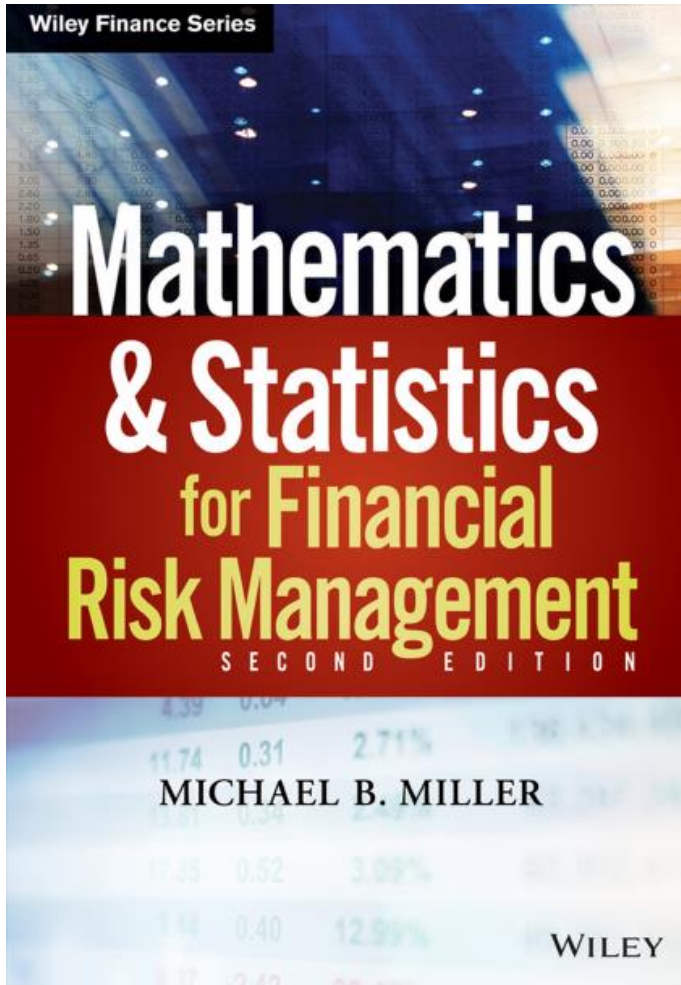
FN211 Basic Matrices

Part 2

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Outline

- *Introduction and Types of Matrices*
- *Matrix Operations*
- *Inverse of a Matrix*
- **Application**

Reading:

Chapter 8

Example: Sales

The A-Plus auto parts store chain has two outlets, one in Vancouver and one in Quebec. Among other things, it sells wiper blades, windshield cleaning fluid, and floor mats. The monthly sales of these items at the two stores for two months are given in the following tables:

January Sales

	Vancouver	Quebec
Wiper Blades	20	15
Cleaning Fluid (bottles)	10	12
Floor Mats	8	4

February Sales

	Vancouver	Quebec
Wiper Blades	23	12
Cleaning Fluid (bottles)	8	12
Floor Mats	4	5

Use matrix arithmetic to calculate the change in sales of each product in each store from January to February.

Example: Sales (2)

The revenue generated by sales in the Vancouver and Quebec branches of the A-Plus auto parts store (see Example 2) was as follows:

January Sales in Canadian Dollars

	Vancouver	Quebec
Wiper Blades	140.00	105.00
Cleaning Fluid	30.00	36.00
Floor Mats	96.00	48.00

If the Canadian dollar was worth \$0.65 U.S. at the time, compute the revenue in U.S. dollars.

Example: Revenue

The A-Plus auto parts store mentioned in examples in the previous section had the following sales in its Vancouver store:

	Vancouver
Wiper Blades	20
Cleaning Fluid (bottles)	10
Floor Mats	8

The store sells wiper blades for \$7.00 each, cleaning fluid for \$3.00 per bottle, and floor mats for \$12.00 each. Use matrix multiplication to find the total revenue generated by sales of these items.

Example: Revenue (2)

January sales at the A-Plus auto parts stores in Vancouver and Quebec are given in the following table.

	Vancouver	Quebec
Wiper Blades	20	15
Cleaning Fluid (bottles)	10	12
Floor Mats	8	4

The usual selling prices for these items are \$7.00 each for wiper blades, \$3.00 per bottle for cleaning fluid, and \$12.00 each for floor mats. The discount prices for A-Plus Club members are \$6.00 each for wiper blades, \$2.00 per bottle for cleaning fluid, and \$10.00 each for floor mats. Use matrix multiplication to compute the total revenue at each store, assuming first that all items were sold at the usual prices, and then that they were all sold at the discount prices.

Solving Systems of Equations

Solve a System of n Linear Equations in n Unknowns

Having used systems of equations and row reduction to find matrix inverses, we will now use matrix inverses to solve systems of equations. Recall that, at the end of the previous section, we saw that a system of linear equations could be written in the form

$$AX = B$$

where A is the coefficient matrix, X is the column matrix of unknowns, and B is the column matrix of right-hand sides.

Example

The system of linear equations

$$2x \quad + z = 9$$

$$2x + y - z = 6$$

$$3x + y - z = 9$$

Solve a System of n Linear Equations in n Unknowns

From: $AX = B$

The object is to solve for the matrix X of unknowns:

Solve a System of n Linear Equations in n Unknowns

Example

Solve the following three systems of equations.

a. $2x + z = 1$

$$2x + y - z = 1$$

$$3x + y - z = 1$$

b. $2x + z = 0$

$$2x + y - z = 1$$

$$3x + y - z = 2$$

c. $2x + z = 0$

$$2x + y - z = 0$$

$$3x + y - z = 0$$

Solve a System of n Linear Equations in n Unknowns

● **Nutrition** A four-ounce serving of Campbell's® Pork & Beans contains 5 grams of protein and 21 grams of carbohydrates.¹⁶ A typical slice of “lite” rye bread contains 4 grams of protein and 12 grams of carbohydrates.

I am planning a meal of “beans-on-toast” and I want it to supply 20 grams of protein and 80 grams of carbohydrates. How should I prepare my meal?

● **Resource Allocation** You manage an ice cream factory that makes three flavors: Creamy Vanilla, Continental Mocha, and Succulent Strawberry. Into each batch of Creamy Vanilla go two eggs, one cup of milk, and two cups of cream. Into each batch of Continental Mocha go one egg, one cup of milk, and two cups of cream. Into each batch of Succulent Strawberry go one egg, two cups of milk, and one cup of cream. Your stocks of eggs, milk, and cream vary from day to day. How many batches of each flavor should you make in order to use up all of your ingredients if you have the following amounts in stock?

- a. 350 eggs, 350 cups of milk, and 400 cups of cream
- b. 400 eggs, 500 cups of milk, and 400 cups of cream
- c. A eggs, B cups of milk, and C cups of cream

Input-Output Models

Input-Output Models

Output Industries Inputs and Pollutants' Output	Industry 1	Industry 2	Pollution Abatement	Final Demand	Total Output
	Industry 1	Industry 2	Pollution Abatement	Final Demand	Total Output
Industry 1	X_{11}	X_{12}	X_{1pa}	Y_1	X_1
Industry 2	X_{21}	X_{22}	X_{2pa}	Y_2	X_2
Physical Output of Pollutant	X_{1p}	X_{2p}	$-X_{pa}$	Y_p	X_p
Primary Inputs	V_1	V_2	V_{pa}	V_y	V
Total Inputs	X_1	X_2	X_{pa}	Y	X

Input-Output Models

Total supply from Sector 1 = Total demand for Sector 1 products

$$\begin{array}{ccccccc} x_1 & = & 0.50x_1 & + & 0.25x_2 & + & 7000 \\ & & \uparrow & & \uparrow & & \uparrow \\ & & \text{Coal required by Sector 1} & & \text{Coal required by Sector 2} & & \text{External demand for coal} \end{array}$$

Total supply from Sector 2 = Total demand for Sector 2 products

$$\begin{array}{ccccccc} x_2 & = & 0.10x_1 & + & 0.25x_2 & + & 14,000 \\ & & \uparrow & & \uparrow & & \uparrow \\ & & \text{Electricity required by Sector 1} & & \text{Electricity required by Sector 2} & & \text{External demand for electricity} \end{array}$$

Input-Output Models

This is a system of two linear equations in two unknowns:

$$x_1 = 0.50x_1 + 0.25x_2 + 7000$$

$$x_2 = 0.10x_1 + 0.25x_2 + 14,000$$

We can rewrite this system of equations in matrix form as follows:

$$\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{Production}} = \underbrace{\begin{bmatrix} 0.50 & 0.25 \\ 0.10 & 0.25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{Internal demand}} + \underbrace{\begin{bmatrix} 7000 \\ 14,000 \end{bmatrix}}_{\text{External demand}}$$

In symbols,

$$X = AX + D$$

How to solve for production vector?

Input-Output Models

Note:

$$A = \begin{bmatrix} 0.50 & 0.25 \\ 0.10 & 0.25 \end{bmatrix}$$

is called the **technology matrix**. The entries of the technology matrix have the following meanings:

a_{11} = units of Sector 1 needed to produce one unit of Sector 1

a_{12} = units of Sector 1 needed to produce one unit of Sector 2

a_{21} = units of Sector 2 needed to produce one unit of Sector 1

a_{22} = units of Sector 2 needed to produce one unit of Sector 2

Input-Output Models: Example (1)

Example **Petroleum and Natural Gas**

Consider two sectors of the U.S. economy: crude petroleum and natural gas (*crude*) and petroleum refining and related industries (*refining*). According to government figures,* in 1998 the crude sector used \$27,000 million worth of its own products and \$750 million worth of the products of the refining sector to produce \$87,000 million worth of goods (crude oil and natural gas). The refining sector in the same year used \$59,000 million worth of the products of the crude sector and \$15,000 million worth of its own products to produce \$140,000 million worth of goods (refined oil and the like). What was the technology matrix for these two sectors? What was left over from each of these sectors for use by other parts of the economy or for export?

Input-Output Models: Example (2)

Example **Rising Demand**

Suppose that external demand for refined petroleum rises to \$200,000 million, but the demand for crude remains \$1000 million (as in Example 1). How do the production levels of the two sectors considered in Example 1 have to change?

Input-Output Models: Example (3-4)

- **Campus Food** The two campus cafeterias, the Main Dining Room and Bits & Bytes, typically use each other's food in doing business on campus. One weekend, the input-output table was as follows.³²

<i>To</i>		Main DR	Bits & Bytes
<i>From</i>	Main DR	\$10,000	\$20,000
	Bits & Bytes	5000	0
Total Output		50,000	40,000

Given that the demand for food on campus last weekend was \$45,000 from the Main Dining Room and \$30,000 from Bits & Bytes, how much did the two cafeterias have to produce to meet the demand last weekend?

- **Plagiarism** Two student groups at Enormous State University, the Choral Society and the Football Club, maintain files of term papers that they write and offer to students for research purposes. Some of these papers they use themselves in generating more papers. In order to avoid suspicion of plagiarism by faculty members (who seem to have astute memories), each paper is given to students or used by the clubs only once (no copies are kept). The number of papers that were used in the production of new papers last year is shown in the following input-output table:

<i>To</i>		Choral Soc.	Football Club
<i>From</i>	Choral Soc.	20	10
	Football Club	10	30
Total Output		100	200

Given that 270 Choral Society papers and 810 Football Club papers will be used by students outside of these two clubs next year, how many new papers do the two clubs need to write?

Question?