

# Chapter 4 Part 2 : Endogenous Growth Model

EE312

Macroeconomics, Stephen Williamson, Chapter 7,8

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- Chapter 4: Growth Model
  - 4.1 Solow Growth Model
  - 4.2 Endogenous Growth Model

## Outline (this part)

- Introduction :
  - Solow growth predictions
  - Growth facts
  - Growth in Solow model
- Endogenous growth

# Endogenous Growth

- 1 Definition : human capital accumulation
- 2 Representative consumer : accumulation of human capital
- 3 The representative firm
  - 1 The firm's profit function
  - 2 Demand for efficiency units of labor
  - 3 Determination of the real wage
- 4 Competitive equilibrium
- 5 Growth of human capital
- 6 Factors in human capital growth
- 7 Consumption and output growth
- 8 Source of growth
- 9 Government policy on growth
- 10 No convergence
- 11 Human capital externality

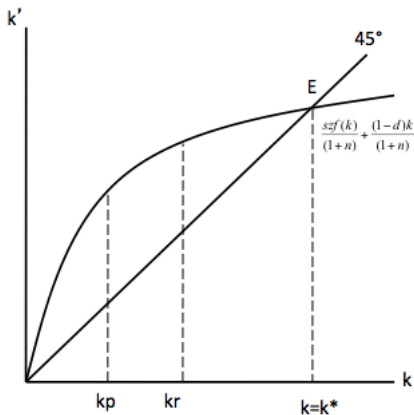
### Solow growth predictions ( (1) Absolute convergence, (2) Conditional Convergence )

#### (1) Absolute convergence

- If two countries start with:
  - the same population growth rate ( $n$ ), saving rate ( $s$ ) and total factor productivity ( $z$ ),
  - but different per capita incomes ( $y$ ), e.g., rich versus poor countries;
  - they will converge to the same steady-state  $k^*$ ,  $y^*$  and  $c^*$  —  
**Absolute convergence.**
- The poor country will have temporary higher growth and catch up with the rich.
- Absolute convergence (convergence in  $k^*$ ,  $y^*$  and  $c^*$ ),
  - Convergence in  $y^*$ , Convergence in output growth path

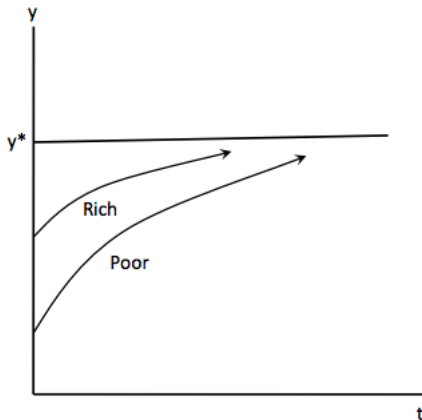
## Absolute convergence

- The rich starts at  $k_r$  while the poor starts at  $k_p$  (with the same  $s$ ,  $n$  and  $z$ ).
- They converge to  $k^*$  and  $y^*$  in the long run.



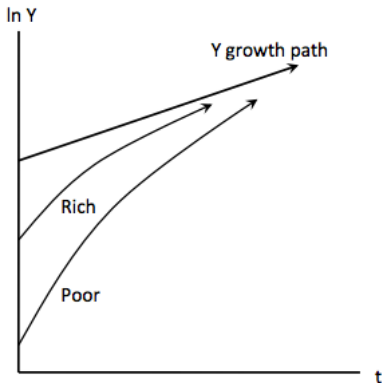
## Convergence in per capita income

- The rich and the poor converge to the same level of  $y^*$ .



## Convergence in output growth path

- The rich and the poor converge to the long-run growth rate ( $n$ ) of aggregate output ( $Y$ ).



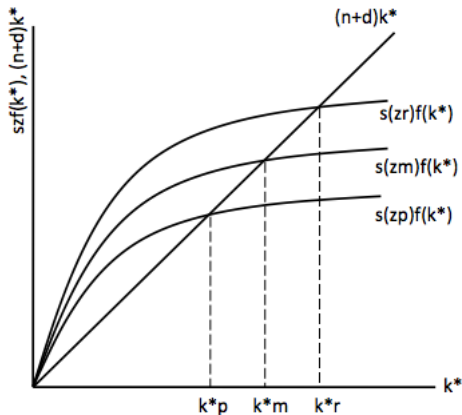
## Solow growth predictions: (2) Conditional convergence

- With differences in  $n$ ,  $z$  and  $s$ , the steady-state  $k^*$ ,  $y^*$ ,  $c^*$  are different.
  - Each country has its own steady state.
  - The steady-state growth rate of aggregates ( $K$ ,  $Y$ ) is still  $n$  for each country.
- Countries are predicted to converge to their own steady state.
- Disparity among countries due to different values of  $n$ ,  $z$  and  $s$ .

### Solow growth predictions Vs. Growth Facts

- Absolute convergence has occurred among rich countries.
- No absolute convergence between rich and poor countries. Exception is East Asia.
- No absolute convergence among poor countries.
- Great diversity among poor countries.
- Why no absolute convergence?
  - Countries have different  $s$ ,  $n$  and  $z$ .
  - Each country has different steady-state  $k^*$ ,  $y^*$ ,  $c^*$ .
  - Each country is moving towards its own steady-state — **Conditional convergence.**
  - But differences in  $s$  and  $n$  are not large enough to explain all international disparity.
  - **Difference in access to technology ( $z$ )?**

- Countries with different  $z$ 's will not converge to the same  $k^*$  and  $y^*$ .
- $p$  = poor
- $m$  = medium
- $r$  = rich



## Disparity due to different $z$ 's

- Different levels of total factor productivity ( $z$ ) will perpetuate differences in capital per worker ( $k^*$ ), per capita income ( $y^*$ ) . . .
- despite the same saving rate ( $s$ ) and population growth ( $n$ ).

## Barriers to technology adoption

- **Labor legislation:** strong labor unions obstruct adoption of new technology.
- **Trade protectionism:** domestic firms with market power lack incentives to innovation.
- **Political corruption:** government's protection of inefficient firms.
- **Undeveloped financial system:** poor resource allocation mechanism.

## How to catch up?

- Promotion of more competition among firms.
  - Liberalization and competition policy.
  - More pressure and incentive for firms to innovation.
- Free trade for greater international competition.
- Privatization of state enterprises.
  - State enterprises guarantee employment at the expense of efficiency.

- The Solow model does not explain the mechanism of growth itself.
  - Growth depends on exogenous factors.
  - Total factor productivity ( $z$ ) is exogenously determined.
  - $z$  depends on R&D by firms, education, training.
  - These are partly affected by government policy.
  - Government policy to raise  $z$  and long-term growth?

- Explanation of growth within the model.
- Total factor productivity ( $z$ ) depends on ‘human capital accumulation’.
- **Human capital:** the accumulated stock of skills and education workers have at a point in time.
  - Higher human capital; more production; more production of new human capital — faster growth over time.

# 1. Human capital accumulation

- The higher human capital, the more efficiency the production of human capital has.
  - Better schooling, more future production, better passing on skills and knowledge.
- **Human capital is an investment.**
  - Opportunity cost of education and training — sacrifice of current consumption.
  - Benefits: more future production and consumption.

- Knowledge is '**non-rivalry**': one's acquisition of knowledge does not reduce others' ability to acquire the same knowledge.
- Human capital accumulation is **NOT subject to diminishing marginal returns**.
- No limit on how productive a person can become, given increasing knowledge and skills.
- **Unbounded growth** in endogenous models.
- Growth in Solow model is limited:
  - **Diminishing returns** on physical capital accumulation — rivalry in resource uses.

## 2. The representative consumer

- The consumer allocates time between work and accumulating human capital.
- $H^S$  = efficiency units of current human capital;
- $u$  = time allocated to work;
- $w$  = the real wage;
- $C$  = current consumption;
- The **budget constraint** is total labor earnings:

$$C = uwH^S \quad (1)$$

## Accumulation of human capital

- The consumer trades off current consumption for future consumption by accumulating human capital:
- $H^{s'}$  = future human capital;
- $(1 - u)$  = time allocated to human capital accumulation;
- $b$  = efficiency of human capital accumulation technology;  $b > 0$ .

$$H^{s'} = (1 - u)bH^s \quad (2)$$

### 3. The representative firm

- The firm's production function using efficiency units of labor:
- $Y$  = current output;
- $z$  = marginal product of efficiency units of labor, where  $z > 0$ ;
- $uH^d$  = current input of efficiency units of labor:

$$Y = zuH^d \quad (3)$$

## The firm's profit function

- $uH^d$  is also the firm's demand for the efficiency units of labor.
- The function is characterized by **constant returns to scale** (CRS)  
— only one input.

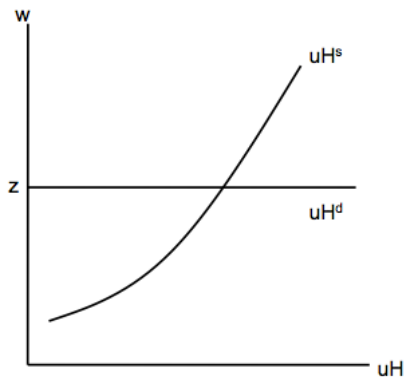
$$\begin{aligned}\pi &= Y - wuH^d \\ \pi &= zuH^d - wuH^d \\ \pi &= (z - w)uH^d\end{aligned}$$

$$\pi = (z - w)uH^d \quad (4)$$

- $(z-w) < 0$ ,  $\pi < 0$ ; the firm hires no units of labor; or  $uH^d = 0$ .
- $(z-w) > 0$ ,  $\pi > 0$ ; the firm hires infinite units.
- $z = w$ ,  $\pi = 0$ ; the firm is indifferent.
- The demand curve is infinitely elastic at  $w = z$ .

## Determination of the real wage

- $uH^d$  is horizontal at  $w = z$ .
- The real wage equals  $z$ , the marginal product of  $uH^s$ . Assume  $uH^s$  with slope  $> 0$ .



## 4. Competitive equilibrium

- The market clears at  $w = z$  where  $uH^d = uH^s$ .
- Equilibrium consumption and growth of human capital accumulation:

$$C = zuH.$$

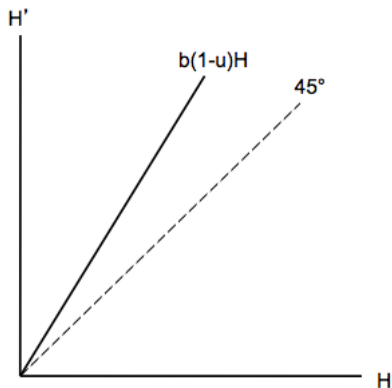
$$H' = b(1 - u)H.$$

$$\frac{H'}{H} - 1 = b(1 - u) - 1,$$

where  $b(1 - u) - 1$  is constant.

## 5. Growth of human capital

- $H'$  is a function of  $H$  where  $H' > H$ .
- Slope =  $b(1-u) = 1 + \text{rate of growth of human capital}$ .



## 6. Factors in human capital growth

$$\frac{H'}{H} - 1 = b(1 - u) - 1, \quad (5)$$

- $\frac{H'}{H}$  is higher if  $b$  increases or  $u$  decreases.
  - $b$  = efficiency of human capital accumulation technology (or efficiency of the education sector).
  - $u$  = time spent on current output production.
  - Falling  $u$  (or rising  $1-u$ ) = more time spent on human capital accumulation.

## 7. Consumption and output growth

- Current consumption  $C = zuH$  also holds for future consumption  $C' = zuH'$ .
  - So consumption grows at the same rate of  $b(1-u)$  as human capital.
- Output also grows at the same rate as  $Y = C$  in every period.

$$\frac{C'}{C} - 1 = \frac{zuH'}{zuH} - 1 = \frac{H'}{H} - 1 = b(1 - u) - 1 \quad (6)$$

## 8. Source of growth

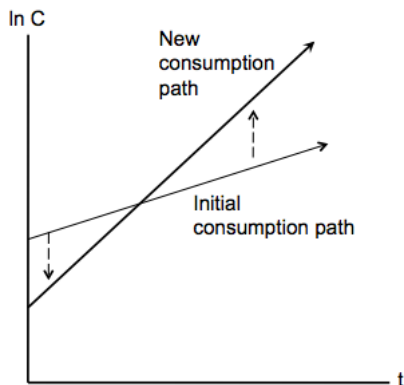
- **b and z are fixed:** constant technology.
- No population growth.
- Growth is determined inside the model, by the value of  $b$  and  $u$ .
- **Growth is unbounded** because human capital accumulation is not subject to diminishing returns to scale.
  - Output grows in proportion to human capital, given  $u$ .

## 9. Government policy on growth

- Government can increase growth:
  - Increases in  $b$ , the efficiency of human capital accumulation technology (education policy).
  - Reduction in  $u$ , taxes or subsidies to education.
  - Higher  $b(1-u)$ , higher growth of human capital, consumption and output.
- But current consumption must be sacrificed as  $u$  is lower, given initial human capital ( $H$ ).

## Lower $u$ and consumption

- a lower  $u$  results in lower current consumption but higher consumption in the long run.



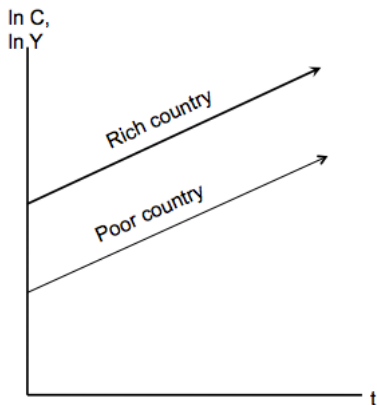
- Government's education policy (raising  $b$ ) involves expenses of current resources and lower current consumption.
- Higher long-run growth is desirable?
  - This depends on the consumer's preference on current and future consumption.
  - The consumer may be worse off if current consumption is actually preferred.

## 9. No convergence

- Countries with all identical characteristics except differences in initial human capital will not converge on the levels of consumption and income.
  - **Poor countries:** low  $Y = C = zuH$ ;
  - **Rich countries:** high  $Y = C = zuH$ .
  - But their  $C$  and  $Y$  grow at the same rate of  $b(1-u)$ .

## Rich and poor do not converge.

- The  $Y$  and  $Y$  time paths do not converge despite the same growth rate of  $b(1-u)$ .



## 10. Human capital externalities

- The endogenous model explains the lack of convergence among poor countries and between rich and poor countries.
- But convergence occurs among rich countries, why? — Human capital externalities.
  - Contact with others with higher human capital increases our own human capital.
  - Capital and labor are highly mobile; skills are more easily transferred in rich countries.

- More opportunities and contact make levels of human capital in rich countries converge.
  - Convergence of income per worker.
- Lack of human capital externalities in poor countries.
  - Less contact with developed countries.
  - People with high human capital move to developed countries (i.e., brain drains).
  - Differences in human capital persist.