EE431/438 Economics of Financial Markets and Institutions

Topic 6: Debt Market and Structure of Interest Rate

A note on the Term Structure of Interest Rate

Federic Mishkin, The Economics of Money, Banking and Financial Markets Chapter 4 - 6
(available at the reserve section of the library, HG173 .M57 2007)

Term Structure of Interest Rate

- Bonds with identical risk, liquidity, and tax characteristics may have different interest rates because the time remaining to maturity is different
- Yield curve: a plot of the yield on bonds with differing terms to maturity but the same risk, liquidity and tax considerations
- Generally, when talking about the yield curve, it refers to the yield on government bonds
- In most cases (normal situations),

  1. Yield curve slopes upwards
  2. Interest rates on bonds of different maturities move together over time

- three theories explain why bonds with differing terms to maturity have different yields to maturity: which one can explain the two findings the best
- note that what information the yield curve does contain remains a topic of debate in economics

(See Fixed income fortune teller, http://www.economist.com/blogs/freeexchange/2009/01/fixed_income_fortune_teller. (Assignment 2))

1. Segmented Market Theory

- Assumption : bonds of different maturities are not substitutes at all
- Result
  - market for bonds with different maturies are totally separated from one another
  - interest rates (YTM) and bond prices for each bonds with different maturities are determined by the demand and supply of the bonds in each market only
- Does segmented market theory can explain the first fact, the yield curve usually slopes upward?
  - yields on bonds with longer maturity is high
  - yieldson bonds with shorter maturity is low
  - It is possible that the demand for bonds with shorter maturity is higher

- Does segmented market theory can explain the second fact, the yields on bond with different maturity usually move together?
  - No. The market for bonds with different maturies are totally separated. There is no reason for the yields on bonds with different maturity to move together. Long-term and short-term rates are determined independently.

2. Expectation Theory

Assumption: bonds of different maturities are perfect substitutes (with the same level of risk, liquidity, taxation)

Result : For a given investment period, at equilibrium, is it possible to have the returns from investment in bonds with different maturities? No. Bonds with different maturities are perfectly substitute. An investors have two strategies for a given investment period.

Consider a two-years investment period. An investor will have two stategies.

Strategy 1: Buy a one-year bond, and when it matures in one year, buy another one-year bond. (Roll over)

Strategy 2: Buy a two-year bond and hold it until maturity. (Buy and hold)

Therefore, for this investor, he may invest in a one-year bond or a two-year bond. As any investor will be indifferent between these two strategies, the returns from these two strategies have to be equal. Arbitrage should result in the returns from the two strategies being the same. For example, if the returns from "Buy and hold" strategy is greater than Rollover strategy. People will buy long-term bonds and sell short-term bonds.

Let $i_{n,t}$ is the (annual) interest rate on bonds with n years time to maturity, at time t. Then, $i_{2,t}$ is the (annual) interest rate on a two-year bond today (at time t). $i_{1,t}$ is the (annual) interest rate on a one-year bond today (at time t). $i^e_{1,t+1}$ is the interest rate on a one-year bond next year (at time t+1). It has superscript e, indicating that the number is an expected value. The interest rate on a one-year bond next year (at time t+1) is an expected value because we never know its exact value today.

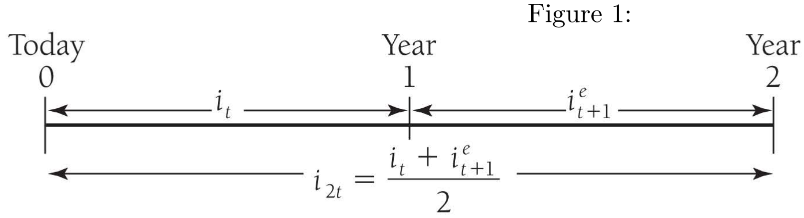
With Buy and hold strategy, you buy a two-year bond and hold it until maturity. After two years, the \$1 investment will have grown to \$1 $\times(1+i_{2,t})(1+i_{2,t})$.

With Rollover stategy, you buy a one-year bond today and hold it until it matures in one year. At that time, you buy a second one-year bond, and you hold it until it matures at the end of the second year. After two years, you will expect your \$1 investment to have grown to \$1 $\times(1+i_{1,t})(1+i^e_{1,t+1})$. With the rollover strategy, you must form an expectation of what the interest rate on the one-year bond will be.

$$\begin{aligned}
\text{Returns from buy and hold strategy} &= \text{Returns from roll over strategy} \\
(1+i_{2,t})^2 &= (1+i_{1,t})(1+i^e_{1,t+1}) \qquad (1)\\
1+2i_{2,t}+i^2_{2,t} &= 1+i_{1,t}i^e_{1,t+1}+i_{1,t}+i^e_{1,t+1}.
\end{aligned}$$

Assuming that interest rates are small decimal numbers. For example, $10\% = 0.1$ and $0.1 \times 0.1 = 0.01$. The product of interest rates are small numbers. Therefore, it is assumed that they approach to zero. Equation (1) reduces to

Figure 1:



$$1 + 2i_{2,t} = 1 + i_{1,t} + i^e_{1,t+1}. \tag{2}$$

Then, we can write

$$i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2} \tag{3}$$

This equation tells us that the interest rate on the two-year bond is an average of the interest rate on the one-year bond today and the expected interest rate on the one-year bond one year from now. Consider a n years investment period, we can then write

$$i_{n,t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n}.$$

so that the interest rate on a n-year bond is the average of the one-year interest rates that are expected to prevail over the next n years.

Interpreting the Term Structure Using the Expectations Theory

Example: Given the following information, in 2011

- Government bond, ttm = 3 years, YTM = 6%
- Government bond, ttm = 2 years, YTM = 4%
- Government bond, ttm = 1 years, YTM = 2%

What is the expected YTM on a government bond with 1 year time to maturity in 2012 and 2013?

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According to the given information, we can write $i_{3,t} = 6\%$, $i_{2,t} = 4\%$, $i_{1,t} = 2\%$, where t= 2011. Then, the expectation theory predicts that

$$i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2}.\ 4\% = \frac{2\% + i^e_{1,t+1}}{2},\ i^e_{1,t+1} = 8\% - 2\% = 6\%.$$
$$i_{3,t} = \frac{i_{1,t} + i^e_{1,t+1}, +i^e_{1,t+2}}{3}.\ 6\% = \frac{2\% + 6\% + i^e_{1,t+2}}{3},\ i^e_{1,t+2} = 18\% - 2\% - 6\% = 10\%.$$

Example: Given the following information, in 2011

- Government bond, ttm = 3 years, YTM = 5%
- Government bond, ttm = 2 years, YTM = 6%
- Government bond, ttm = 1 years, YTM = 8%

What is the expected YTM on a government bond with 1 year time to maturity in 2012 and 2013?

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According to the given information, we can write $i_{3,t} = 5\%$, $i_{2,t} = 6\%$, $i_{1,t} = 8\%$, where t= 2011. Then, the expectation theory predicts that

$$i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2}.\ 6\% = \frac{8\% + i^e_{1,t+1}}{2},\ i^e_{1,t+1} = 12\% - 8\% = 4\%.$$
$$i_{3,t} = \frac{i_{1,t} + i^e_{1,t+1}, +i^e_{1,t+2}}{3}.\ 5\% = \frac{8\% + 4\% + i^e_{1,t+2}}{3},\ i^e_{1,t+2} = 15\% - 8\% - 4\% = 3\%.$$

According to Expectation Theory, an upward-sloping yield curve is the result of investors expecting future short-term rates to be higher than the current short-term rate. A flat yield curve is the result of investors expecting future short-term rates to be the same as the current short-term rate. A downward-sloping yield curve is the result of investors expecting future short-term rates to be lower than the current short-term rate.

How well the theory can explain the two findings?

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The expectations hypothesis can explain fact (2) "the yields on bond with different maturity usually move together". Since interest rates at all maturities depend on today's short-term rate $i_{1,t}$, then they will tend to move together, rising when it rises and falling when it falls.

How's about fact (1) the yield curve usually slopes upward? Since short-term interest rates are as likely to rise as they are to fall, the expectations hypothesis predicts that the yield curve is as likely to slope upward as it is to slope down. It cannot explain why most of the time the yield curve slopes up.

3. Preferred Habitat Theory

Assumption : Bonds of different maturities are partial (not perfect) substitutes.

Result: Investors have a preference for bonds of one maturity over another. They will be willing to buy bonds of different maturities only if they earn a somewhat higher expected return

To see how this theory works, recall that according to the expectations hypothesis:

$$i_{n,t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n}.$$

Liquidity premium theory modifies this equation to

$$i_{n.t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n} + \eta_{n,t}.$$

$\eta_{n,t}$ is term premium for bonds with n time to maturity at time t. η is low for bonds with the time to maturity that the investor likes the most, it is high bonds with a different time to maturity

Notice that the first part of this equation comes from the expectations hypothesis. It implies that investors still care about the interest rates on bonds of different maturities, so they will not let the expected returns on different investment strategies to get too far out of line. But the second part of this equation comes from segmented markets theory. It tells us that investors do have preferences for some maturities over others.

4. Liquidity Premium

Assumption : Bonds of different maturities are partial (not perfect) substitutes. Investors prefer short-term bonds over long-term ones because short-term bonds are more liquid –that is, their "preferred habitat" is in short-term bonds

Notice that the expectations hypothesis and segmented markets theory make assumptions that are somewhat extreme. The expectation theory assumes that bonds of different maturities are perfect substitutes. The segmented markets theory assumes that bonds of different maturities are not substitutes at all. Preferred Habitat Theory and Liquidity Premium theory have less extreme assumption.

Result: As investors prefer short-term bonds, they will be willing to buy bonds of longer term maturities only if they earn a somewhat higher expected return. The demand for short-term bonds will be greater than the demand for long-term bonds. Thus, the interest rate on long-term bonds will be higher than the interest rate on short-term bonds. In other words, if investors prefer short-term bonds, then the liquidity or term premium lnt will be positive and will tend to rise as n increases.

$$i_{n.t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n} + \eta_{n,t}$$

Again, the first part of this equation comes from the expectations hypothesis and the second part of this equation comes from segmented markets theory. It tells us that investors do have preferences for some maturities over others. Therefore, η is high for long-term bonds, it is low for short-term bonds. Term premium is the additional interest investors require in order to be willing to buy a long-term bond rather than a comparable sequence of short-term bonds.

Why might most investors prefer short-term bonds?

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They may wish to avoid the risk of incurring capital losses if they need to sell long- term bonds before maturity. They may also wish to take advantage of the greater liquidity of short-term bonds.

Example : Suppose that investors' preferred habitat is in short-term bonds,

$\eta_{2,t}$ = liquidity premium on 2-year bonds = 0.25%

$\eta_{5,t}$ = liquidity premium on 5-year bonds = 1%

Case 1 : If short-term interest rates are expected to rise. $i_{1,t}$ = 5%, $i^e_{1,t+1}$= 6 %, $i^e_{1,t+2}$ = 7 %, $i^e_{1,t+3}$= 8 %, $i^e_{1,t+4}$ = 9 %. Then, find $i_{2,t}$ and $i_{5,t}$.

$$\Rightarrow i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} = \frac{5\% + 6\%}{2} + 0.25\% = 5.5\% + 0.25\% = 5.75\%$$

$$\Rightarrow i_{5,t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + i^e_{1,t+4}}{5} + \eta_{5,t} = \frac{5\% + 6\% + 7\% + 8\% + 9\%}{5} + 1\% = 7\% + 1\% = 8\%$$

Hence, when short-term interest rates are expected to rise, the yield curve slopes up.

Case 2 : If short-term interest rates are expected to fall slightly. $i_{1,t}$ = 9%, $i^e_{1,t+1}$= 8.75 %, $i^e_{1,t+2}$ = 8.5 %, $i^e_{1,t+3}$= 8.25 %, $i^e_{1,t+4}$ = 8 %. Then, find $i_{2,t}$ and $i_{5,t}$.

$$\Rightarrow i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} = \frac{9\% + 8.75\%}{2} + 0.25\% = 9.125\%$$

$$\Rightarrow i_{5,t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + i^e_{1,t+4}}{5} + \eta_{5,t} = \frac{9\% + 8.75\% + 8.5\% + 8.25\% + 8\%}{5} + 1\% = 9.5\%$$

Hence, when short-term interest rates are expected to fall slightly, the yield curve still slopes up.

Case 3 : If short-term interest rates are expected to fall sharply. $i_{1,t} = 9\%$, $i^e_{1,t+1}$= 8 %, $i^e_{1,t+2} =$ 7 %, $i^e_{1,t+3}$= 6 %, $i^e_{1,t+4} = 5$ %. Then, find $i_{2,t}$ and $i_{5,t}$.

$$\Rightarrow i_{2,t} = \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} = \frac{9\% + 8\%}{2} + 0.25\% = 8.75\%$$

$$\Rightarrow i_{5,t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + i^e_{1,t+4}}{5} + \eta_{5,t} = \frac{9\% + 8\% + 7\% + 6\% + 5\%}{5} + 1\% = 8\%$$

Hence, only when short-term rates are expected to fall sharply does the yield curve slope down.

How well the theory can explain the two findings?

1. yield curve usually slopes upwards $\rightarrow$ Since the yield curve slopes down only when short-term interest rates are expected to fall sharply, most of the time the yield curve will slope up.

2. the yields on bond with different maturity usually move together? $\rightarrow$ Since interest rates at all maturities depend on today's short-term rate i_t, then they will tend to move together, rising when it rises and falling when i_t falls.