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Group Homework 1

Semester 2/2022 EE320 Introductory mathematical economics

Due date: Feb 3rd 2022 (before midnight /B.E. moodle).

Note: Late homework will not be accepted. Use the format of filename as required; this will cost you two points if you don't follow the instruction.

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1. Consider the market for good x. Suppose that the market demand equation is given by

$$P_x = 8 - bQ_x^d + cP_y; \quad b > 0 \; and \; c > 0$$

and the equation for market supply is given by

$$P_x = 20 + dQ_x^S; \quad d > 0$$

where P_x is price of good X, P_y is price of good Y, Q_x^d is quantity demanded for good X and Q_x^S is quantity supplied for good X. Answer the following questions.

a) What is(are) the endogenous variable(s) in the model? What is(are) the exogenous variable(s) in the model?

b) Specify condition(s) under which the market equilibrium for good x is guaranteed to exist. What restrictions do we need to place on the values of P_y so that the market equilibrium exists?

c) Solve for the equilibrium price (P_x^*) and equilibrium quantity (Q_x^*) of the market for good x.

d) Calculate the magnitude of the response of equilibrium quantity to the change in exogenous variable(s).

2. (an old midterm exam question) Consider a market with *10 identical consumers*. Each of the consumer's demand function is given by:

$$P = 10 + k_1 P_x + k_2 Y - Q_j^d,$$

where P is the unit price of the product sold in this market, Q_j^d is the amount of quantity demanded by the j-th consumer, P_x is the price of product x, and Y is the level of income. Assume further that the industry is controlled by two producers, each of whom has the following supply function:

$$P = 5 + k_3 W + Q_1^s, \qquad \text{and}$$

$$P = 20 + k_4 T + 2Q_2^s,$$

where Q_1^s and Q_2^s are the amount of quantity supplied by the first and second producer, respectively. W is the price of gasoline and T is the level of technology. All the parameters are STRICTLY positive.

Use the information given to answer the following questions:

a. Is "product x" considered as a *substitute* product or a *complementary* product?
b. Derive the market demand equation.

Now, I supplement two pieces of information to be used in the remaining parts of this question. That is, I assume that

(i) $0 < (20 + k_4 T) < (5 + k_3 W)$ and (ii) $10 + k_1 P_x + k_2 Y > 5 + k_3 W$

c. What does the first condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.
d. Given (i) and (ii), state the condition under which the market ceases to have only *single* firm that stays active in the business?

Continue with the information given above, but now consider a specific case where the value of coefficients and exogenous variables are given in the following table:

Coefficients	k_1	k_2	k_3	k_4
Value	2	2	3	1

Variables	Y	P_x	w	T
Value	5	10	10	5

e. Solve for the equilibrium price *and* quantity.

f. Suppose that the government provides a subsidy of \$5 for each unit of output that the consumers have purchased. Calculate the benefit that consumers and each of the two producers receive under the subsidy program.

3. Consider a simple macroeconomics model given below

$C = 0.3Y_d - k_1 r;$ $\qquad k_1 > 0.$

$I = 0.5Y - k_2 r;$ $\qquad k_2 > 0.$

$Y_d = Y - T$

$G = G_0$

$T = T_0$

$M^d = k_3 Y - k_4 r;$ $\qquad k_3 > 0 \;\; and \;\; k_4 > 0.$

$M^s = M_0$

where $Y = GDP, \;\; C = consumption, \;\; I = investment, \;\; G = government\ purchase, \;\; T = tax, \;\; r = interest\ rate, \;\; M^d = money\ demand, \; and \; M^s = money\ supply$

a. Write the system of linear equations in matrix form with two variables included, namely Y and r.
b. Solve for the equilibrium solution (Y^*, r^*)
c. Has fiscal policy, for example, the change in government purchase, become *more effective* if both k_1 and k_2 are getting bigger? Show your numerical result, and explain your result with some economic intuitions. (Hint: how do we measure the effectiveness of a policy?)

1. Consider the market for good x. Suppose that the market demand equation is given by

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$$P_x = 20 + dQ_x^S; \quad d > 0$$

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a) What is(are) the endogenous variable(s) in the model? What is(are) the exogenous variable(s) in the model?

exo : P_y , Q^d_x, Q_y^d

endo : P_x

b) Specify condition(s) under which the market equilibrium for good x is guaranteed to exist. What restrictions do we need to place on the values of P_y so that the market equilibrium exists?

c) Solve for the equilibrium price (P_x^*) and equilibrium quantity (Q_x^*) of the market for good x.

d) Calculate the magnitude of the response of equilibrium quantity to the change in exogenous variable(s).

b.) $P_x = 8 - bQ^d_x + cP_y$ $\qquad \frac{4}{\frac{1}{2}}$ $\qquad 4\times2$

$Q_x^d = \dfrac{8 - P_x + cP_y}{b}$

if P_y is positive than product x & y need to be substitute product

so $\frac{c}{b} > 0$

c.) $Q_x^d = Q_x^S$

$\dfrac{8 - P_x + CP_y}{b} = \dfrac{P_x - 20}{d}$

$8d - dP_x - CdP_y = bP_x - 20b$

$8d - CdP_y + 20b = bP_x + dP_x$

$8d - CdP_y + 20b = (b+d)P_x$

$\dfrac{8d - CdP_y + 20b}{(b+d)} = P_x$; $b+d > 0$

$Q_x = \dfrac{\dfrac{8d - CdP_y - 20b - 20}{(b+d)}}{d}$

D) $\dfrac{Q^*x}{\Delta Y} = -20 - 8 = -28$ #

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$I = 0.5Y - k_2 r;$ $\quad k_2 > 0.$

$Y_d = Y - T$

$G = G_0$

$T = T_0$

$M^d = k_3 Y - k_4 r;$ $\quad k_3 > 0$ and $k_4 > 0.$

$M^s = M_0$

where $Y = GDP,\ C = consumption,\ I = investment,\ G = government\ purchase,\ T = tax,\ r = interest\ rate,\ M^d = money\ demand,\ and\ M^s = money\ supply$

a. Write the system of linear equations in matrix form with two variables included, namely Y and r.

a.) → goods market

$Y = C + I + G$

$Y = 0.3(Y-T) - k_1 r + 0.5Y - k_2 r + G_0$

$Y = 0.3Y - 0.3T - k_1 r + 0.5Y - k_2 r + G_0$

$Y = \frac{1}{0.2}(-0.3T - k_1 r - k_2 r + G_0)$

$Y = -1.5T_0 - 5k_1 r - 5k_2 r + 5G_0$

$Y = -1.5T_0 - 5r(k_1 + k_2) + 5G_0$

$Y + r(5k_1 + 5k_2) = -1.5T_0 + 5G_0$

→ Money market

$M^d = M^s$

$k_3 Y - k_4 r = M_0$

$$\begin{bmatrix} 1 & 5k_1 + 5k_2 \\ k_3 & -k_4 \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} -1.5T_0 + 5G_0 \\ M_0 \end{bmatrix}$$

b) $\frac{M^s}{P} = L^d$

$M^s = M^d$

$M_0 = k_3 Y - k_4 r$

$r^* = \frac{k_3 Y - M_0}{k_4}$

Equilibrium: $Y = -1.5T + 5G_0 - 5 \cdot \left(\frac{k_3 Y}{k_4}\right) \cdot (k_1 + k_2)$

$= -1.5T + 5G_0 - 5k_1\left(\frac{k_3 Y - M_0}{k_4}\right) - 5k_2\left(\frac{k_3 Y - M_0}{k_4}\right)$

$= -1.5T + 5G_0 - \frac{5k_1 k_3 Y + 5k_1 M_0}{k_4} - \frac{5k_2 k_3 Y + 5k_2 M_0}{k_4}$

$= -1.5T + 5G_0 - \frac{5k_1 M_0}{k_4} + \frac{5k_2 M_0}{k_4} - \frac{5k_2 k_3 Y}{k_4} + \frac{5k_2 M_0}{k_4}$

$Y + \frac{5k_1 k_3 Y}{k_4} + \frac{5k_2 k_3 Y}{k_4} = -1.5T + 5G_0 + \frac{5k_1 M_0}{k_4} + \frac{5k_2 M_0}{k_4}$

$Y\left(1 + \frac{5k_1 k_3}{k_4} + \frac{5k_2 k_3}{k_4}\right) = -1.5T + 5G_0 + \frac{5k_1 M_0}{k_4} + \frac{5k_2 M_0}{k_4}$

$$Y^* = \frac{-1.5T + 5G_0 + \frac{5k_1 M_0}{k_4} + \frac{5k_2 M_0}{k_4}}{1 + \frac{5k_1 5k_3}{k_4} + \frac{5k_2 k_3}{k_4}}$$

$$Y^* = \frac{\frac{-1.5Tk_4 + 5G_0 k_4 + 5k_1 M_0 + 5k_2 M_0}{k_4}}{\frac{k_4 + 5k_1 k_3 + 5k_2 k_3}{k_4}}$$

$$Y^* = \frac{-1.5Tk_4 + 5G_0 k_4 + 5k_1 M_0 + 5k_2 M_0}{k_4 + 5k_1 k_3 + 5k_2 k_3}$$

c. Has fiscal policy, for example, the change in government purchase, become *more effective* if both k_1 and k_2 are getting bigger? Show your numerical result, and explain your result with some economic intuitions. (Hint: how do we measure the effectiveness of a policy?)

$$Y^* = \frac{-1.5Tk_4 + 5G_0 k_4 + 5k_1 M_0 + 5k_2 M_0}{k_4 + 5k_1 k_3 + 5k_2 k_3}$$

$$\frac{\Delta Y^*}{\Delta G_0} = \frac{5k_4}{k_4 + 5k_1 k_3 + 5k_2 k_3}$$

= If k_1 and k_2 is getting larger, Y^* will fall.
So, No it hasn't fiscal policy became more efficitive

4. Let the demand function be $P = 14 - 3Q$ and the supply function be $P = 4 + 2Q$. Suppose that the government imposes tax by $t per unit of output. This tax is assumed to impose on consumer. Answer the following questions.

   a. Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero.
   b. State the condition that links between consumer's and producer's price
   c. Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)
   d. Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?
   e. Find the expression of the revenue that the government can collect from the market under the equilibrium.
   f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?

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   a. Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero.
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a)

$$P^d = 14 - 3Q^d \qquad P^s = 4 + 2Q^s$$

$$\frac{P^d - 14}{-3} = \frac{P^s - 4}{2}$$

$$2P^d - 28 = -3P^s + 12$$

$$2P^d + 3P^s = 40$$

$$5P = 40$$

$$P = 8$$

$$P = 8; \quad P^d = 14 - 3Q^d$$

$$8 = 14 - 3Q^d$$

$$Q = 2$$

equilibrium = $P = 8$, $Q = 2$ #

b)

$$P^d = 14 - 3Q^d \rightarrow Q^d = \frac{14 - P}{3} \quad ; 0 \leq P \leq 14$$

$$P^s = 4 + 2Q^s \rightarrow Q^s = \frac{P - 4}{2} \quad ; P \geq 4$$

c)

$$Q^d = Q^s$$

$$P^s + t = P^d$$

$$4 + 2Q + t = 14 - 3Q$$

$$5Q + t = 10$$

$$5Q = 10 - t$$

$$Q = 2 - \frac{t}{5}$$

$$P^d = 14 - 3\left(2 - \frac{t}{5}\right) = 8 + \frac{3t}{5}$$

$$P^s = 4 + 2\left(2 - \frac{t}{5}\right) = 8 - \frac{2}{5}t$$

d) Tax revenue = $2t - \frac{t^2}{5}$

Tax burden on Producer = $\frac{2t}{5} \times 2 - \frac{t}{5} = \frac{4t}{5} - \frac{t^2}{25}$ #

Tax burden on consumer $\frac{3t}{5} \times 2 - \frac{t}{5} = \frac{6t}{5} - \frac{3t^2}{25}$ #

The consumer has higher tax burden

t Q

e) tax revenue = t per unit × unit output sold

$$Q^* = 2 - \frac{t}{5}$$

$$(t)\left(2 - \frac{t}{5}\right)$$

$$\text{tax revenue} = 2t - \frac{t^2}{5}$$

f) appropriate level = $T(t) = 2t - \dfrac{t^2}{5}$

$$T(t) = 2t - \frac{t^2}{5}$$

$$T'(t) = 2 - \frac{2t}{5}$$

$$\frac{2t}{5} = 2$$

$$t = 5 \ \#$$

e. Find the expression of the revenue that the government can collect from the market under the equilibrium.

f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, “t”, that it should impose to the market?