EE320 PRACTICE PROBLEM SET

Semester 2/2021

BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

Introductory Mathematical Economics

PRACTICE PROBLEM SET 1
Function and equilibrium analysis

Question 1 (Easy): Simple break-even analysis

1.1 Product A has a fixed cost at 5,000 Baht and variable cost for 7.5 baht per unit and price for 10 Baht per unit.

a) Construct the profit function of the producer of product A.

b) Determine break-even quantity and illustrate by the graph

1.2 Let TC = 2,000 + 20Q and price per unit is 40 Baht per unit. Determine the following:

a) Total revenue

b) Break-even quantity

c) If the company requires the minimum profit of 2,000 Baht, how many products company should produce?

Question 2 (Moderate): Individual vs market demand

Alex, Zander, and Clark are the only three consumers in the market for oatmeal. They have the following individual demand curves for oatmeal. The market supply curve for oatmeal is also provided to you.

Alex: $P = 20 - Q_d$

Zander: $Q_d = 40 - 2P$

Clark: $Q_d = 20 - 4P$

Supply: $Q_s = \frac{3}{2}P - 12$

Answer the following questions:

a. Derive equation for market demand. Go step-by-step slowly. You can use whichever methods that we discuss in the class.
b. What is the equilibrium price and quantity of oatmeal?
c. Under equilibrium, are there any consumers rationed or excluded from the market?

Question 3 (Easy)

In June, KFC lowers the price of fried chicken from 50 Baht per piece to 30 Baht per piece. Then, KFC can sell more fried chicken from 600 pieces to 1,800 pieces and the sale of the drinks increases from 300 to 1,500 cups.

From the above information, answer the following questions

a. Find "price elasticity" of demand for fried chicken with respect to price of fried chicken.
b. Find "cross-price elasticity" of demand for the drinks with respect to price of fried chicken.
c. Holding other things remain constant, if the fried chicken's price drops to be 25 Baht per piece, would the total revenue from selling fried chicken and drinks increase or not? Explain.

Question 4 (Tedious): Given the following information,

Price	**6**	**4**	**2**
Q_d	0	4	8
Q_s	120	80	40

Answer the following questions:

a. Demand equation for each individual consumer
b. Supply equation for each individual producer
c. Suppose there are 10,000 identical consumers in the market, find the market demand equation.
d. Suppose there are 10,000 identical producers in the market, find the market supply equation.
e. Equilibrium price and quantity (also illustrate by graph)
f. If government provides subsidy (to sellers) for each unit sold, 0.1 Baht per unit, find the total amount of money required for the subsidy program.

Question 5 (moderate): The IS-LM model

Consider the following IS-LM model

$$C = 48 + 0.8Y^*$$
$$I = 98 - br^*$$
$$\frac{M_S}{P} = 250$$
$$L_d = 52 + 0.3Y^* - 150r^*$$

where

C = consumption

Y = income

I = investment

r = interest rate

$\frac{M_S}{P}$ = real money supply

L_d = real money demand

a. Derive IS and LM equation. Graph the equations.
b. Suppose that b = 75, find the equilibrium income (Y*) and equilibrium interest rate (r*)
c. Now suppose instead that b = 0. Discuss the effectiveness of monetary policy.

Question 6 (moderate): Consider the ice cream market in Bangkok. In July, the ice cream market demand and supply curves are given by the following equations where Q is the quantity of ice cream units and P is the price in dollars per unit of ice cream:

Demand: Q = 14000 – 10P

Supply: Q = 2000 + 20P

a) Find the equilibrium price and quantity of ice cream in July.

b) Calculate the price elasticity of demand and supply at the equilibrium price in July. Use the point elasticity formula to compute the values of these two elasticity.

Suppose that the city of Bangkok imposes on producers an excise tax of B 15 per unit of ice cream.

c) Calculate the new equilibrium price and quantity in July for this ice cream market.

Question 7 (moderate)

The demand and supply functions of a two-commodity model are as follows:

$$\begin{aligned} Q_{d1} &= 18 - 3P_1 + P_2 & \quad Q_{d2} &= 12 + P_1 - 2P_2 \\ Q_{s1} &= -2 + 4P_1 & \quad Q_{s2} &= -2 + 3P_2 \end{aligned}$$

Find the equilibrium of the model.

Question 8 (easy)

Let the national-income model be:

$$\begin{aligned} Y &= C + I_0 + G & & \\ C &= a + b(Y - T_0) & \quad & (a > 0, 0 < b < 1) \\ G &= gY & \quad & (0 < g < 1) \end{aligned}$$

(a) Identify the endogenous variables.

(b) Give the economic meaning of the parameter g.

(c) Find the equilibrium national income.

(d) What restriction(s) on the parameters is needed for an economically reasonable solution to exist?

Question 9 (HARD)

A study has shown that there are three groups of iPhone users, namely, *crazy, love-it, and just-live-with-it.* Demand for iPhone of each group can be given by:

crazy: $Q_c = 100 - P$;

Love-it: $P = 50 - Q_L$;

Just-live-with-it: $Q_j = 20 - P$;

where Q_c is the quantity demanded by *c*razy group, Q_L is quantity demanded by *l*ove-it group, and Q_j is the quantity demanded by *j*ust-love-with-it group.

a) Find the domain set of prices that justifies the demand equation for each group of iPhone user. And, rewrite each demand function in a more appropriate way.

b) At what domain set of prices, do all the three types of iPhone users stay active in the market?
c) Find the function for market demand for iPhone. Be precise about what is needed to make your equation justified.

Now, suppose that market supply equation is given by

$p = 4 + 3w + \frac{3}{8}Q.$

d) Find the equilibrium when w = 1/3 where w is wage rate for each unit of labor hired.
e) How much does each type of consumer consume in the equilibrium?
f) What is the likely effect on market equilibrium when wage drops? State your prediction and develop intuition for your result. (Note: Answer to this question could be made in qualitative sense. You don't need to get into algebraic solution with numbers solved.)

Question 10: (Tricky) *Nonlinear* macroeconomics model

Find the equilibrium Y and C from the following:

$$Y = C + I_0 + G_0, \qquad C = 25 + 6Y^{1/2}, \qquad I_0 = 16, \qquad G_0 = 14.$$

Hint: let $Y^{\frac{1}{2}} = A$, $Y = A^2$

Question 11 (tedious)

In a 2-good market equilibrium model, the **inverse** demand functions are given by

$$P_1 = Q_1^{\frac{-2}{3}} Q_2^{\frac{1}{3}}, \quad P_2 = Q_1^{\frac{1}{3}} Q_2^{\frac{-2}{3}}.$$

(a) Find the demand functions $Q_1 = D^1(P_1, P_2)$ and $Q_2 = D^2(P_1, P_2)$.

(b) Suppose that the supply functions are

$$Q_1 = a^{-1}P_1, \qquad Q_2 = P_2.$$

Find the equilibrium prices (P_1^*, P_2^*) and quantities (Q_1^*, Q_2^*) as functions of a.

(Hint: Tricky. It might be useful to apply some logarithm rule)

Question 12 (Easy)

Find the equilibrium solution of the following model:

$$Q_d = 3 - P^2, \qquad Q_s = 6P - 4, \qquad Q_s = Q_d.$$

Question 13 (moderate)

Price support V.S. Price guarantee

a. Suppose that the supply and demand for rice are given by the following equations where Q is the quantity in units of rice and P is the price per unit of rice:

Supply of rice: $Q = \frac{1}{2}P - 3$

Demand for rice: $Q = 27 - \frac{1}{3}P$

What are the equilibrium price and quantity in the rice market without government intervention? Illustrate your answer with a well labeled graph.

b. The government tries to raise rice prices using a *price support program*. It sets the price at $54 per unit of rice, and *commits to buy any leftover rice*. Given this program, how much rice do consumers buy? How much rice does the government buy? What is the cost of the program to the government?

c. Now, suppose the government still wants to keep the price of rice at $54 per unit of rice, but instead of implementing a price support program the government decides to enact a price guarantee program that will *subsidize* the rice producers. Under this scenario, how much rice will consumers buy? What is the cost of the program to the government? Illustrate your answer with a well labeled graph.

d. Given the market and the programs described in this problem answer the following questions and provide a rationale for your answer to each question. Which program will the consumers prefer? Which program will the producers prefer? Which program will the government prefer?

e. Could you think of any reason for the government to prefer the price guarantee program?

Question 14 (Moderate)

*This question doesn't require your prior knowledge in calculus. Use the Quadratic formula

Consider the milk market in the U.S. There are a number of companies selling milk, so that the market is perfectly competitive. Let's look at Kemps, a firm that produces milk. Assume that Kemp's total cost is given by the following equation:

$$TC = q^2 + 8q + 5$$

where q denotes units of milk.

The market price is $50 per unit of milk.

a) Given the above information, how many gallons of milk will Kemps produce in the short run?

b) Given your answer in (a), find the short run profit for Kemps. Show your work.

PRACTICE PROBLEM SET 2
Chapter 4 Matrix algebra and its applications in economics

Question 1 Basic operations on matrix

1.1 Find $A^{-1}B$ when $A = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

1.2 If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 3 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$ Find $(ABC)^T$.

1.3 If $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ Find $A^{-1}B$.

1.4 Let $A = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix}$ Find determinant of A

1.5

If $A = \begin{bmatrix} 3 & -1 \\ -4 & 0 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ -1 & -2 \\ 1 & 1 \end{bmatrix}$ Find $[A^T B]^{-1}$.

1.6) Find adjoint and inverse of the following matrices

(a) $A = \begin{bmatrix} 3 & -3 & 2 \\ 4 & 7 & 1 \\ 0 & 10 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} a & b & c \\ c & b & a \\ b & a & c \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -5 & 1 & 0 & 0 \\ 10 & 8 & 6 & 4 \\ -3 & 7 & 0 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & 2 & 6 & 8 \\ -5 & 7 & 1 & 0 \\ 0 & 0 & 3 & 4 \\ 7 & 1 & 1 & 2 \end{bmatrix}$

Question 2

Consider a simple macroeconomic model.

$C = a + bY_d;\ 0 < b < 1$

$I = I_a + iY;\ 0 < i < 1$

$G = G_0$

$T = T_0 + tY;\ 0 < t < 1$

$R = R_0$

$Y_d = Y - T + R$

where R is the government transfer and G is the government purchase. All the remainings are defined as usual.

a) Determine *all* the endogenous and exogenous variables in the model.

b) State the condition that characterizes the equilibrium of this model.

Simplify the model into a 3-variable system of equations that only includes on Y, C and I. Rewrite the system of equations in 2.3 in the form of matrix.

c) Solve for the solution of Y, C and I. Use the Cramer's rule method.

d) Compare the multipliers of G and R. Which one has a bigger impact? Why?

Question 3

Consider the following simple Keynesian macroeconomic model

$$Y = C + I + G$$
$$C = 200 + 0.8Y$$
$$I = 1000 - 2000R,$$

where the endogenous variables include national income (Y), consumption (C), and investment (I), and the exogenous variables include government spending (G) and the interest rate (R).

(a) Set up this model with a 3×3 matrix of parameters, a 3×1 vector of endogenous variables, and a conformable vector of exogenous variables.

(b) Find the inverse of the 3×3 matrix of parameters.

(c) Evaluate the effect of a \$50 billion decrease in government spending on income. Compare this result to the one obtained in Section 4.1. Does your answer here differ, and if so, can you provide an economic rationale for this difference?

Question 4

Assume that we have three markets in the economy. Each market can be characterized by demand and supply equations as given below.

$$\text{Demand for goods A} : q_A^d = 3 - P_A + P_B \qquad \text{Supply for goods A} : q_A^s = P_A - 2$$
$$\text{Demand for goods B} : q_B^d = 3 - 2P_B + P_C \qquad \text{Supply for goods B} : q_B^s = P_B - 1$$
$$\text{Demand for goods C} : q_C^d = 6 + 2P_A - P_C \qquad \text{Supply for goods C} : q_C^s = 2P_C - 2$$

a. State the equilibrium conditions for this multi-market economy, and write the model in the form of matrix.

b. Solve for the equilibrium solution by using the *Cramer's rule*.

Question 5

Consider a simple global economy model. There are but two countries, namely A and B. Model equations for each country are given below.

$$\begin{aligned} \text{Country A} : C_A &= 150 + 0.4Y_A \\ M_A &= 0.6Y_A \\ I_A &= 250 \end{aligned}$$

$$\begin{aligned} \text{Country B} : C_B &= 100 + 0.5Y_B \\ M_B &= 0.4Y_B \\ I_B &= 200 \end{aligned}$$

where Y is national income, C is consumption, M is import, and I is investment.

4.1) Let "X" be a new variable representing export of a country. That is, when I write X_A, this is referred to the value of export of country A. Based on the information given above, can we find the export function of both country A and country B?

4.2) State all the endogenous and exogenous variables.

4.3) State the equilibrium condition<u>s</u> for the global economy model.

4.4) Simplify the model into a 2-variable system of equations where only Y_A *and* Y_B are included. Then rewrite the simplified model in the matrix form.

4.5) Solve for the equilibrium income using the *inverse matrix method*.

4.6) Under the equilibrium, how much is the *net export* in both countries?

Question 6

Given the following supply and demand functions:

$$Q_D = 100 - 3P$$

$$Q_S = 80 + 2P$$

a) Write the equilibrium condition for this market, and translate the system of equations into matrix notation.

b) Use matrix inversion to solve for the equilibrium quantity and equilibrium price.
c) Suppose that the government subsidizes the consumption of this good by giving the consumer \$5 per unit of the goods consumed. Use Cramer's rule to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

Question 7

Examine for what values of the constants a and b the system of equations

$$ax + y = 3$$
$$x + z = 2$$
$$y + az + bu = 6$$
$$y + u = 1$$

has a unique solution in the unknown x, y, z, and u. Find the unique solution (expressed in terms of a and b).

Question 8

Given the IS equation $0.3Y + 100r - 252 = 0$ and the LM equation $0.25Y - 200r - 176 = 0$. Use matrix inversion to solve for the equilibrium of national income and rate of interest.

Question 9: (*Tinbergen 1952: "On the theory of economic policy"*)

This question guides you to understand an important foundation on the principle of economic policy laid down by Jan Tinbergen. (Tinbergen was the first Noble laureate in economics science. His main contribution is the advancement in developing methodological frameworks for the centrally-planned policy and government interventions analysis.) In his classical manuscript published in 1952, he argues that authority might have multiple economic objectives/targets, given the policy instrument on hands. (Target is generally defined a variable that authority ultimately would like to achieve. Instrument is defined as a variable that authority can control with its high precision, and has certain relationship with the targets.) He claims that *the number of targets should be equal to number of instruments; otherwise, all the targets cannot simultaneously achieved.*

Following the model described in class, consider the following macroeconomic model that takes into account a dynamic feature of

Y = C + I + G + X - M (a)

$C = 0.75Y_d$ (b)

$Y_d = 0.75(Y - T)$ (c)

$T = G$ (d)

$G = G_c + G_I$ (e)

$I = 0.25(Y - Y_{-1}) + 0.1G_I$ (f)

$M = 0.2C + 0.08I + 0.06G_I + 0.03X$ (g)

$N = 0.6Y$ (h)

$B = P_x X - P_M M = 1.05X - 1.07M$ (j)

where Y – GDP, Y_{-1} – GDP of the previous year, C – Private Consumption, I – Private Investment, M – Imports, X – Exports, Y_d – Disposable Income, T – Taxes, G–Government Expenditure, G_c –Public Consumption Expenditures, G_I – Public Investment Expenditures, N – Employment, B – Current Account of the Balance of Payments. All variables, except N , are measured in $ billion. Employment (N) unit of measurement is 1,000 persons.

The first three equations are three standard equations used in Keynesian cross model. The fourth equation (d) suggests that government always use the balanced budget policy. Equation (e) explains about the composition of government spending which can be divided into government consumption and government investment. Equation (f) and (g) represent the investment function and the import function. The assumed form of the investment function is a generalization of the commonly known accelerator-based investment behavior developed by Samuelson. Equation h links that level of employment to the level of real economic activity (Y). Lastly, the equation (j) provides the definition for the balance of payment where economy has only opened up the international trade account.

Suppose the following data is given: $Y_{-1} = 200$ and X = 200. Consider the following questions.

i. Identify the endogenous and exogenous variables of this model.
ii. Suppose that the government wants to target balance of payments and employment. Is the structure of the model suitable for this purpose? Why?
iii. Can government use private investment (I) as a policy instrument? Why?
iv. Can government use only public investment to reach its balance of payments and employment targets? Why?
v. Following (iv), eliminate the irrelevant endogenous variables and express the remaining target variables in terms of the exogenous ones.
vi. Suppose that the government aims at achieving $160 billion surplus in the balance of payments and an employment level of 1,100 by using public consumption and public investment. How much should government spend?

Question 10

The profit functions of two airplane manufacturers, the European firm Airbus and the American firm Boeing, are

$$A = -\frac{1}{2}B + F$$

$$B = -\frac{1}{2}A + G,$$

where A is the profits of Airbus, B is the profits of Boeing, F is a subsidy from the governments of Europe to Airbus, and G is a subsidy from the U.S. government to Boeing.

(a) Express these two equations as a matrix system with A and B as the endogenous variables and F and G as the exogenous variables.

(b) Determine the solution if each government provides a subsidy of \$100 million.

(c) What happens to the profits of each firm if the European governments increase their subsidy to \$200 million, while the U.S. government keeps its subsidy equal to \$100 million?

PRACTICE PROBLEM SET 3

Differential Calculus and its applications

Question 1. Differentiate the following functions:

a) $y = \frac{x^2-9x+20}{x-5}$

b) $y = \sqrt{x + \sqrt{x}}$

c) $y = ln(e^x + x)$

d) $y = x^4e^x$

Question 2.

Use the chain rule to find the derivative, $f'(x)$, of each function.

(a) $f(x) = (x + 1)^3 + (x^2 - 2x)^2 - 5$

(b) $f(x) = (2x + 4)^{99}$

(c) $f(x) = (5x^2 + 10x + 3)^{20}$

(d) $f(x) = [e^x]^{ab}$

(e) $f(x) = [e^{x^a}]^b$

(f) $f(x) = (e^{a+bx+cx^2})^{10}$

Question 3.

Use the quotient rule to differentiate each function.

(a) $f(x) = \frac{(2x + 7)}{(x^2 - 1)}$

(b) $f(x) = \frac{(bx^3 + cx^2 + x - 4)}{x}$

(c) $f(x) = \frac{e^{2x}}{x^2}$

(d) $f(x) = \frac{(3x + 2)^2}{x}$

Question 4.

Use the rules for differentiating logarithms to find the derivative of each function.

(a) $f(x) = x^{-4} + \ln ax$

(b) $f(x) = 4x^3 \ln x^2$

(c) $f(x) = \ln x - \ln(1 + x)$

(d) $f(x) = \ln\left(\dfrac{2x^2}{5x}\right)$

(e) $f(x) = \log_2(6x)$

Question 5.

Consider a total cost function $f(x)$ and its average cost function $A(x) = f(x)/x$.

(a) Using the quotient rule, take the derivative of the average cost function $A'(x)$ to obtain a relationship between average cost and marginal cost, $f'(x)$.

(b) When $A'(x) < 0$, is marginal cost greater or less than average cost? Is the average cost function decreasing or increasing in this scenario?

(c) When $A'(x) > 0$, is marginal cost greater or less than average cost? How does this affect the slope of the average cost function?

(d) If marginal cost equals average cost, what does this suggest about the slope of $A'(x)$?

Question 6. Find the marginal and the average functions for each of the following total functions.

a) $TC = 2Q^2 + 5Q + 11$

b) $\pi = Q^2 - 10Q + 68$

Question 7. Given the total-product function:

$$Q = 3L + 2L^2 - L^3$$

a) Find the average product (AP) function.

b) Find the marginal product (MP) function.

c) Determine the slopes of the AP and MP functions. What can you conclude about their relative slopes?

Question 8. A monopolist faces the demand curve given by $P = kQ^{-b},\ \ k, b > 0$. Answer the following questions.

a. Find the absolute value of elasticity of demand. Does the value decrease as the quantity of output increases? Interpret your result.
b. Find the expression for the revenue function
c. What is the average revenue function? Describe the property of average revenue as the quantity of output increases.
d. Derive the marginal revenue function. How does the value of "b" determine the property of marginal revenue function?
e. Based on the functional form of market demand equation, prove/disprove the following argument. The statement is *"Raising up the price always increases the revenue of firm, since an increase in price would always make-up with the loss in the quantity sold."* (Hint: your proof has to make uses some mathematical results regarding to the marginal revenue function.)

Question 9. Let the production function be $Q = -3(7 - L)^5 + k$, where k is a very large value constant. determine the level (interval) of L that the production function exhibits the law of diminishing returns.

Question 10. Suppose that the consumption function can be given by, $C = 5\left(\frac{2\sqrt{Y^3}+3}{Y+10}\right)$, find the value of MPC and MPS when Y = 100

Question 11. Suppose that total output of a firm can be given by $Q = 10\frac{L^2}{\sqrt{L^2+19}}$, where is L is the number of workers hired. Answer the following questions

a. Find the marginal product of labor (worker).
b. Suppose this firm can only charge for a constant fixed price equal to "P". Find the expression for the revenue function (TR) in term of L.
c. Using the function derived above, Find the marginal revenue product of labor or $\frac{dTR}{dL}$. (Hint: you may just treat "P" as a constant term in your equation.)

Continue with the same production function given above, but replace the assumption that the firm can only charge for a constant fixed price equal to "P" with that the firm can charge price based on the quantity of output that it sells. Assume the the pricing equation is given by $P = \frac{900}{Q+9}$. Consider the following questions.

d. Find the express of the revenue function (TR) in term of L.
e. Applying the chain rule and derive the expression of marginal revenue product of labor or $\frac{dTR}{dL}$. Then find the value of marginal revenue product of labor when L = 9.

Question 11.

11.1) Find the elasticity of consumption with respect to income when the consumption function is given by, $C = 10 + \frac{5}{8}Y - \frac{1}{2}\sqrt{Y}$ and Y = 16.

11.2)

A person's productivity at a particular job may change with experience. Consider the following model of effective labor input, L,

$$L = 10(1 - e^{-0.1t}) + 2,$$

where L is a measure of the "effective labor input" of a particular worker and t is the worker's tenure (that is, years spent in a particular job). Use differentiation to determine the marginal change in the effective labor input with respect to time on the job, t. Provide a brief economic interpretation of the differences in these values of dL/dt.

11.3)

Assume that Country X has no domestic production of widgets. Its demand for widgets, therefore, is based exclusively on imports. Consider the import demand function

$$Q_M = 20 - 0.5P_M,$$

where P_M is the price of imports.

(a) At what quantity does this import demand curve exhibit unit elasticity, that is, $\epsilon_{q,p} = -1$ (or $|\epsilon_{q,p}| = 1$)? If $Q_M = 8$, is import demand elastic or inelastic? what if the quantity demanded rises to 11?

(b) Now assume that the import demand function is written as a log-linear specification, $\ln(Q) = 20 - 0.5\ln(P)$. What is the elasticity of this function, and how would you characterize it (that is, elastic, inelastic, unit elastic)? What happens to the elasticity if the quantity demanded changes?

11.4)

Consider the following function, which links the level of consumption of a good, x, to the utility it provides:

$$U(x) = -\left(\frac{1}{\alpha}\right)e^{-\alpha x}.$$

Find the coefficient of relative risk aversion

$$\gamma = -\frac{U''(x) \cdot x}{U'(x)},$$

which determines the function's degree of risk aversion. Find the coefficient of absolute risk aversion

$$\theta = -\frac{U''(x)}{U'(x)}$$

for this utility function.

11.5)

Using the Taylor series expansion formula, find the linear and quadratic approximations to each function, expanded around the point $x = 2$. Evaluate the Taylor series expansions at $\Delta x = 0.1$, and determine the deviation of the approximations from each function's actual value.

(a) $f(x) = 3x^2 - 5x + 1$

(b) $f(x) = \ln(2x)$

(c) $f(x) = e^{3x}$

Question 12. Suppose that market demand is $p = 300 - q^2$

a. Determine the point elasticity of demand when q = 5.

b. For q = 5, is demand elastic, inelastic, or does it have unit elasticity?

c. For what value of q does demand have unit elasticity?

Question 13. Suppose the average total cost function is $ATC = \frac{10}{q^2+2q} + \frac{q}{q+1} + \frac{1000}{q}$.

a. What is the fixed cost?

b. What is the variable cost? What is the average variable cost?

c. Find the marginal cost function.

Practice problem set 4
Single-variable optimization

Question 1 Total product function

Suppose that $TP(L) = 120L^2 - L^3$, where TP is the total output. Answer the following questions

a. Find the expression for AP and MP.
b. Find the value of L that maximizes AP, MP and TP, respectively.
c. Define the domain set of L that justifies the production function.

Question 2

Let the total cost function be:

$$TC(Q) = 2Q^2 - 8Q + 10.$$

a) Determine whether $TC(Q)$ is a convex or concave function.

b) Find the quantity Q^* that minimizes the total cost.

c) Verify that $TC(Q^*)$ is the lowest cost by using the second derivative test.

Question 3

Let the demand function be given by $Q = 60 - 2P$ where P is the unit price and Q is the amount of quantity. Assume that the production function of a firm is given by $Q = 5\sqrt{L}$ where L is the number of workers hired. Consider the following questions.

a. Suppose that wage is \$50 per each worker hired. This is fixed, regardless to how many workers hired by a firm. Write down the equation that summarizes the total cost of hiring labor by "L" workers.
b. Normally, our cost function is written in terms of Q. Based on "a", can you find a way to represent the total cost equation in the terms of Q, rather than L?
c. Use the cost function in "b" and solve for the level of profit-maximizing output/price. Verify your answer
d. At the level of profit-maximizing output, how many workers should the firm hire?

<u>Question 4</u> *Deriving the market supply*

Suppose the cost function of a representative firm can be given by: $C(Q) = 3Q^2 + 5Q + 75$ where Q is the level of output produced. Answer the following question

a. Derive the expression for MC, AVC, and ATC.
b. Find the level of Q that results in the lowest total cost.
c. Find the supply equation of the representative firm. Make sure you specify the range of price that justifies your equation as the one representing the supply equation.
d. If there are 60 identical firms in the market, derive the market supply curve.

<u>Question 5</u>

Given the following function

$$f(x) = 2x^3 + 8x^2 - 32x - 50$$

a. Find the critical value(s) of x and the corresponding stationary value(s) of *f(x)*.

b. Evaluate whether the stationary value(s) found in part a) are relative maxima or minima or inflection points by using *the first-derivative test*.

Question 6

A competitive firm receives a price p for each unit of its output and pays a price w for each unit of its only variable input. It also incurs set up costs of F. Its output from using x units of variable input is $f(x) = \sqrt{x}$. Determine the firm's revenue, cost, and profit functions.

Question 7

The price a firm obtains for a commodity varies with demand Q according to the formula $P(Q) = 18 - 0.006Q$. Total cost is $C(Q) = 0.004Q^2 + 4Q + 4500$.

a. Find the firm's profit and the value Q which maximizes profit.

b. Find a formula for the elasticity of $P(Q)$ w.r.t. Q and find the particular value Q^* of Q at which the elasticity is equal to -1.

c. Show that the marginal revenue is 0 at Q^*.

Question 8 Monopoly and Subsidy program

A monopolist firm faces the market demand equation given by $P = 150 - 0.5Q$ and operates under a technology with cost function given by $TC = 100 + 3Q + 7Q^2$. Consider the following questions

a. By using the derivative method, find the level of profit-maximizing output and price. Verify that your answer is the correction solution that results in maximized profit.

Continue with all the information given above, but now add another assumption to the questions. That is, we now assume that government subsidizes the monopolist for $3 for each unit of output.

b. Write the cost function of the monopolist when subsidization is taken into account.
c. Find the level of profit-maximizing output and price under the subsidy program.

Question 9

A monopolist is operating under a cost function given by,

$$TC = 2Q^3 - 12Q^2 + 93Q + 50,$$

where Q is the quantity of output.

Suppose that the market demand for the goods produced by this monopolist is given by: $P = 81 - Q^2$, where P is the price per unit of output.

Consider the following problems.

a. Determine the *absolute* value of the price elasticity of demand when P = $54.
b. Without solving for the optimization, determine the revenue-maximizing level of output.
c. Derive the profit function and solve for the profit-maximizing level of output. Verify your answer by using the second-order derivative.
d. Would the monopolist change the level of output if government taxes the monopolist based on the profit level that he earns?

Question 10. Given firm i, a perfect competition in a product market, with a short-run total cost function of $STC_i = Q_i^3 - 8Q_i^2 + 20Q_i + 40$ where Q_i is the output of firm i (in hundreds of units per day).

The market demand and supply schedules for the product are:

$Q_i^d = 600 - 20P$ and $Q_i^S = -150 + 30P$

where ${Q_i}^d$ and ${Q_i}^S$ are the quantities demanded and supplied in the market (in hundreds of thousands of units per day) and P is the market price (in dollars per unit). Consider the following problems.

a. What is the market equilibrium price?
b. Find the profit-maximizing level of output for firm i. Check the second order condition.
c. Calculate the level of profits (losses).
d. Given the level of economic profits, discuss the long-run adjustment in the market and for this firm. (Hint: your answer to the question could be only stated in a narrative way. You don't need solve out any numbers; provide economic intuition to support your claim.)

Question 11. A sugar refiner is located in an isolated geographic area. It is a price-taker in the output market. The price facing the firm equals 9, and its total revenue is TR = 9Q, where Q is the rate of output. The refiner's production function is $Q = 3L^{\frac{1}{3}}$, where L is the amount of labor employed. Because of its isolated location, the amount of labor the firm employs have an effect on the level of compensation paid. The firm's compensation function is $wL = 6L^{\frac{4}{3}}$, where w equals wages. The refiner's profit function is

$$\pi(L) = TR\big(Q(L)\big) - wL.$$

a) Set up the first-order condition, and determine the optimal amount of labor the refiner should employ in order to maximize profits.
b) Use the second-order condition to confirm that this value represents a maximum. What is the economic interpretation of the second-order condition in this example?

Question 12. The compensation firms pay workers typically includes benefits as well as salary. The cost of one particular benefit, health insurance, has been rising rapidly. Consider a firm that uses only labor as its input and has the production function

$$f(L) = \alpha L^{\beta}$$

where L is the number of workers employed, $0 < \alpha$ <1, and $0 < \beta < 1$. The profits of this firm, π,equal

$$\pi(L) = p * f(L) - (w + b)L$$

where p is the price of the firm's product, w is the wage paid to a worker, and b is the cost of providing a worker with other fringe benefits.

(a) Find the number of workers the firm will demand in order to maximize its profits. Assume that the firm takes as given p, w, and b. How does the optimal number of workers demanded vary with the value of b?

(b) Show that your solution represents the maximization of profits as opposed to the minimization of profits.

Question 13. A monopolist operates with a cost function given by, $C(q) = aq^2 + bq$, and faces with the market demand given by, $P = -cQ + d$. Suppose that the government imposes a unit tax on each unit of firm's production. The tax rate is set $ t per each unit. Calculate the level of unit tax that maximizes the government revenue.

Practice problem set 5
Multivariate calculus: *basis derivative and applications*

Question 1: Determine $Z_x, and\ Z_y$

a. $Z = (x^2y+2)^2$
b. $Z = x^{1/2}y^{1/3}$
c. $Z = x\,\mathrm{y} - ln(xy)$

Question 2:

Consider a function $Z = f(x,y) = x^3 + 3x^2y + 6xy^2 - y^3$

a. Evaluate the gradient matrix when x = 0 and y = 2
b. Derive the Hessian matrix.
c. Evaluate the value of the Hessian matrix where x = 2 and y =3

Question 3:

a. Given that $Z = \frac{x^3-y^3}{x^2y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

b. Given that $Z = \frac{x-y}{x+y}$, find $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y}$ =?

c. If z = $2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (c.1a) find ∂z/∂s and ∂z/∂r ; (c.2) evaluate when r = 1 and s = 0

d. For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

e. Given that $ln\,(x+y+z) + xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

Question 4

Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2.$$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

a. What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative
b. Is the product X considered an inferior product?
c. What is the level of quantity demanded if $P_x = 10, P_y = 25 \ and \ I = 10$?
d. Calculate the own-price elasticity of demand, and evaluate the value when $P_x = 10, P_y = 25 \ and \ I = 10$.
e. Calculate the cross-price elasticity of demand when $P_x = 10, P_y = 25 \ and \ I = 10$.
f. Calculate income elasticity of demand when $P_x = 10, P_y = 25 \ and \ I = 10$. Is the product a necessary or luxurious product?

Question 5: Consider a simple market model where

Demand: $q = -p + \sqrt{I}$
Supply: $q = 3p - w^2$

where q is quantity of output, p is price, I is the level of income, and w is the price of factor input

a) Derive the solution of all the endogenous variables.

b) Use the partial derivative to show that, under the equilibrium, an increase in W causes a decrease in quantity, but an increase in price.

c) Predict the impact of an increase in income on equilibrium quantity and price.

d) Suppose initial wage and income are \$2 and \$4 respectively. Find the approximate changes in equilibrium price and equilibrium quantity if income and wage both equally increase by \$1.

Question 6:

6.1) Find the total differentials for the following functions:

a. $z = 4x^3 - 13xy - 6y^5$
b. $z = (2x^2 - y)(3x - 4y^3)$
c. $z = 8x^{\frac{1}{2}}y^{\frac{1}{4}}w^{\frac{1}{4}}$

6.2)

For each function, use the total differential to approximate the change in y due to the given changes in x and z.

(a) $y = x^2 + 4x - z^2 - 2xz$, where $x = 1, z = 4, \Delta x = 2, \Delta z = -2$
(b) $y = e^{x^2+3z}$, where $x = 1, z = 2, \Delta x = 2, \Delta z = -2$
(c) $y = \ln x^3 - 4z + 2xz$, where $x = 1, z = 2, \Delta x = 2, \Delta z = 4$
(d) $y = x^2 + e^{z/2}$ for $x = 2, z = 2, \Delta x = 1$, and $\Delta z = -1$.

Question 7: Suppose that Mr. A's utility depends on two commodities: x_1 and x_2. His utility function is given by

$$U = 25x_1 + 27x_2 - 3x_1^2 - 7x_1x_2 - 4x_2^2$$

(a) Determine the marginal utility function of each commodity.

(b) Find the marginal rate of substitution (MRS) between the two commodities when $x_1 = 2$ and $x_2 = 1$.

Question 8: Given the equation for the production isoquant

$$18[0.2K^{-0.4} + 0.8L^{0.4}]^{-2.5} = 1936$$

Use the implicit function rule to find the marginal rate of technical substitution (MRTS) of L for K.

Question 9: The demand for money, *M*, in the United States for the period 1929-1952 has been estimated as

$$M = 0.14Y + 76.03(r - 2)^{-0.84} \quad , \quad (r > 2)$$

where Y is the annual national income, and r is the interest rate measured in percent per year. Find $\frac{\partial M}{\partial Y}$ and $\frac{\partial M}{\partial r}$ and discuss their signs.

Question 10:

Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using L units of labor. Assume that $f'(L) > 0$ and $f''(L) < 0$, so f is strictly increasing and strictly concave. If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$. Show that

Question 11:

Let the demand for a commodity be

$$Q_d = D(P, Y_0, t_0), \qquad \frac{\partial D}{\partial P} < 0; \frac{\partial D}{\partial Y_0} > 0; \frac{\partial D}{\partial t_0} < 0$$

$$Q_s = S(P, T_0), \qquad \frac{\partial S}{\partial P} > 0; \frac{\partial S}{\partial T_0} < 0$$

where P is the price, Y_0 is the consumer's income, t_0 is the taste for the commodity and T_0 is the tax on the commodity. All derivatives are continuous. Use implicit function rule to find $\frac{\partial P^*}{\partial t_0}, \frac{\partial P^*}{\partial T_0}, \frac{\partial Q^*}{\partial Y_0}$, and $\frac{\partial Q^*}{\partial T_0}$. Discuss their economic implications.

Question 12:

A joint-cost function is defined implicitly by the equation

$$c + \sqrt{c} = 12 + q_A\sqrt{9 + q^2{}_B}$$

where c is denoted as the total cost for producing q_A and q_B units of product A and B, respectively.

a) If $q_A = 6$ and $q_B = 4$, find the corresponding value of c.

b) Determine the marginal costs with respect to q_A and q_B, when $q_A = 6$ and $q_B = 4$.

Question 13:

Suppose a production function is given by $P = \frac{kl}{k+l}$

a. Determine the marginal productivity functions.
b. Show that when $k = l$, the marginal productivities are equal.

Question 14:

Suppose the demand equations for related products A and B are

$$q_A = e^{-(p_A+p_B)} \quad \text{and} \quad q_B = \frac{16}{p_A^2\, p_B^2}$$

where q_A and q_B are the number of units of A and B demanded when the unit prices (in thousands of dollars) are p_A and p_B, respectively.

a. Classify A and B as competitive, complementary, or neither.
b. Calculate elasticity of demand for good A and good B, respectively. (Get me all whatsoever you can do for the elasticities.)
c. If the unit prices of A and B are \$1000 and \$2000, respectively, estimate the change in the demand for A when the price of B is decreased by \$20 and the price of A is held constant.

Question 15:

Suppose the cost c of producing q_A units of product A and q_B units of product B is given by $c = (3q_A^2 + q_B^3 + 4)^{1/3}$. And the coupled demand functions for the products are given by

$$q_A = 10 - p_A + p_B^2 \qquad \text{and} \qquad q_B = 20 + p_A - 11p_B$$

Use a chain rule to evaluate $\frac{\partial c}{\partial p_A}$ and $\frac{\partial c}{\partial p_B}$ when $p_A = 25$ and $p_B = 4$.

Practice problem set 6

Multivariate calculus: Unconstrained optimization

Question 1:

Define f(x,y) for all (x,y) by

$$f\,(x,y) = e^{x+y} + e^{x-y} - \frac{3}{2}x - \frac{1}{2}y$$

a. Derive the Hessian matrix of $f\,(x,y)$.
b. Show that f(x,y) is monotonically convex function. That's, the function is convex for the entire domain sets defining f(x,y).
c. Find the *global* extrema of f(x,y)

Question 2:

Consider a function $f\,(x,y) = x^2 - y^2 - xy - x^3$

a. Find and classify the stationary points of $f\,(x,y)$
b. Find the domain set of $f\,(x,y)$ where $f\,(x,y)$ is concave, and find the largest value of $f\,(x,y)$ in that domain set.

Question 3: Suppose that there are two firms in the industry, and they are competing in quantities. The amount of the commodity sold by firm i is q_i, $i = 1, 2$. The market demand function is given by $P = 50 - 3q$, where $q = q_1 + q_2$. The cost functions for each firm is given by $TC_i = 25 + 5q_i, i = 1,2$.

a. Find the profit-maximizing quantity for each firm, and determine each firm's profit level.
b. Suppose that both firms merge. Compute the new profit-maximizing quantity and the new profit of the merged firm. Do firms have incentive to merge, and why?

Question 4: Given the production function

$$\boldsymbol{Q = f(K, L) = 8K^{1/2}L^{1/4}}$$

Suppose that the price per unit of Q is \$$P$, and the per unit input prices for K and L are \$$r$ and \$$w$, respectively. ($P$, w, and r are positive constants.)

a. Solve for the values and that maximizes the profit. Verify that the second-order sufficient condition is met.

b. Find the comparative statics derivatives $\frac{\partial K^*}{\partial r}$ and $\frac{\partial L^*}{\partial P}$, evaluate the signs, and interpret their economic meanings.

Question 5: Each of two firms A and B produces its own brand of a commodity, such as mineral water, in amounts denoted by x and y, and these are sold at prices p and q unit, respectively. Each firm determines its own price and produces exactly as much as is demanded. The demands for the two brands are given by

$$x = 29 - 5p + 4q$$

$$y = 16 + 4p - 6q$$

Firm A has total costs $5 + x$, whereas firm B has total costs $3 + 2y$. (Assume that the functions to be maximized have maxima, and at positive prices.)

a. Initially, the two firms collude in order to maximize their combined profits, as one monopolist would. Find the prices (p, q), the production levels (x, y), and the profits of firms A and B.

b. Then, an antitrust authority prohibits collusion, so each producer maximizes its own profit, taking the other's price as given.
   - If q is fixed, how will A choose p? (Find p as a function $p = p_A(q)$.)
   - If p is fixed, how will B choose q? (Find q as a function $q = q_B(p)$.)

c. Under the assumptions in part b), what constant equilibrium prices are possible? What are the production levels and profits in this case?

Question 6: A firm produces two different kinds A and B of a commodity. The daily cost of producing Q_1 units of A and Q_2 units of B is:

$$C(Q_1, Q_2) = 0.1(Q_1^2 + Q_1Q_2 + Q_2^2)\,.$$

Suppose that the firm sells all its output at a price per unit $P_1 = 120$ for A and $P_2 = 90$ for B.

a. Find the daily production levels that maximize profit.
b. What prices (P_1) per unit of A would imply that the optimal daily production level for A is 400 units?

Question 7:

Let the total cost function depend on goods x, y and z ;

Determine the level of x , y and z which minimize total cost and determine the minimum total cost.

Question 8:

A monopolist produces two products, A, and B. The joint-cost function is $\boldsymbol{C} = \mathbf{5000 + 5\,} \boldsymbol{q_A} \mathbf{+ 3\,} \boldsymbol{q_B}$, where c is the total cost of producing q_A units of A and q_B units of B. The demand functions for these products are given by $p_A = 205 - 2q_A - q_B$ and $p_B = 153 - q_A - q_B$, where p_A and p_B are the prices of A and B, respectively. Consider the following problem.

a. Determine the profit-maximizing level output for both products.
b. How much should the monopolist set the price of the two products?

Question 9:

A manufacturer produces products A and B for which the average costs of production are constant at 3 and 5 (dollars per unit), respectively. The quantities q_A, q_B of A and B that can be sold each week are given by the joint-demand functions $q_A = 10 - p_A + p_B$ and $q_B = 12 + p_A - 3p_B$, p_A and p_B are the prices (in dollars per unit) of A and B, respectively. Determine the prices of A and B at which the manufacturer can maximize profit.

Question 10:

Consider a market with 2 firms. Each firm sells an identical product, facing the same market demand equation given by $p = 10 - Q$. For the first firm, denoted by firm 1, the cost function is given by $C = c_1 Q_1$. For the second firm, the cost function is given by $C = c_1 {Q_2}^2$. Consider the following problem.

a. Determine the level of output that each firm will choose to produce under the Cournot equilibrium.
b. State the requirement for c_1 in order to ensure that both firms stay active in the equilibrium.
c. Do they share the same market size under the equilibrium? Explain your result with some economic intuitions.
d. Determine the level of output that each firm will produce under the collusion.

Question 11:

The profit function of a firm is $\pi(x,y) = px + qy - \alpha x^2 - \beta y^2$, where p and q are the prices per unit and $\alpha x^2 + \beta y^2$ are the costs of producing x units of the first good and y units of the other. The constants are all positive.

a. Find the values of x and y that maximize profits. Denote them by x^* and y^*. Verify that the second-order conditions are satisfied.
b. Define $\pi^*(p,q) = \pi(x^*, y^*)$) as the optimal profit function. The function generates the level of maximum profit that firm attain under different combination of profit-maximizing output bundles. Verify that $\partial\pi^*(p,q)/\partial p = x^*$ and $\partial\pi^*(p,q)/\partial q = y^*$. Give these results economic interpretations.

c. Show that $\pi(p,q)$ is convex in p and q. That is, you show that Hessian matrix of the optimal profit function is positive definiteness.

Practice problem set 7
Constrained optimization problem

Question 1:

The profit obtained by a firm from producing and selling x and y units of two brands of a commodity is given by

$$P(x, y) = -0.1x^2 - 0.2xy - 0.2y^2 + 47x + 48y - 600$$

a. Find the production levels that maximize profits.
b. A key raw material is rationed so that total production must be restricted to 200 units. Find the production levels that now maximize profits.

Question 2

Suppose that Mr. Bean's utility depends on two commodities: x_1 and x_2. His utility function is given by:

$$U = x_1x_2 + 2x_1 + x_2 .$$

Suppose that Mr. Bean's income is $76, and the per-unit prices for the commodities x_1 and x_2 are $2 and $3, respectively.

a. Determine the values x_1^* and x_2^* that maximize Mr. Bean's utility, given that Mr.Bean spends all of his income on these two commodities.
b. Show the second-order sufficient condition for the constrained utility maximization.

Question 3:

Suppose that a monopolistic firm sells a single product in three separate markets (say, three different countries), and therefore faces three different demand functions:

$$P_1 = 32 - 1.5Q_1$$
$$P_2 = 68 - 3Q_2$$
$$P_3 = 40 - 2Q_3$$

The total cost function is given by:

$$TC = 60 + 20Q \text{ where } Q = Q_1 + Q_2 + Q_3$$

a. Suppose that there is a shortage of a key raw material so that the total production must be restricted to $\underline{Q} = 12$units. Find the profit-maximizing output levels of Q_1, Q_2, and Q_3, and determine the maximum profit.
b. Use the bordered Hessian matrix to verify that the second-order sufficient condition is met.

Question 4

A firm produces and sells two commodities. By selling x tons of the first commodity the firm gets a price per ton given by $p = 96 - 4x$. By selling y tons of the other commodity the price per ton is given by $q = 84 - 2y$.

The cost of producing and selling x tons of the first commodity and y tons of the second is given by:

$$C(x, y) = 2x^2 + 2xy + y^2 .$$

a. Show that the firm's profit function is: $P(x, y) = -6x^2 - 3y^2 - 2xy + 96x + 84y$
b. Compute the first-order partial derivatives of profit (P) with respect to x and y, and find its only stationary point.
c. Suppose the production causes pollution, and that the authorities for this reason require the firm the produce only 11 tons in total. Solve the firm's maximization problem in this case. Verify that the production restrictions do reduce the maximum possible value of $P(x, y)$.

Question 5:

Given the utility maximization problem

$$Max_{x,y}U(x,y) = \sqrt{x} + y$$

$$\text{subject to } x + 4y = 100$$

a. Find the quantities demanded of the two goods using the Lagrange method. Determine the maximum utility level and the value of the Lagrange multiplier.
b. Suppose income increases from 100 to 101. What is the exact increase in the optimal value of U(x, y)? Compare with the value found in (a) for the Lagrange multiplier.
c. Suppose we change the budget constraint to $px + qy = m$, but keep the same utility function. Derive the quantities demanded of the two goods if $m > \frac{q^2}{4p}$.

Question 6:

A firm has an order of 10,000 units of its product and has two plants at which to manufacture these units. Let q_1 be the number of units to be produced at the first plant and q_2 denote the number to be manufactured at the second plant. Suppose that the cost function is given by

$$C = 48{q_1}^3 + 3\,{q_2}^3 + 25{,}000.$$

a. Use the method of Lagrange multipliers to determine how many units should be produced at each plant to minimize this cost function.
b. Confirm your result by checking the second-order condition.

Question 7: *Consumption-Leisure choice*

An individual has a Cobb-Douglas utility function $U\,(M,L) = AM^{a}L^{b}$, where m is income and l is leisure. Here A, a, and b are positive constants, with $a + b \leq 1$. A total of T_0 hours are allocated between work (W) and leisure (L), so that $W + L = T_0$. If the hourly wage is w, then $M = \underline{w}W$, and the individual's problem is

$$max_{M,L} \;\; AM^{a}L^{b} \; st. \;\; M/\underline{w} \; + \; L \; = \; T_0$$

a. Solve for optimal leisure that this individual will choose.
b. Confirm your result by checking the second-order condition.
c. How does the change in hourly wage affect the decision to work?

Question 8: *Intertemporal consumption problem*

Consider a consumer who lives in two periods of time. In the first period, the consumer earns income Y_0. This income can be allocated for consumption in the current period (C_0) or saving (S) for the future use. The amount of saving that the consumer chooses can be used in the subsequent period for consumption. Every unit of saving would yield this consumer $(1 + r)$ of proceeds that can be used for the consumption in the future (C_1).

Let C_0 be the amount of today's consumption, C_1 be the amount of future's consumption, S be the amount of savings, and $1 + r$ be gross yield from saving. The problem that this consumer faces is to choose for different level of consumption between the two periods, maximizing the following utility-based criteria function given by,

$U = ln\,(C_0) + \beta ln\,(C_1)$ subject to $C_1 = (1 + r)(Y_0 - C_0)$
where $0 < \beta < 1$.

Consider the problem.

a. Solve for the optimal consumption in the two periods using the LaGrange multiplier method.
b. Confirm your result in a by checking the second-order condition.
c. What happen to C_0 if β increases. Explain the intuition of your result.
d. How does the interest rate affect the amount of saving? Explain the intuition of your result.

Question 9: *Cost minimization problem.*

Consider a representative firm with the production function taking the form: $Q = K^a + L^a$, where K is the capital, L is the labor input, Q is the amount of output, and $a \in (0,1)$. This firm purchases capital and labor for the market at the price per unit of r and w, respectively. Consider the following problem

a. Does the production function exhibit returns to scale technology? If yes, state the type of returns to scale.
b. Use the second-order derivative matrix and show that the production function is concave.
c. Derive the cost-minimizing bundle of factor inputs.
d. Confirm your result in "c" that your proposed solution is the least cost combination of factor inputs.
e. Show the expression for LaGrange multiplier.
f. Derive the minimized cost function.
g. Show that marginal cost is the LaGrange multiplier.
h. Show that the minimized cost function is concave in the prices of factor inputs. That is, show that the Hessian of minimized cost function is negative definite.

Question 10:

Solve for the following constrained optimization problem.

a. $min\ \ x^2 + y^2 + z^2\ \ st.\ \ x + y + z = 1$
b. $max\ U(x, y, L) = 2xy(24 - L)\ \ st. p_x x + p_y y\ \ = wL$

Practice problem set 8

Integration and its application in economics.

Question 1:

In the manufacture of a product, the marginal cost of producing x units is $C'(x)$ and fixed cost are $C(0)$. Find the total cost function $C(x)$ when:

a. $C'(x) = 3x + 4$, $C(0) = 40$.

b. $C'(x) = ax + b$, $C(0) = C_0$.

Question 2

Let $K(t)$ denote the capital stock of an economy at time t. Then net investment at time t, denoted by $I(t)$, is given by the rate of increase $\frac{dK}{dt}$ of $K(t)$.

a. If $I(t) = 3t^2 + 2t + 5, t \geq 0$, what is the total increase in the capital stock during the interval from $t = 0$ to $t = 5$?

b. If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time $t = t_0$ to $t = T$ when the investment function $I(t)$ is as in part (a).

Question 3:

Given the following demand and supply curves, compute the consumer and producer surplus.

a. Demand: $P = 200 - 0.2Q$; Supply: $P = 20 + 0.1Q$.

b. Demand: $P = \frac{6000}{Q+50}$; Supply: $P = Q + 10$.

Question 4

Suppose that the profit of a firm as a function of its output x is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, x > 0$$

a. Find the level of output that maximizes profit. Sketch the graph of f.

b. The actual output varies between 1000 and 3000 units. Compute the average profit $I = \frac{1}{2000}\int_{1000}^{3000} f(x)\, dx$.

Question 5:

Evaluate the following integrals by using integrations by substitution:

a. $\int_0^1 x\sqrt{1+x^2}\ dx$

b. $\int_1^e \frac{lny}{y} dy$

Question 6:

Evaluate the following integrals by using integrations by using integrations by parts (r $\neq$0).

a. $\int_0^T bte^{-rt}\, dt$

b. $\int_0^T (a+bt)e^{-rt}\, dt$

Question 7:

a. Evaluate $\int_0^1 x^{p}(x^{q} + x^{r} + x^{s})\, dx$ where $p, q, r, and\ s$ are positive numbers.

b. Let $F(x) = \int_0^x (t^2 + 2)\, dt$ and $G(x) = \int_0^{x^2} (t^2 + 2)\, dt$. Find F'(x) and G'(x).

Question 8:

Let the demand and supply of goods Q in a perfectly competitive market be the followings;

Demand Function : $P = 25 - Q^2$

Supply Function : $P = 2Q + 1$

a. Determine the consumer surplus as the equilibrium.
b. If the government imposes tax on consumers for $4 per unit of production, calculate the deadweight loss.

Question 9:

Let $P = 274 - Q^2$ be the demand function in a monopoly market. Suppose further that marginal cost of the monopolist is given by $MC = 4 + 3Q$.

a. Determine consumer's surplus at the profit-maximizing production level.
b. Calculate deadweight loss under monopoly.

Question 10:

A company has MC = 80 where the demand function is P = 1400-6Q. At zero production, the company faces loss by $1,500. Determine the maximum profit by using integral calculus and prove your answer.