

CH11. Long-Term Economic Growth

EE312 (for Section 046402)

Read: Williamson Ch. 7 - 8

March 2018

Contents

I	Importance of Growth	3
II	The Solow growth model	6
1	The Solow Growth Model	6
1.1	The population growth	6
1.2	Consumers:	6
1.3	The representative firm	7
1.4	Equilibrium Output	8
1.5	The Steady-State Capital Per Worker	9
1.6	Analysis of Steady State	10
1.7	Growth accounting	15
III	Endogenous Growth Model	16
2	Endogenous Growth Model	16
2.1	Introduction	16
2.2	Human capital accumulation	19
2.3	The representative consumer	19
2.4	Accumulation of human capital	20
2.5	The representative firm	20
2.6	The firm's profit function	20
2.7	Determination of the real wage	21
2.8	Competitive equilibrium	21
2.9	Growth of human capital	21
2.10	Factors in human capital growth	21
2.11	Consumption and output growth	22
2.12	Source of growth	22
2.13	Government policy on growth	22
2.14	Human capital externalities	23
IV	Comparison	24

Course Outline

1. Macroeconomics Measurement, Business Cycles VS. Trend

Part I Business cycles and Economics fluctuations: Short-run analysis

2. IS-LM model and labour market
3. Open economy macroeconomics

Part II Business cycles and Economics fluctuations: Medium run analysis

4. Keynes and Monetarist
5. New Classicals
6. New Keynesian

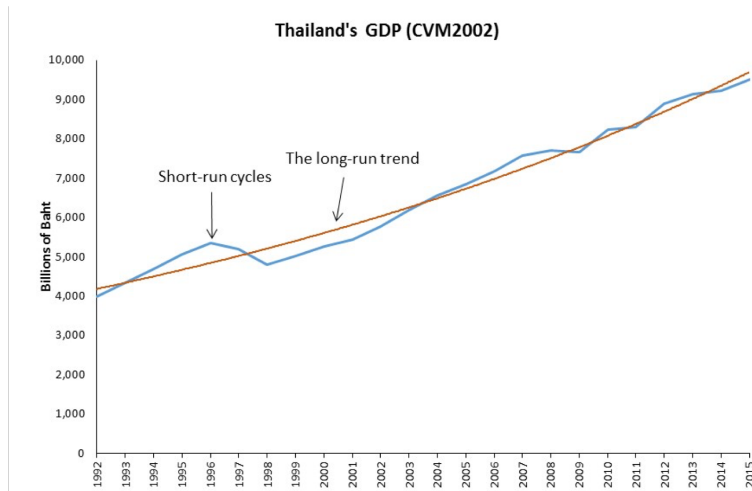
Part III Business cycles and Economics fluctuations: Micro-founded macroeconomic approach

7. **A Closed-Economy One-Period Macroeconomic Model: Optimizing-agent decision**
8. **A Closed-Economy One-Period Macroeconomic Model: Equilibrium**
9. **Two-Period Model: the Consumption-Savings Decision**
10. **A Real Intertemporal Model with Investment**
11. **Long-term Economic Growth**
 - (a) Solow Growth Model
 - (b) Endogenous Growth Model

Part I

Importance of Growth

- The standards of living in the long term depend on economic growth. Short-run fluctuations tend to cancel out in the long run.



- **Economic growth facts**

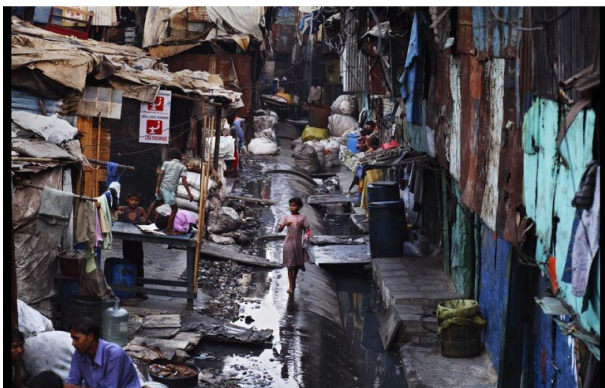
- Before the Industrial Revolution in about 1800, standards of living differed little over time and across countries.
- Since the Industrial Revolution in about 1800, per capita income growth has been sustained in the richest countries (e.g., about 2% in US since 1869).
- Across countries, we observe that
 - * The higher the rate of investment, the higher output per worker.
 - * The higher population growth, the lower output per worker. High population growth corresponds with low living standards.
- Are standard of living converging across countries of the world?
 - * International differences in living standards widen between developed and developing countries (except East Asia).
 - * Growth convergence occurs among developed countries.
 - * No growth convergence among the poorest countries.

- **World Bank classification 2016**

- Gross National Income (GNI) per capita (Atlas method). High income (79) > \$12,235 Upper middle income (55) \$3,956 - \$12,235 Lower middle income (52) \$1,006 - \$3,955 Low income (31) < \$1,006 World average \$10,302

* <https://data.worldbank.org/indicator/NY.GNP.PCAP.CD>

High income (79)	> \$12,235
Upper middle income (55)	\$3,956 - \$12,235
Lower middle income (52)	\$1,006 - \$3,955
Low income (31)	< \$1,006
World average	\$10,302



- **The rags and the rich**

- Richest: Monaco, Liechtenstein, Switzerland, Luxembourg, Australia, Scandinavia, USA, Canada, Germany, UK, France.
- Poorest: Congo, Niger, Liberia, Malawi, Central Africa Rep, Burundi, Eritrea, North Korea, Somalia

- **Rich and poor in Asia**

Japan	38,000	Singapore	51,880
S. Korea	27,600	Malaysia	9,850
Hong Kong	43,420	Thailand	5,640
China	8,260	Philippines	3,580
Bhutan	2,510	Indonesia	3,400
India	1,680	Vietnam	2,050
Pakistan	1,510	Lao	2,150
Bangladesh	1,330	Myanmar	1,190
Nepal	730	Cambodia	1,140

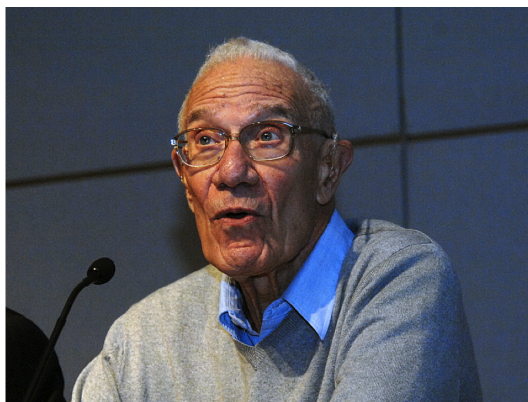
Will this income disparity last forever?

- **Gross National Income (GNI) per capita (Atlas method)**

Country	1986	1996	2006	2016
Thailand	830	2,950	3,100	5,640
India	310	400	790	1,670
Singapore	7,320	25,640	32,080	51,880
China	310	650	2,060	8,250
Malaysia	1,860	4,500	5,850	9,860

Source: world bank

- “ONCE you start thinking about growth, it’s hard to think about anything else.” ,
[said Robert Lucas, a Nobel prize-winning economist, “On the Mechanics of Economic Development”,
Journal of Monetary Economics, 1988]
- “Judging by their rhetoric, the world’s policymakers are indeed thinking about little else. The statement released after the most recent meeting of G20 leaders in Toronto in June mentioned the word “growth” 29 times in nine pages. Mr Obama says his economic policy is all about “laying the foundations for long-term growth”. Britain’s prime minister, David Cameron, used his first speech in office to lay out a “strategy for economic growth”. Japan’s government unveiled a ten-year “new growth strategy” in June.”
The Economist, 2010



Robert M. Solow (b.1924), Nobel Prize 1987

- **What determines economic growth?**

- Models of economic growth.

1. The Solow growth model :

- * sustainable growth based on technological progress.
- * Exogenous growth: technology is determined outside the model.
- * Growth convergence among countries. Endogenous growth

2. Endogenous growth models.

- * sustainable growth based on human capital.
- * Growth engine is endogenous.
- * No certainty in growth convergence.

Part II

The Solow growth model

1 The Solow Growth Model

- The basis of all modern theories of growth.
- Long-term economic growth depends on one single factor — **technological progress**.
 - Rising total factor productivity (z).
 - Sustained improvement in living standards (real per capita income or output per worker).

1.1 The population growth

- Assume population grows exogenously at a constant rate.
- N = population (or workers) in the current period.
- N' = population in the future period.
- n = rate of population growth in the current period

$$N' = (1 + n)N$$

$$n > -1$$

1.2 Consumers:

- Consumers = population = workers.
- Consumers supply labor in production.
- Consumers receive real output (Y) as (wage and dividend) income.
- Spend on consumption goods (C) and save a constant fraction (s) of Y as saving (S).

$$Y = C + S$$

$$S = sY$$

$$C = (1 - s)Y$$

1.3 The representative firm

1.3.1 The Neoclassical Production Function

- The firm produces output using the current capital stock (K) and the current labor input (N).
- Assuming
 1. Constant returns to scale.
 $F(\lambda K, \lambda N) = \lambda F(K, N)$; for all $\lambda > 0$
 2. Positive and diminishing returns to private inputs:
For all $K > 0$ and $L > 0$, F exhibits positive and diminishing marginal products with respect to each input.
 3. Inada conditions:
The marginal product of capital (or labor) approaches infinity as capital (or labor) goes to 0 and approaches 0 as capital (or labor) goes to infinity.

- **Production function :** $Y = zF(K, N)$

- **Per worker production function:**

Let $y = \frac{Y}{N}$ = output per worker and $\frac{K}{N} = k$ = capital per worker

$$\frac{Y}{N} = \frac{1}{N} zF(K, N)$$

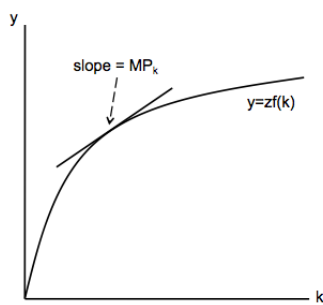
$$\text{CTS : } F(\lambda K, \lambda N) = \lambda F(K, N)$$

$$\frac{Y}{N} = zF\left(\frac{K}{N}, 1\right)$$

$$\text{Let } f(k)F\left(\frac{K}{N}, 1\right) = F(k, 1)$$

$$y = zf(k)$$

- Marginal Product of Capital



Production function, fixed labor

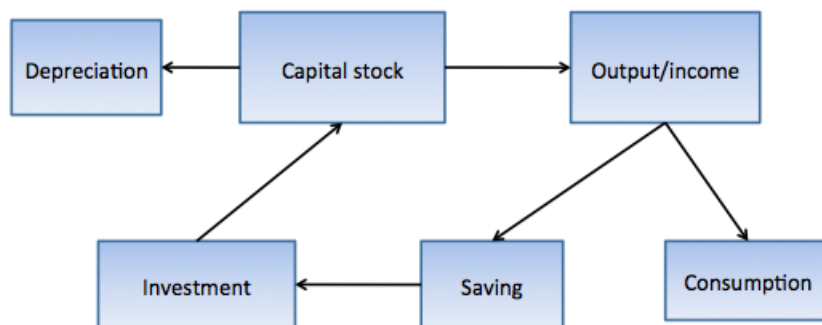
- Output per worker (y) increases at a decreasing rate as capital per worker (k) rises.
- Slope is the marginal product of capital (MP_K).

1.3.2 Growth of Capital Stock

- Assume capital wears out over time at the rate of d (or depreciation), where $0 > d > 1$.
 - I = (gross) investment = addition to capital stock.
 - K' = capital stock in the future period.

$$K' = (1 - d)K + I$$

1.3.3 The Working of Growth



1.4 Equilibrium Output

- At equilibrium, saving equals investment so that output consists of consumption and investment.

$$S = I$$

$$S = Y - C$$

$$Y = C + S$$

$$Y = C + I$$

- Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving).

$$Y = C + I$$

$$C = (1 - s)Y$$

$$I = K' - (1 - d)K$$

- Substitute C and I in the Y equation
- Per worker formulation

$$Y = (1-s)Y + K' - (1-d)K$$

rearranging the term

$$K' = Y - (1-s)Y + (1-d)K$$

$$= sY + (1-d)K$$

From $Y = zF(K, N)$,

$$K' = szF(K, N) + (1-d)K$$

divided by N

$$\frac{K'}{N} = sz \frac{F(K, N)}{N} + (1-d) \frac{K}{N}$$

$$\frac{K'}{N} = szf(k) + (1-d)k$$

- Future capital per worker function ($k' = \frac{K'}{N'}$)

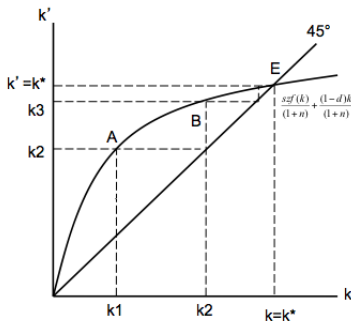
$$\frac{K'}{N} \times \frac{N'}{N'} = sz \frac{F(K, N)}{N} + (1-d) \frac{K}{N}$$

$$\text{Let } k' = \frac{K'}{N'} \text{ and } \frac{N'}{N} = (1+n)$$

$$k'(1+n) = szf(k) + (1-d)k$$

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

1.5 The Steady-State Capital Per Worker



- At A, $k_2 > k_1$; k is growing.
- At B, $k_3 > k_2$; k is growing.
- $k = k^*$; steady-state capital per worker.

- Diminishing returns on k
- At E, $k = k' = k^*$ so that k^* is steady.
- To the left of k^* , $k' > k$ so that k is increasing.

- To the right of k^* , $k' < k$ so that k is decreasing.
- As k is increasing, MP_k is falling so that y is **increasing at a decreasing rate**.
- Finally, investment (or new capital) is just sufficient to keep up with population growth and depreciation, so that k (and y) is stagnant.

Steady-state aggregates

- With k^* at the steady state, y^* , c^* and $szf(k^*)$ are all at the steady-state.
 - No further improvement in output per worker (y).
- Given population growth (n), total factor productivity (z) and the saving rate (s), the steady-state growth rate is ‘ n ’ for aggregate quantities:
 - Capital stock (K) and output (Y);
 - Consumption (C), saving (S) and investment (I).

1.6 Analysis of Steady State

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

$$k^* = \frac{szf(k^*)}{(1+n)} + \frac{(1-d)k^*}{(1+n)}$$

Multiply both sides by $(1+n)$

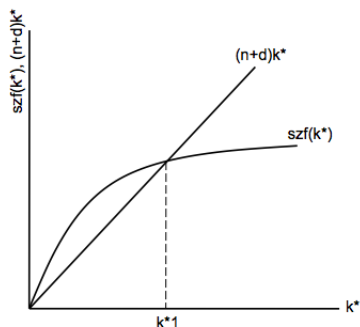
$$(1+n)k^* = szf(k^*) + (1-d)k^*$$

$$(n+d)k^* = szf(k^*)$$

$$\text{Steady State Investment} = \text{Steady State Saving}$$

- $szf(k^*)$ = saving per worker;
- $(n+d)k^*$ = investment per worker needed to keep up with population growth and depreciation.
- At k^* , the capital stock is still growing, but just sufficient to equip each worker with the same k and depreciation (so k^* is steady).
 - **‘Capital widening’**: growing K just to keep the steady k and y .

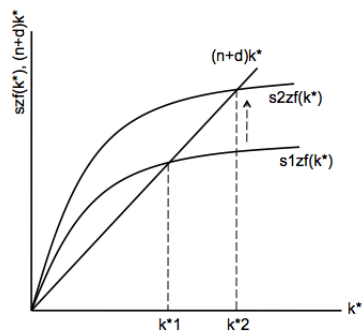
1.6.1 Determination of steady-state k^*



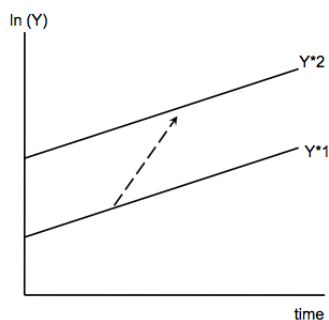
- $szf(k^*)$ is concave due to $zf(k^*)$.
- $(n+d)k^*$ has the slope = $(n+d)$.

1.6.2 Effect of an increase in s

- s may increase due to changes in consumers' propensity or government policy.
- Assume a permanent increase in s :
 - $szf(k^*)$ rotates upwards.
 - Higher steady-state k^* and y^* (on a different 'growth path').
 - Higher growth of K and Y is transitional.
 - Convergence to the same steady-state growth rate of ' n '.



- Higher saving rate results in a higher k^* and y^* .
- **A rise in s raises k^* .**



- **Temporary gain in growth rate**
- K and Y move to new 'growth paths'.
- Higher growth rates of K and Y are transitional, converging to n .

1.6.3 Steady-state consumption per worker

$$y^* = zf(k^*)$$

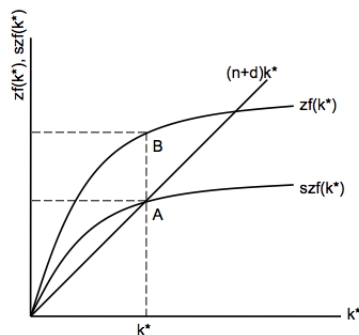
$$\frac{S}{N} = szf(k^*)$$

$$c^* = zf(k^*) - szf(k^*)$$

$$= (1 - s)zf(k^*)$$

$$\text{At } k^*, szf(k^*) = (n + d)k^*$$

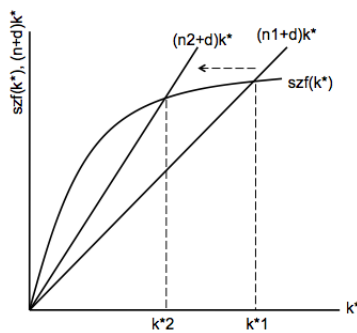
$$c^* = zf(k^*) - (n + d)k^*$$



- $c^* = y^* - szf(k^*) = zf(k^*) - (n + d)k^*$.
- $AB = c^*$

1.6.4 Effect of an increase in n

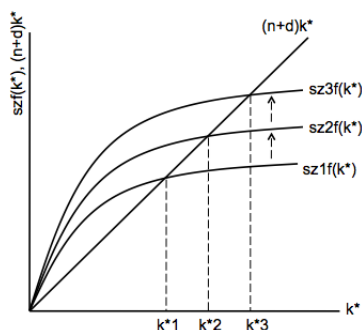
- The increase in population growth (n_1 to n_2) rotates $(n+d)k^*$ upwards.
- Decreased steady-state capital (k^*) and output per worker (y^*).
 - More workers (N^*) produce larger output (Y^*).
 - But falling productivity of labor results in lower output per worker (y^*).
- The steady-state growth rate is higher at n_2 for the capital stock (K) and total output (Y).



- Higher population growth (n) results in lower k^* and y^* .
- **A higher n with lower k^***

1.6.5 Effect of an increase in z

- A rising s or falling n raises steady-state output per worker (living standards).
 - But the improvement will cease at some point (s cannot exceed 1; n cannot fall indefinitely).
- An increase in total factor productivity (z) raises steady-state capital (k^*) and output per worker (y^*).
 - Sustained increases in z cause sustained increases in output per worker (y).



- Sustained increases in z cause sustained improvements in y^* .

Sources of sustained growth

- Growth from increases in productive inputs:
 - * Physical capital accumulation, $F(K, N)$. Human capital accumulation, $F(K, H)$.
- Growth from total factor productivity (z):
 - * Technical progress, inventions, better management and organization.
 - * Weather, improved government regulations, falling input prices.

Solow model predictions

- In the long run, higher savings rate results in higher income per worker.
 - * Fact: positive correlation between GDP per capita and the ratio of investment to GDP.
- An increase in population growth causes a decrease in income per worker.
 - * Fact: negative correlation between population growth and GDP per capita

1.6.6 Golden Rule

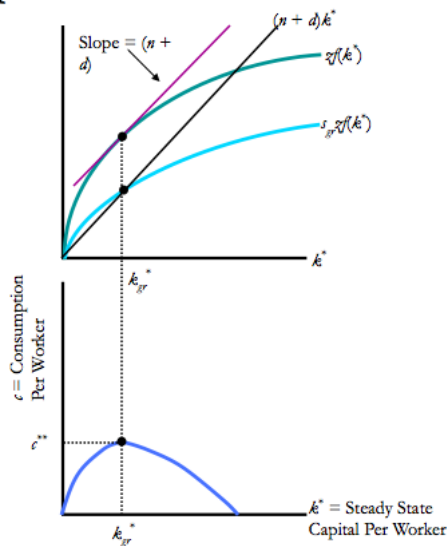
- Consumption per worker in the steady state is $c^* = y^* - szf(k^*) = zf(k^*) - (n + d)k^*$.
- Golden rule quantity of capital per worker, k^{**} , gives the maximum consumption per worker, c^{**} .
- Maximized c^{**}

$$c^* = zf(k^*) - (n + d)k^*$$

Set $\frac{dc^*}{dk^*} = 0$

$$\frac{dzf(k^*)}{dk^*} = (n + d)$$

$$MP_k = n + d$$



- In this economy, there are two markets in the current period.

1. consumption goods are traded for current labor
2. consumption goods are traded for capital

- Consumers save by accumulating capital.

At equilibrium, $S = I$ so that $Y = C + I$.

Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving)

$$\begin{aligned} K &= k^* N \\ Y &= y^* N = zf(k^*) N \\ I &= sY = szf(k^*) N \\ C &= (1 - s)Y = (1 - s)zf(k^*) N \end{aligned}$$

- Given population growth (n), total factor productivity (z) and the saving rate (s), the steady-state growth rate is 'n' for aggregate quantities.

“while the growth rate of per worker variable = 0 at steady state”

- Solow model tells us that growth in key macroeconomic aggregates is determined by exogenous labor force growth when the saving rate, the labor force growth rate, and total factor productivity are constant.

- Solow's model states that investment in capital cannot drive long run growth in GDP per worker.
- Policy lesson: don't advise poor countries to invest without due regard for technology and incentives. Capital deepening (an increase in capital per worker) cannot lead to a sustained economic growth in the long run.

1.7 Growth accounting

- Growth since the Industrial Revolution has come mainly from rising z .
 - Will this continue indefinitely into the future?
- Growth accounting: identify sources of growth. Increases in productive inputs (K , N) or in total factor productivity (z).
- Calculation based on the production function and the Solow residual.

Thailand's production function

$$Y = zK^{\alpha}N^{(1-\alpha)},$$

where $0 < \alpha < 1$. Note that $\alpha + (1 - \alpha) = 1$

- Assume constant returns to scale (CRS).
- α = share of the capital input in GDP.
- $1 - \alpha$ = share of the labor input in GDP.

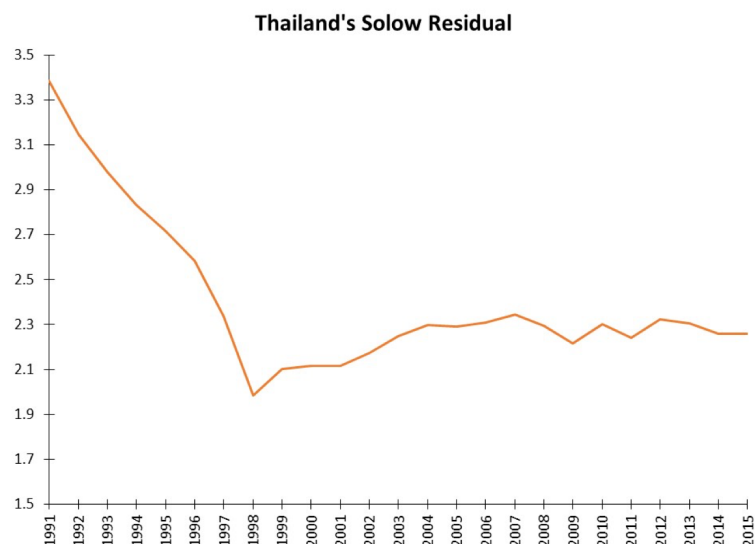
$$Y = zK^{0.6}N^{0.4}.$$

The Solow residual for Thailand

$$Y = zK^{0.6}N^{0.4}$$

$$z = \frac{Y}{K^{0.6}N^{0.4}}$$

- Estimated z = Solow residual.
- It measures the level of total factor productivity for Thailand.



Year	Output	Capital	Labor	Solow residual
1990-1997	5.40%	18.00%	0.90%	-6.10%
1998-2015	3.80%	5.00%	0.20%	0.70%

Part III

Endogenous Growth Model

2 Endogenous Growth Model

2.1 Introduction

2.1.1 Solow growth predictions

Solow growth predictions ((1) Absolute convergence, (2) Conditional Convergence)

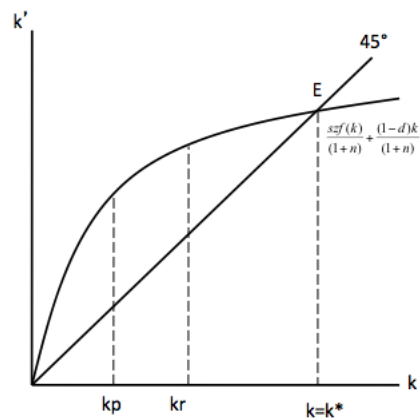
1. Absolute convergence

- If two countries start with:
 - the same population growth rate (n), saving rate (s) and total factor productivity (z),
 - but different per capita incomes (y), e.g., rich versus poor countries;
 - they will converge to the same steady-state k^* , y^* and c^* — **Absolute convergence**.
- The poor country will have temporary higher growth and catch up with the rich.

(a) Absolute convergence (convergence in k^* , y^* and c^*),

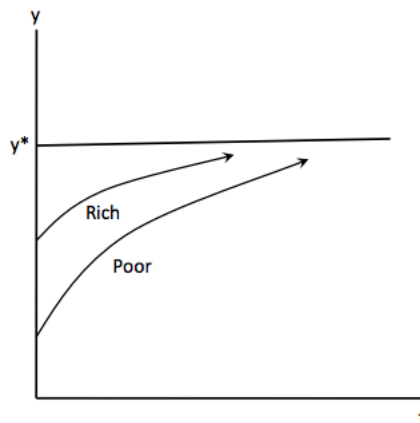
- Convergence in y^* , Convergence in output growth path

- The rich starts at k_r while the poor starts at k_p (with the same s , n and z).
- They converge to k^* and y^* in the long run.



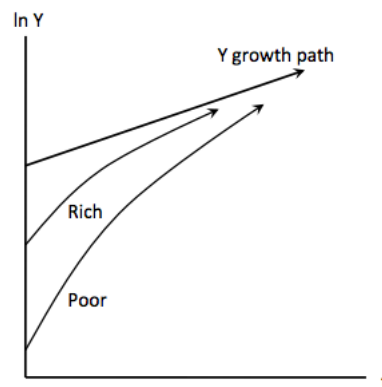
(b) Convergence in per capita income

- The rich and the poor converge to the same level of y^* .



(c) Convergence in output growth path

- The rich and the poor converge to the long-run growth rate (n) of aggregate output (Y).



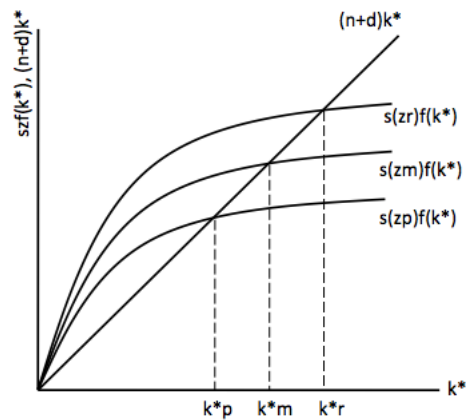
2. Conditional convergence

- With differences in n , z and s , the steady-state k^* , y^* , c^* are different.
 - Each country has its own steady state.
 - The steady-state growth rate of aggregates (K , Y) is still n for each country.
- Countries are predicted to converge to their own steady state.
- Disparity among countries due to different values of n , z and s .

2.1.2 Solow growth predictions Vs. Growth Facts

- Absolute convergence has occurred among rich countries.
- No absolute convergence between rich and poor countries. Exception is East Asia.
- No absolute convergence among poor countries.
- Great diversity among poor countries.
- Why no absolute convergence?
 - Countries have different s , n and z .
 - Each country has different steady-state k^* , y^* , c^* .
 - Each country is moving towards its own steady-state — **Conditional convergence**.
 - But differences in s and n are not large enough to explain all international disparity.
 - **Difference in access to technology (z)?**

- Countries with different z 's will not converge to the same k^* and y^* .
- p = poor
- m = medium
- r = rich



Disparity due to different z 's

- Different levels of total factor productivity (z) will perpetuate differences in capital per worker (k^*), per capita income (y^*) ...
- despite the same saving rate (s) and population growth (n).

Barriers to technology adoption

- **Labor legislation:** strong labor unions obstruct adoption of new technology.
- **Trade protectionism:** domestic firms with market power lack incentives to innovation.
- **Political corruption:** government's protection of inefficient firms.
- **Undeveloped financial system:** poor resource allocation mechanism.

How to catch up?

- Promotion of more competition among firms.
 - * Liberalization and competition policy.
 - * More pressure and incentive for firms to innovation.

- Free trade for greater international competition.
- Privatization of state enterprises.
 - * State enterprises guarantee employment at the expense of efficiency.

2.1.3 Growth in Solow Model

- The Solow model does not explain the mechanism of growth itself.
- Growth depends on exogenous factors.
- Total factor productivity (z) is exogenously determined.
- z depends on R&D by firms, education, training.
- These are partly affected by government policy.
- Government policy to raise z and long-term growth?

2.2 Human capital accumulation

- The higher human capital, the more efficiency the production of human capital has.
 - Better schooling, more future production, better passing on skills and knowledge.
- **Human capital is an investment.**
 - Opportunity cost of education and training — sacrifice of current consumption.
 - Benefits: more future production and consumption.
- Knowledge is '**non-rivalry**': one's acquisition of knowledge does not reduce others' ability to acquire the same knowledge.
- Human capital accumulation is **NOT subject to diminishing marginal returns**.
- No limit on how productive a person can become, given increasing knowledge and skills.
- **Unbounded growth** in endogenous models.
- Growth in Solow model is limited:
 - **Diminishing returns** on physical capital accumulation — rivalry in resource uses.

2.3 The representative consumer

- The consumer allocates time between work and accumulating human capital.
- H^S = efficiency units of current human capital;
- u = time allocated to work;
- w = the real wage;

- C = current consumption;
- The **budget constraint** is total labor earnings:

$$C = uwH^S \quad (1)$$

2.4 Accumulation of human capital

- The consumer trades off current consumption for future consumption by accumulating human capital:
- $H^{S'}$ = future human capital;
- $(1 - u)$ = time allocated to human capital accumulation;
- b = efficiency of human capital accumulation technology; $b > 0$.

$$H^{S'} = (1 - u)bH^S \quad (2)$$

2.5 The representative firm

- The firm's production function using efficiency units of labor:
- Y = current output;
- z = marginal product of efficiency units of labor, where $z > 0$;
- uH^d = current input of efficiency units of labor:

$$Y = zuH^d \quad (3)$$

2.6 The firm's profit function

- uH^d is also the firm's demand for the efficiency units of labor.
- The function is characterized by **constant returns to scale** (CRS) — only one input.

$$\begin{aligned} \pi &= Y - wuH^d \\ \pi &= zuH^d - wuH^d \\ \pi &= (z - w)uH^d \end{aligned}$$

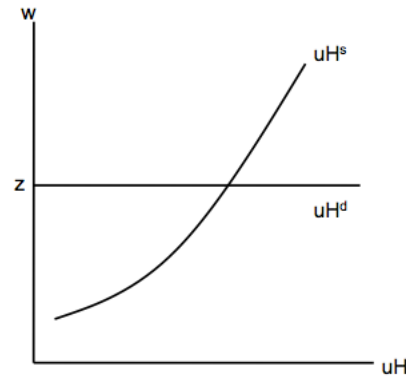
Demand for efficiency units of labor

$$\pi = (z - w)uH^d \quad (4)$$

- $(z - w) < 0$, $\pi < 0$; the firm hires no units of labor; or $uH^d = 0$.
- $(z - w) > 0$, $\pi > 0$; the firm hires infinite units.
- $z = w$, $\pi = 0$; the firm is indifferent.
- The demand curve is infinitely elastic at $w = z$.

2.7 Determination of the real wage

- uH^d is horizontal at $w = z$.
- The real wage equals z , the marginal product of uH^s . Assume uH^s with slope > 0 .



2.8 Competitive equilibrium

- The market clears at $w = z$ where $uH^d = uH^s$.
- Equilibrium consumption and growth of human capital accumulation:

$$C = zuH.$$

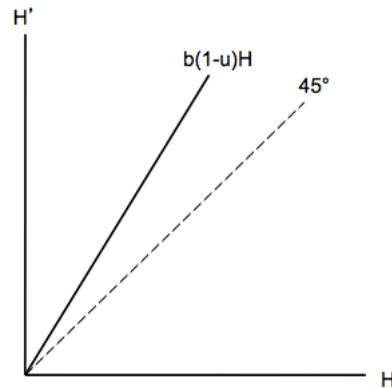
$$H' = b(1 - u)H.$$

$$\frac{H'}{H} - 1 = b(1 - u) - 1,$$

where $b(1 - u) - 1$ is constant.

2.9 Growth of human capital

- H' is a function of H where $H' > H$.
- Slope = $b(1 - u) = 1 + \text{rate of growth of human capital}$.



2.10 Factors in human capital growth

$$\frac{H'}{H} - 1 = b(1 - u) - 1, \quad (5)$$

- $\frac{H'}{H}$ is higher if b increases or u decreases.
 - b = efficiency of human capital accumulation technology (or efficiency of the education sector).

- u = time spent on current output production.
- Falling u (or rising $1-u$) = more time spent on human capital accumulation.

2.11 Consumption and output growth

- Current consumption $C = zuH$ also holds for future consumption $C' = zuH'$.
 - So consumption grows at the same rate of $b(1-u)$ as human capital.
- Output also grows at the same rate as $Y = C$ in every period.

$$\frac{C'}{C} - 1 = \frac{zuH'}{zuH} - 1 = \frac{H'}{H} - 1 = b(1-u) - 1 \quad (6)$$

2.12 Source of growth

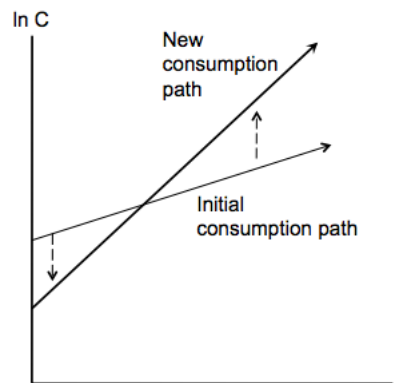
- **b and z are fixed:** constant technology.
- No population growth.
- Growth is determined inside the model, by the value of b and u .
- **Growth is unbounded** because human capital accumulation is not subject to diminishing returns to scale.
 - Output grows in proportion to human capital, given u .

2.13 Government policy on growth

- Government can increase growth:
 - Increases in b , the efficiency of human capital accumulation technology (education policy).
 - Reduction in u , taxes or subsidies to education.
 - Higher $b(1-u)$, higher growth of human capital, consumption and output.
- But current consumption must be sacrificed as u is lower, given initial human capital (H).

2.13.1 Lower u and consumption

- a lower u results in lower current consumption but higher consumption in the long run.



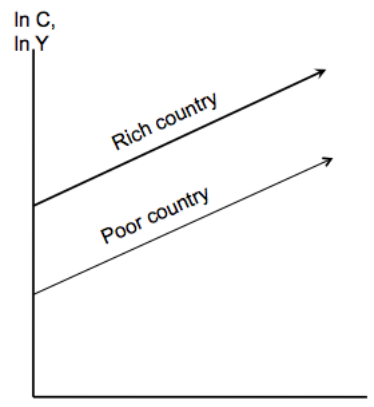
2.13.2 Consumer preference

- Government's education policy (raising b) involves expenses of current resources and lower current consumption.
- Higher long-run growth is desirable?
 - This depends on the consumer's preference on current and future consumption.
 - The consumer may be worse off if current consumption is actually preferred.

2.13.3 No convergence

- Countries with all identical characteristics except differences in initial human capital will not converge on the levels of consumption and income.
 - **Poor countries:** low $Y = C = zuH$;
 - **Rich countries:** high $Y = C = zuH$.
 - But their C and Y grow at the same rate of $b(1-u)$.
- Rich and poor do not converge.

- The Y and Y time paths do not converge despite the same growth rate of $b(1-u)$.



2.14 Human capital externalities

- The endogenous model explains the lack of convergence among poor countries and between rich and poor countries.
- But convergence occurs among rich countries, why? — Human capital externalities.
 - Contact with others with higher human capital increases our own human capital.
 - Capital and labor are highly mobile; skills are more easily transferred in rich countries.
- More opportunities and contact make levels of human capital in rich countries converge.
 - Convergence of income per worker.
- Lack of human capital externalities in poor countries.
 - Less contact with developed countries.
 - People with high human capital move to developed countries (i.e., brain drains).
 - Differences in human capital persist.

Part IV

Comparison

Solow's model	An endogenous growth model (A Simplified version of Lucas's)
production function : $Y = zF(K, N)$ $y = zf(k)$ Production function is <u>subject to diminishing marginal returns</u> in capital per worker.	$Y = uzH^d$ Production function <u>does not subject to diminishing marginal returns</u> in human capital.
Source of growth is physical capital accumulation.	Source of growth is human capital accumulation
There is a limit for growth as physical capital per worker increases. Output will go to steady state output. After it reaches the steady state, it will stay there. Eventually, growth of per capita output goes down to zero.	Output continues to grow at a constant rate. as human capital increases. There is no limit for growth. Output growth is always positive. (growth rate is equal to $b(1 - u) - 1$).
Economic reason for bounded growth : Physical capital is <u>rivalry</u> (once one company is using it the other cannot). production function is <u>subject to diminishing marginal returns</u> in capital per worker As physical capital increases, output increases at a decreasing rate. Therefore, we cannot always generate growth by raising physical capital.	Economic reason for unbounded growth : This is because knowledge is '<u>non-rivalry</u>', production function <u>does not subject to diminishing marginal returns</u> in human capital As human capital increases, output increases at a constant rate. Therefore, we can always generate growth by raising human capital.