Group members

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0.1) Given that $Z = \frac{x^3 - y^3}{x^2 y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

$$\frac{\partial Z}{\partial x} = \frac{x^2y^2(3x^2) - (x^3-y^3)(2xy^2)}{x^4y^4}$$

$$= \frac{3x^4y^2 - (2x^4y^2 - 2xy^5)}{x^4y^4}$$

$$= \frac{x^4y^2 + 2xy^5}{x^4y^4}$$

$$\frac{\partial Z}{\partial y} = \frac{x^2y^2(-3y^2) - (x^3-y^3)(2x^2y)}{x^4y^4}$$

$$= \frac{-3x^2y^4 - (2x^5y - 2x^2y^4)}{x^4y^4}$$

$$= \frac{-x^2y^4 - 2x^5y}{x^4y^4}$$

$$x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = x\left(\frac{x^4y^2 + 2xy^5}{x^4y^4}\right) + y\left(\frac{-x^2y^4 - 2x^5y}{x^4y^4}\right)$$

$$= \frac{x^5y^2 + 2x^2y^5 - x^2y^5 - 2x^5y^2}{x^4y^4}$$

$$= \frac{x^2y^5 - x^5y^2}{x^4y^4}$$

$$= \frac{x^2y^2(y^3 - x^3)}{x^4y^4}$$

$$= \frac{y^3 - x^3}{x^2y^2}$$

$$= -\left(\frac{x^3 - y^3}{x^2y^2}\right) = -Z$$

0.2) Given that $Z = \frac{x-y}{x+y}$, use the total differential and calculate the change in Z when x= 1 and y =1. What would happen to Z if X increases by 2 units while Y decreases by 2 units?

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$$

$$\frac{\partial z}{\partial x} = \frac{(x+y) - (x-y)}{(x+y)^2}$$

$$= \frac{2y}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x+y)(-1) - (x-y)}{(x+y)^2}$$

$$= \frac{-x-y-x+y}{(x+y)^2}$$

$$= \frac{-2x}{(x+y)^2}$$

$$dz = \frac{2y}{(x+y)^2} dx - \frac{2x}{(x+y)^2} dy$$

Find z when $dx = 2$, $dy = -2$

$$dz\Big|_{x=1,\, y=1} = \frac{2(1)}{(1+1)^2}(2) - \frac{2(1)}{(1+1)^2}(-2)$$

$$= 1 + 1$$

$$= 2$$

0.3) If z = $2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (0.3a) find ∂z/∂s and ∂z/∂r ; (0.3b) evaluate when r = 1 and s = 0

$$x = 1^2 + 2(1)(0) = 1$$

$$y = 2(1) - 4(0) = 2$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= (4xy + 3y)(2r) + (2x^2 + 3x + 2y)(-4)$$

$$= [4(1)(2) + 3(2)](2) + [2(1)^2 + 3(1) + 2(2)](-4)$$

$$= 28 + 9(-4)$$

$$= -8 \quad \#$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$= (4xy + 3y)(2r + 3s) + (2x^2 + 3x + 2y)(2)$$

$$= [4(1)(2) + 3(2)][2(1) + 3(0)] + [2(1)^2 + 3(1) + 2(2)](2)$$

$$= (14)(2) + (9)(2)$$

$$= 46 \quad \#$$

0.4) For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

$$2x^2 + 3y^2 + 2z^2 - 16 = 0$$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$$

$$F_y = 6y$$

$$F_z = 4z$$

$$\left.\frac{\partial z}{\partial y}\right|_{x=1,\, y=2,\, z=-1} = \frac{-F_y}{F_z} = \frac{-6y}{4z}$$

$$= \frac{-6(2)}{4(-1)}$$

$$= \frac{-12}{-4}$$

$$= 3$$

0.5) Given that $ln\,(x+y+z)+xyz=ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

$$\ln(x+y+z)+xyz-ze^{x+y+z}=0$$

$$\frac{\partial z}{\partial x}=\frac{-F_x}{F_z}$$

$$F_x=\frac{1}{x+y+z}+yz-ze^{x+y+z}$$

$$F_z=\frac{1}{x+y+z}+xy-\left[ze^{x+y+z}+e^{x+y+z}\right]$$

$$\left.\frac{\partial z}{\partial x}\right|_{x=0,\,y=1,\,z=0}=\frac{-F_x}{F_z}=\frac{-\left[\frac{1}{0+1+0}+(1)(0)-(0)(e^1)\right]}{\left[\frac{1}{0+1+0}+(0)(1)-\left[(0)(e^1)+e^1\right]\right]}$$

$$=\frac{-1}{1-e}$$ ※

2.1) To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

According to the proposition, suppose $n > 0$ the restriction that need to add is $0 < n < 1$ because production function exhibits a decreasing return to scale, so $\alpha + \beta < 1$ from the degree of homogenity.

2.2) Under the assumption used in (2.1), show that the production function satisfies the law of diminishing returns.

$$\frac{\partial Q}{\partial L} = \frac{AnL^n}{L} \qquad \frac{\partial Q}{\partial K} = \frac{AnK^n}{K}$$

$$\frac{\partial^2 Q}{\partial L^2} = (n-1)AnL^{n-2} \; ; n < 1 \qquad \frac{\partial^2 Q}{\partial K^2} = (n-1)AnK^{n-2} \; ; n < 1$$

∴ if 1 more unit of labor is added, MP_L will decrease by $(n-1)AnL^{n-2}$

∴ if 1 more unit of capital is added, MP_K will decrease by $(n-1)AnK^{n-2}$

2.3) Calculate the marginal rate of technical substitution (MRTS) of labor (L) for capital (K).

$$Q = f(K, L) = A[K^n + L^n]$$

$$= AK^n + AL^n \; ; n > 0$$

$$MP_K = \frac{\partial Q}{\partial K} = AK^n + AL^n = nAK^{n-1}$$

$$MP_L = \frac{\partial Q}{\partial L} = AK^n + AL^n = nAL^{n-1}$$

$$MRTS = \frac{MP_L}{MP_K} = \frac{\cancel{nA}K^{n-1}}{\cancel{nA}L^{n-1}} = \frac{L^{n-1}}{K^{n-1}}$$

2.4) how that MRTS is a decreasing function in L. That is, as labor increases, the value of MRTS decreases.

$$\frac{\partial MRTS}{\partial L} = \frac{1}{K^{n-1}}(n-1)L^{n-2}$$

$$= \frac{(n-1)L^{n-2}}{K^{n-1}} \; ; n < 1$$

2.5) Show that Q is increasing over time.

$$Q = A[u^n + L^n]$$

$$= A\left[\left(\tfrac{1}{2}t^2 + 2t + 3\right)^n + \left(e^t + 3\right)^n\right]$$

$$= A\left(\frac{t^2}{2} + 2t + 3\right)^n + A\left(e^t + 3\right)^n$$

$$\frac{\partial Q}{\partial t} = An\left(\frac{t^2}{2} + 2t + 3\right)^{n-1}(t+2) + An\left(e^t + 3\right)^{n-1}(e^t)$$

$$= An\left[\left(\frac{t^2}{2} + 2t + 3\right)^{n-1}(t+2) + \left(e^t + 3\right)^{n-1}(e^t)\right]$$

2.6) Compute $\frac{dQ/Q}{dt}$ when t = 0, i.e. growth of output in the initial period.

$$Q = A\left[u^n + L^n\right]$$

$$\frac{\frac{dQ}{Q}}{dt} = \frac{dQ}{Q} \cdot \frac{1}{dt}$$

$$= \frac{dQ}{dt} \cdot \frac{1}{Q}$$

$$= An\left[\left(\frac{t^2}{2} + 2t + 3\right)^{n-1}(t+2) + \left(e^t + 3\right)^{n-1}(e^t)\right] \cdot \frac{1}{Q}$$ from 2.5

$$= \frac{\cancel{A}n\left[\left(\frac{t^2}{2} + 2t + 3\right)^{n-1}(t+2) + \left(e^t + 3\right)^{n-1}(e^t)\right]}{\cancel{A}\left[u^n + L^n\right]}$$

$$= \frac{n\left[\left(\frac{0}{2}\right)^2 + 2(0) + 3)^{n-1}(0+2) + \left(e^0 + 3\right)^{n-1}(e^0)\right]}{u^n + L^n}$$

$$= \frac{2n(3)^{n-1} + 4^{n-1}}{u^n + L^n}$$

$$= \left[\frac{2n(3)^n}{3} + \frac{4^n}{4}\right]\frac{1}{u^n + L^n}$$

$$= \left[\frac{8n(3)^n + 3(4)^n}{12}\right]\frac{1}{u^n + L^n}$$

$t = 0$; $L(0) = 4$, $u(0) = 3$

$$= \left[\frac{8n(3)^n + 3(4)^n}{12}\right]\frac{1}{3^n + 4^n} = \frac{8n(3)^n + 3(4)^n}{12(3^n + 4^n)}$$ ##

Question 4: Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using L units of labor. If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$. Solve for L^* and calculate $\frac{\partial L^*}{\partial w}$ *and* $\frac{\partial L^*}{\partial P}$.

Profit function = TR − TC

$$\pi = PQ - WL$$

$$= PL^{\frac{1}{2}} - WL$$

Profit maximization at $L^* > 0$

$$\frac{d\pi}{dL} = P\left(\frac{1}{2}L^{-\frac{1}{2}}\right) - W$$

$$0 = \frac{P}{2\sqrt{L}} - W$$

$$W = \frac{P}{2\sqrt{L}}$$

$$\sqrt{L} = \frac{P}{2W}$$

$$L^* = \frac{P^2}{4W^2}$$

$$= \frac{P^2W^{-2}}{4}$$

$$\frac{\partial L^*}{\partial W} = \frac{1}{4}P^2(-2W^{-3})$$

$$= \frac{-P^2}{2W^3} \quad ※$$

$$\frac{\partial L^*}{\partial P} = \frac{1}{4}(2P)W^{-2}$$

$$= \frac{P}{2W^2} \quad ※$$