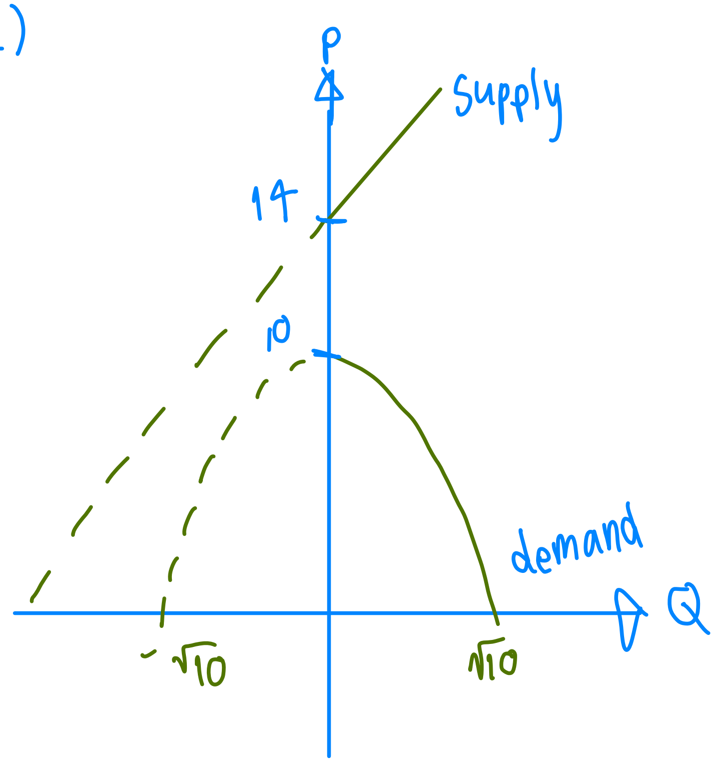
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1)



1.2)

$P = 10 - Q^2$ — ①

$Q = a + P$

$Q = -14 + P$

$P = Q + 14$ — ②

① = ②

$10 - Q^2 = Q + 14$

$0 = Q^2 + Q + 4$

$b^2 - 4ac$; $y = ax^2 + bx + c < 0$

$1^2 - 4(1)(4) = -15$

1.3) supply curve would shif to the right but we cannot predict new market equilibrium and price.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right) \quad , Q \geqslant 0$$

$$R'(Q) = \frac{1}{(Q^2+1)}(2Q) + 3\left(\frac{(Q+1)-Q}{(Q+1)^2}\right)$$

$$R'(Q) = \frac{1}{(Q^2+1)}(2Q) + \frac{3}{(Q+1)^2} \quad \#$$

$$Q \geqslant 0$$

$$Q^2+1 \geqslant 1$$

set $R'(Q) = 0$

$$\frac{2Q}{(Q^2+1)} + \frac{3}{(Q+1)^2} = 0$$

$$Q = 0$$

$\therefore$ $R'(Q) \geqslant 0$, $R'(Q \geqslant 1$

There for $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right)$ is inchesed function.

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$$

$$\pi'(Q) = -(3)\left(\frac{1}{3}\right)Q^2 - 2Q + 8$$

$$\pi'(Q) = -Q^2 - 2Q + 8$$

Critical point; suppose $\pi'(Q) = 0$

$$\pi'(Q) = -Q^2 - 2Q + 8$$

$$-Q^2 - 2Q + 8 = 0$$

$$Q^2 + 2Q - 8 = 0$$

$$(Q - 2)(Q + 4) = 0$$

$$Q = 2, -4$$

$-$ $+$ $-$

-4 2

$$\pi(2) = -\frac{1}{3}(2)^3 - (2)^2 + 8(2) - 1$$

$$= \frac{25}{3}$$

2nd derivative

$$\pi''(Q) = -2Q - 2$$

$\pi''(-4) = 6$; $6 > 0$ min, concave up

$\pi''(2) = -6$; $-6 < 0$ max, concave down

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B → ∴ could not calculate
↳ A and B have different size ※

4.2 A*B

$A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $A \times B = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$ ※

4.3 det(A)

$\begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} = 88 - 90 = \det(A) = -2$ ※

4.4 det(B)

B is not square matrix ※

4.5 det(C)

$\left| \begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{array} \right| = 45 + 84 + 96 - 105 - 48 - 72$

$= 0$

∴ $|C| = 0$ ※

5. Suppose that $U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial U}{\partial x} = ax^{a-1}y^b + \frac{x+y}{x}\left[\frac{(x+y)(1)-(x)(1)}{(x+y)^2}\right]$$

$$= ax^{a-1}\cdot y^b + \frac{(x+y)y}{x(x+y)^2} \quad \divideontimes$$

$$\frac{\partial U}{\partial y} = by^{b-1}x^a + \frac{x+y}{y}\left[\frac{(x+y)(0)-(x)(1)}{(x+y)^2}\right]$$

$$= bx^a y^{b-1} + \frac{(x+y)(x)}{y(x+y)^2} \quad \divideontimes$$