

H.W.

Find $\frac{dy}{dx}$ Approximate Δy when $x=2$, $\Delta x = 0.1$
 and $\Delta x = -0.2$
 Compare the actual Δy to find the errors.

$$y = 10 + \sqrt{x}$$

x	y	$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$
0	10	undefined
1	11	0.5
2	11.4	0.35
3	11.7	0.29

Ans

$$\begin{aligned} \frac{dy}{dx} &= \frac{d(10 + x^{\frac{1}{2}})}{dx} \\ &= \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \\ \therefore \frac{1}{2\sqrt{2}} &= 0.35 \end{aligned}$$

① when $x = 2$, $\Delta x = 0.1$

$$y = 10 + \sqrt{x}, \quad y = 10 + \sqrt{2}$$

$$x = 2 \rightarrow x = 2.1$$

$$x = 2.1, \quad y = 10 + \sqrt{2.1}$$

$$\Delta y_1 = (10 + \sqrt{2.1}) - (10 + \sqrt{2})$$

$$(10 + \sqrt{2.1}) - (11.4) = 0.04$$

(slope)

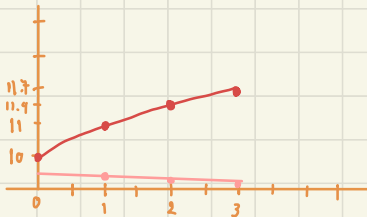
$$\Delta y = \frac{dy}{dx} \cdot \Delta x$$

$$\Delta y_2 = 0.35 \cdot (0.1)$$

$$= 0.035$$

$$\therefore 0.04 - 0.035 = 0.005$$

The first error is 0.005.



It means as x increases the slope is decreasing.

② when $x = 2$, $\Delta x = -0.2$

$$y = 10 + \sqrt{x}, \quad y = 10 + \sqrt{2}$$

$$x = 2 \rightarrow x = 1.8$$

$$x = 1.8, \quad y = 10 + \sqrt{1.8}$$

$$\Delta y_1 = (10 + \sqrt{2}) - (10 + \sqrt{1.8})$$

$$= 11.41 - 11.34$$

$$= 0.07$$

$$\Delta y_2 = m \Delta x$$

$$= 0.35(-0.2)$$

$$= -0.07$$

$$\therefore 0.07 - (-0.07)$$

$$= 0.07 + 0.07$$

$$= 0.14$$

The second error is 0.14.

H.W

Find 2nd order derivative of $y = 10 + \sqrt{x}$

and plot the graph of y and $\frac{dy}{dx}$.

Is the slope of slope a constant.

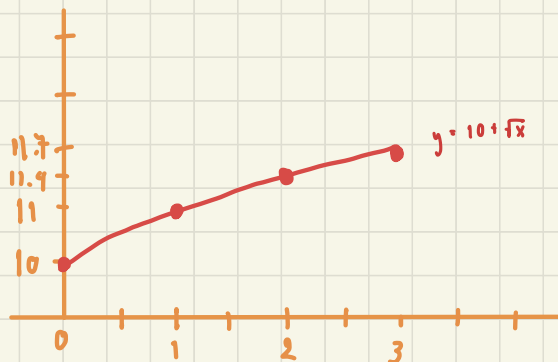
Ans

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

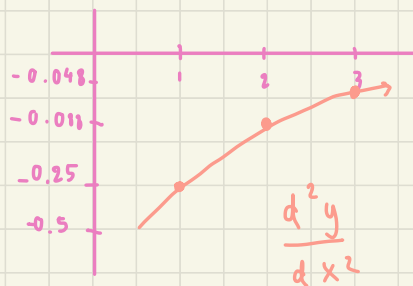
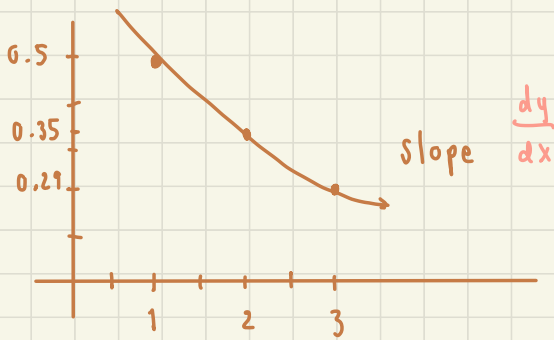
$$\frac{d(10 + \sqrt{x})}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$\frac{d\left(\frac{1}{2} x^{-\frac{1}{2}}\right)}{dx}$$

$$= -\frac{1}{4} x^{-\frac{3}{2}}$$



x	y	$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$	$\frac{d^2y}{dx^2} = -\frac{1}{4} x^{-\frac{3}{2}}$
0	10	undefined	undefined
1	11	0.5	-0.25
2	11.4	0.35	-0.088
3	11.7	0.29	-0.049



\therefore Slope of Slope is Not constant.