

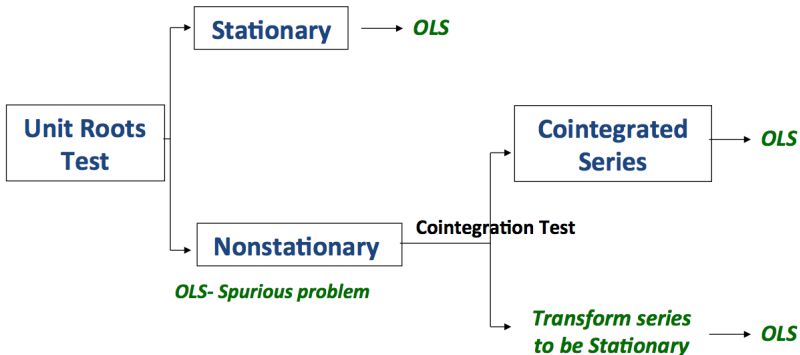
Time Series

EE426 Semester 2/2014

Part 2: Unit Root Test and Spurious Regression

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Basic Concept of Time Series



Unit root process

- ▶ A random walk is a special case of a unit root process
- ▶ AR(1): $y_t = \alpha + \rho y_{t-1} + e_t$ (1)
 - ▶ $E(e_t | y_{t-1}, y_{t-2}, \dots, y_0) = 0$
 - ▶ $\{y_t\}$ has a unit root if, and only if, $\rho = 1$.
 - ▶ If $\alpha = 0$ and $\rho = 1$, $\{y_t\}$ follows a random walk without drift.
 - ▶ If $\alpha \neq 0$ and $\rho = 1$, $\{y_t\}$ follows a random walk with drift.

Integrated series

- ▶ Nonstationary series can be integrated series if the series is differentiated one or more times, the resulting series will be stationary.
- ▶ Stationary process (weakly dependent process) is integrated of order zero, $I(0)$.
- ▶ A random walk is integrated of order one, $I(1)$, meaning that a first difference is weakly dependent, hence $I(0)$.
 - ▶ A time series that is $I(1)$ is a difference-stationary process.
- ▶ If y_t is nonstationary, but its d^{th} difference is stationary, the series is integrated of order d , or $I(d)$.

Testing for Unit roots

Model 1

- ▶ Null hypothesis: $\rho = 1$; Alternative hypothesis: $\rho < 1$ (one-sided test)
 - ▶ A test of whether $\{y_t\}$ is $I(1)$ against alternative that $\{y_t\}$ is $I(0)$
- ▶ A convenient equation for carrying out the unit root test is $\Delta y_t = \alpha + \theta y_{t-1} + e_t$, where $\theta = \rho - 1$ (2)
- ▶ Then, $H_0 : \theta = 0$, against $H_1 : \theta < 0$
- ▶ Under H_0 , y_{t-1} is $I(1)$, so the asymptotic standard normal distribution for t stat does not apply. We need new critical values.
- ▶ We use the Dickey-Fuller (DF) test for a unit root.

Testing for Unit roots

- ▶ Table of critical values for DF test:

Asymptotic Critical Values for Unit Root t Test: No Time Trend

Significance level	1%	2.5%	5%	10%
Critical value	-3.43	-3.12	-2.86	-2.57

- ▶ Still use the usual t statistic ($t_{\hat{\theta}}$), but compare to this table for asymptotic critical values.
- ▶ Reject H_0 if $t_{\hat{\theta}} < c$
- ▶ When we fail to reject a unit root, we should only conclude that the data do not provide strong evidence against H_0

Testing for Unit roots

Model 2

- ▶ We might need to test for unit roots in models with more complicated dynamics
- ▶ If $\{y_t\}$ follows AR(1) with $\rho = 1$, then Δy_t is serially uncorrelated.
- ▶ We can allow $\{\Delta y_t\}$ to follow an AR model by augmenting eq. (2) with additional lags
$$\Delta y_t = \alpha + \theta y_{t-1} + \gamma_1 \Delta y_{t-1} + e_t, \text{ where } |\gamma_1| < 1 \quad (3)$$
- ▶ That is, we can add p lags of Δy_t to the equation to account for the dynamics in the process.
 - ▶ How many lags depend on the frequency of the data.
- ▶ This extended version is called the augmented Dicky-Fuller test. Still use the same critical values as in Model 1.

Testing for Unit roots

Model 3

- ▶ Trend-stationary vs. unit root with trend
- ▶ $\Delta y_t = \alpha + \delta t + \theta y_{t-1} + e_t$ (4)
- ▶ $H_0 : \theta = 0$, $H_1 : \theta < 0 \rightarrow \{y_t\}$ is a trend-stationary process under alternative.
- ▶ If y_t has a unit root, then $\Delta y_t = \alpha + \delta t + e_t$
 - ▶ A change in y_t has a mean linear in t unless $\delta = 0$
- ▶ With time trend, the critical values of the unit root test change.

Asymptotic Critical Values for Unit Root t Test: Linear Time Trend

Significance level	1%	2.5%	5%	10%
Critical value	-3.96	-3.66	-3.41	-3.12

Spurious regression

- ▶ A situation of finding a relationship between two or more trending variables simply because each is growing over time.
 - ▶ Find high R^2 or statistically significant t-stat, but no meaning of this significant relationship
 - ▶ Check: $R^2 > \text{Durbin} - \text{Watson}$ → likely to have a spurious regression
- ▶ If the series are weakly dependent about their time trends, we can solve by including time trend in the regression model.
- ▶ We are likely to find a significant t-stat on a simple regression involving two independent $I(1)$ series even if these two series are not trending.
- ▶ If the series are $I(1)$ or higher order of integration, check for cointegration.