Homework

6304640284

1.) $P = a - bQ$, $Q = q_1 + q_2 + q_3$

$C = C_1 = C_2 = C_3$ Assume $C = TC$

What is equilibrium price? P^*

What are firm's profit? $\pi_1 = \pi_2 = \pi_3$?

$\pi_i = TR - TC = [a - b(q_1 + q_2 + q_3)]Q - c$

Max π_i: $\frac{\partial \pi_i}{\partial q_i} = 0 \rightarrow \frac{\partial \pi_i}{\partial q_1} = a - b(q_1 + q_2 + q_3)Q - c = 0$

$q_1 = a - bq_1 - bq_2 - bq_3$

$0 = a - 2bq_1 - bq_2 - bq_3$

$2bq_1 = a - bq_2 - bq_3$

$q_1^* = \frac{a - c - bq_2 - bq_3}{2b}$ — BR of firm 1 $\quad q_2^* = \frac{a - c - bq_1 - bq_3}{2b}$ — ② $\quad q_3^* = \frac{a - c - bq_1 - bq_2}{2b}$ — ③

sub (2) in (1); $q_1^* = \frac{a - c - b\left(\frac{a - c - bq_1 - bq_3}{2b}\right)}{2b}$ sub (1) in (2); $q_2^* = \frac{a - c - bq_3}{3b}$

$q_1^* = \frac{a - c - bq_3}{3b}$

sub q_1^* and q_2^* in (3); $q_3^* = \frac{a - c - b\left(\frac{a - bq_3}{3b}\right) - b\left(\frac{a - bq_3}{3b}\right)}{2b}$

$= \frac{a + 2bq_3}{6b}$

$6bq_3^* = a + 2bq_3$

$4bq_3^* = a \rightarrow q_3^* = \frac{a}{4b}$

sub q_3^*; $q_1^* = \frac{a - b\left(\frac{a}{4b}\right)}{3b}$

$q_1^* = \frac{a}{4b}$

$q_2^* = \frac{a}{4b}$

$P = a - bQ = a - 3b\left(\frac{a}{3b}\right)$

$= a - \frac{3a}{4}$ $\quad \therefore P = 0.25a$

$\pi_i = (P - C)q_i$

$\pi_1 = (0.25a - c)\frac{a}{4b}$ $\quad \therefore \pi_1 = \frac{a^2}{16b} - c, \quad \pi_2 = \frac{a^2}{16b} - c, \quad \pi_3 = \frac{a^2}{16b} - c$

2.) If there are N Firms

$q_i^* = f(N)$, $P = f(N)$, $\Pi_i = f(N)$

Assume $q_1 + q_2 + \dots + q_n = A$

$P = a - b(q_1 + q_2 + \dots + q_n)$

$= a - bq_1 - bq_2 - \dots - bq_n$

$\Pi_i = (P - C)\,q_i$ $\quad \Pi_1 = (a - bq_1 - bq_2 - \dots - bq_n)q_1 - c_1$

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \rightarrow q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n)$$

$$\vdots$$

$$q_n = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_{n-1})$$

$$\therefore q_1^* = q_2^* = \dots = q_n^* = \frac{a}{b} - A$$

$A = q_1 + q_2 + \dots + q_n$

$= n\left(\frac{a}{b} - A\right)$

$= \frac{na}{b} - nA$

sub A into (1); $q_i = \frac{a}{(n+1)b}$

$A + nA = n\left(\frac{a}{b}\right)$

$A(n+1) = n\left(\frac{a}{b}\right) \rightarrow \frac{na}{(n+1)b}$

$P = a - bA$

$= a - b\left(\frac{na}{(n+1)b}\right)$

$= a - \left(\frac{n}{n+1}\right)a$

$= \frac{na + a - na}{n+1} \longrightarrow P = \frac{a}{n+1}$

$\Pi_i = P \cdot q_i + c_i$

$= \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - c_i \longrightarrow \Pi_i = \frac{a^2}{(n+1)^2 b} - c_i$

3. From question 2, what happen if $N \to \infty$

$N \to 1$

Usually

If $N \to \infty$, it means the market output (Q) goes to a competitive level and the price (p^*) converges to marginal cost.

The market is perfectly competitive market.

In cournot model, there must be 2 or more firms competing in the market. So if $n = 1$, firm will become monopolist.

If $n \to \infty$

$q_i = \frac{a}{(n+1)b}$ will be nearly to 0 and each firm will set at q nearly 0 unit

$A = \frac{na}{(n+1)b}$ will be nearly to 0 , q of every firms conbined will be nearly to ∞ units

$P = \frac{a}{n+1}$ will be nearly to 0, supply will increases as P decreases to nearly 0

$\Pi_i = \frac{a^2}{(n+1)^2 b} - c_i$ each firm will be lose their profit

If $n = 1$

$q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ since $q = a/2b < q = na/(n-1)b \longrightarrow$ monopoly will sell less

$A = \frac{na}{(n+1)b} = q$ it mean firm will become a monopolist

$P = \frac{a}{n+1} = \frac{a}{2}$ since $P_m = a/2 > P_c = a/n+1$, monopoly will set higher price

$\Pi_i = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i$ since $\Pi_m > \Pi_c$, monopolist will earn higher profit.