EE312 Macroeconomics, 2/2013 (Sec. 046402)
Chapter 4. Solow Growth Model

1. **Population growth :** $N' = (1+n)N : n > -1$

2. **Consumers :**
$$\begin{aligned} Y &= C + S \\ S &= sY \\ C &= (1-s)Y \end{aligned}$$

3. **Firm: The Neoclassical Production Function**

   (a) Constant returns to scale. $F(\lambda K, \lambda N) = \lambda F(K, N)$; for all $\lambda > 0$

   (b) Positive and diminishing returns to private inputs: For all $K > 0$ and $L > 0$, F exhibits positive and diminishing marginal products with respective to each input.

   (c) Inada conditions: The marginal product of capital (or labor) approaches infinity as capital (or labor) goes to 0 and approaches 0 as capital (or labor) goes to infinity.

4. **Production function :** $Y = zF(K, N)$

5. **Per worker production function:** $y = zf(k)$

   Let y=$\frac{Y}{N}$ = output per worker and $\frac{K}{N} = k$ = capital per worker

6. **Growth of capital stock :** $K' = (1-d)K + I$

7. **Equilibrium Output**

   In this economy, there are two markets in the current period.

   (a) consumption goods are traded for current labor

   (b) consumption goods are traded for captial

   Consumers save by accumulating capital.

   At equilibrium, $S = I$ so that $Y = C + I$.

   Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving).

$$\begin{aligned} C &= (1-s)Y \\ I &= K' - (1-d)K \end{aligned}$$

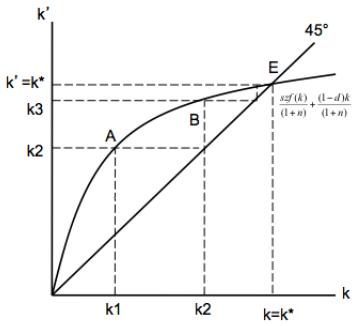
   Per worker formulation

$$\begin{aligned} Y &= \text{.....} \\ K' &= \text{.....} \\ &= \text{.....} \\ \frac{K'}{N} &= \text{.....} \end{aligned}$$

8. **Future capital per worker function** ($k' = \frac{K'}{N'}$)

$$\begin{aligned} \frac{K'}{N} \times \frac{N'}{N'} &= \\ k'(1+n) &= \text{.....} \\ k' &= \text{.....} \end{aligned}$$

9. **The steady-state capital per worker**



- Diminishing returns on k
- At A, k2 > k1; k is growing.
- At B, k3 > k2; k is growing.
- k = k*; steady-state capital per worker.
- As k is increasing, MPk is falling so that **y is increasing at a decreasing rate.**
- To the left of k*, k'> k so that k is increasing.
- To the right of k*, k'< k so that k is decreasing.
- At E, k = k'= k* so that k* is steady. k*; steady-state capital per worker.
- Finally, investment (or new capital) is just sufficient to keep up with population growth anddepreciation, so that k (and y) is stagnant.

$$\begin{array}{lcccc} K & = & k^*.... & & \\ Y & = & y^*.... & = & zf(k^*)N \\ I & = & sY & = & szf(k*)N \\ C & = & (1-s)Y & = & (1-s)zf(k^*)N \end{array}$$

Given population growth (n), total factor productivity (z) and the saving rate (s), the steady-state growth rate is '.....' for aggregate quantities.

Solow model tells us that growth in key macroeconomic aggregates is determined by exogenous labor force growth when the saving rate, the labor force growth rate, and total factor productivity are constant.

Solow's model states that investment in capital cannot drive long run growth in GDP per worker.

Policy lesson: don't advise poor countries to invest without due regard for technology and incentives. Capital deepening (an increase in capital per worker) cannot lead to a sustained economic growth in the long run.

10. **Analysis of the steady-state**

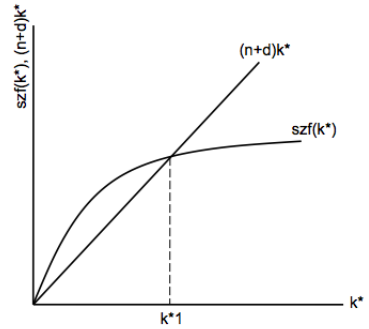
$$k* = ...$$

$$(n+d)k* = ...$$

Steady State Investment = Steady State Saving

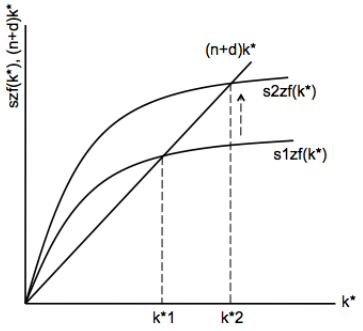
- szf(k*) = saving per worker;
- (n+d)k* = investment per worker needed to keep up with population growth and depreciation.
- At k*, the capital stock is still growing, but just sufficient to equip each worker with the same k and depreciation (so k* is steady).
  - **'Capital widening'**: growing K just to keep the steady k and y.

11. **Determination of steady-state k***

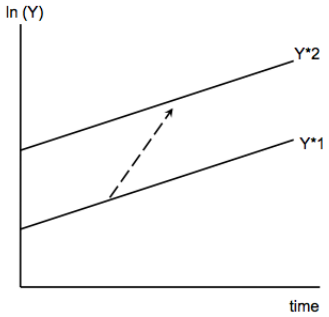


- szf(k*) is concave due to zf(k*).
- (n+d)k* has the slope = (n+d).
- steady-state capital, k^*, is determined by exogenous factor : n, d, s, z

12. **Effect of an increase in s**

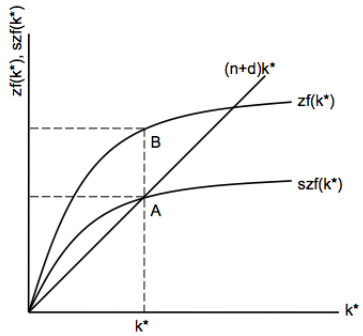


- Higher saving rate results in a higher k* and y*.
- **A rise in s raises k*.**
- consistent with empirical study : $s \uparrow$, $y^* \uparrow$



- **Temporary gain in growth rate**
- K and Y move to new 'growth paths'.
- Higher growth rates of k and y are transitional, converging to n.

13. **Steady-state consumption per worker**

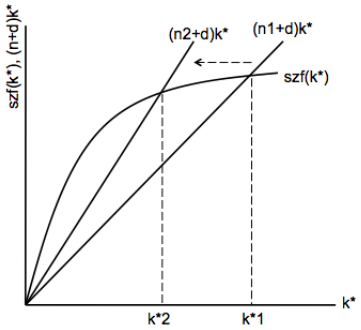


$$c* = zf(k*) - szf(k*)$$

$$c* = zf(k*) - (n+d)k*$$

- $c* = y* - szf(k*) = zf(k*) - (n+d)k*$.
- AB = c*

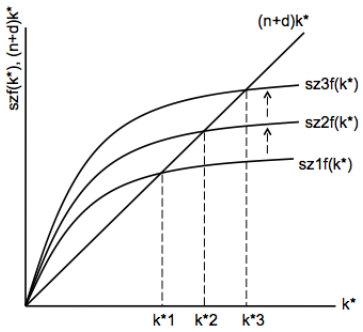
14. **Effect of an increase in n**



- Higher population growth (n) results in lower k* and y*.
- **A higher n with lower k***
- Growth of all aggregate variables .....
- This shows that the higher growth in aggregate income neednot be associated, in the long run, with higher income per worker.
- consistent with the empirical fact: the higher population growth, the lower output per worker. High population growth corresponds with low living standards.

- Intuition : As n increases, k^* decreases. As capital per worker falls, output per worker falls. The reason is that when labor force grows at a higher rate, the current labor force faces a tougher task in bulding capital for the next period's consumers, who are a proportionately a larger group. Thus, output per worker and capital per worker decreases at steady state.

15. **Effect of an increase in z**



- Sustained increases in z cause sustained improvements in y*.

16. Note :

- "Solow's model states that investment in capital cannot drive long run growth in GDP per worker.
- Need technological change (growth in A) to avoid diminishing returns to capital.
- Easterly (2001) argues that "capital fundamentalism" view widely held in World Bank/IMF from 60s to 90s, despite lessons of Solow model.
- Policy lesson: don't advise poor countries to invest without due regard for technology and incentives. Capital deepening (an increase in capital per worker) cannot lead to a sustained economic growth in the long run."
  (borrowed from Chapter 3 in Easterly (2001))
- Solow-Swan, or neoclassical, growth model, implies countries converge to steady state GDP per worker (if no growth in technology)
- Changes in savings ratio causes "level effect", but no long run growth effect higher labour force growth, ceteris paribus, implies lower GDP per worker
- Exact outcome of Solow model does depend on precise functional forms and parameter values.
- With standard production function, Solow model predicts economy moves to steady state because of diminishing returns to capital (assuming no growth in technology z).
- An increase ina country's propensity to save or a decrease in the labor force growth rate imply one time increase in a country's standard of living, but there can be unbounded growth in the standard of living if and only if total factor productivity increases.