Lecture 8
Term Structure of Interest Rates

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FN 312 – INVESTMENTS

Review

- **What is the yield curve?**

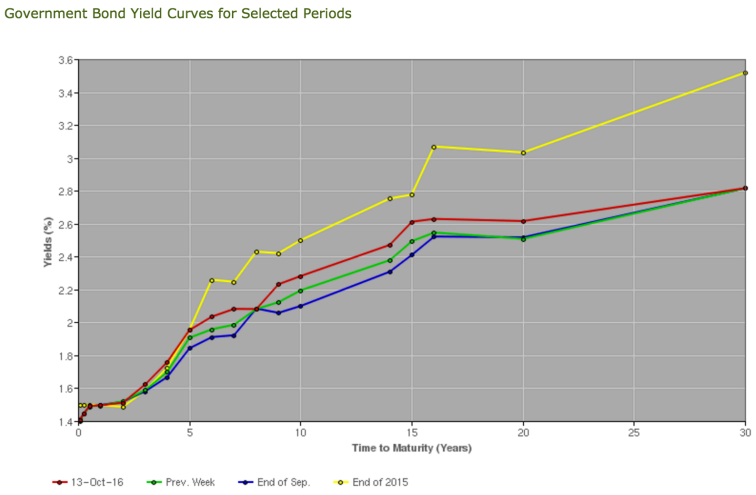
A set of interest rates observed at one point in time for varying maturities

- **What explains the shape of the yield curve?**
  - Upward Sloping
  - Inverted or downward sloping
  - Flat
  - Hump-Shaped
- **Why do interest rates for different maturities move together?**

→ Expectations Hypothesis: The term structure rises or falls because the market expects interest rates in the future to be higher or lower

→ Liquidity Premium Theory: There is a bias toward positively sloped yield curves

Term Structure of Interest Rates



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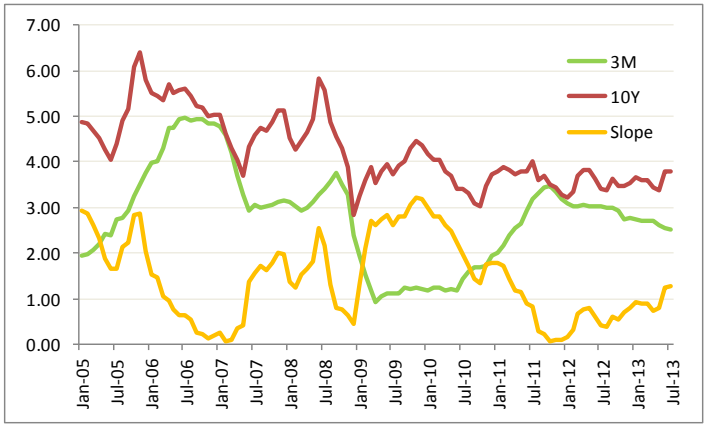
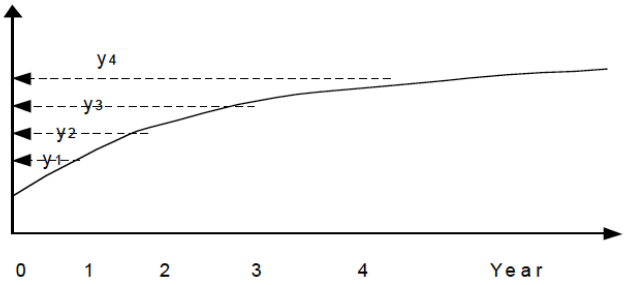
Thailand's Term Spread

- Thailand's yield curve always slopes upwards as yields on the longer term bonds always exceed those of the shorter term ones



The Term Structure

- The term structure of interest rate (yield curve) presents yield of zero-coupon bonds for various investment maturities



Bond Pricing and the Term Structure

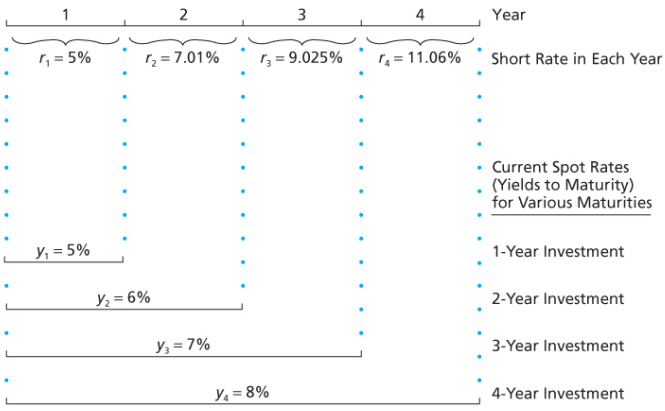
Spot Rates

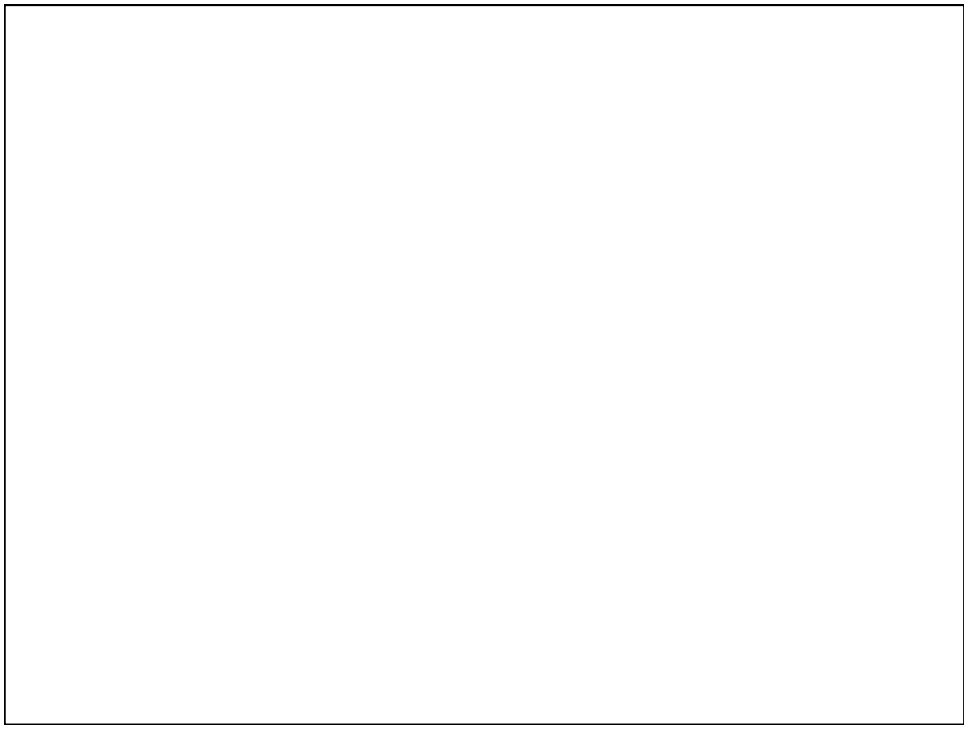
- Spot rates are derived from zero coupon US Treasury bond prices
- Spot rates is the rate that prevails today for a time period corresponding to the zero's maturity
- Spot rates is the yield that is equivalent to the discount rate at time t

→ The yield to maturity of a zero coupon bond maturing in n years = discount rate for a cash flow at year n

Spot rates vs. short rates

- Spot rate – the rate that prevails today for a given maturity
- Short rate – the rate for a given maturity (eg. one year) at different points in time





Arbitrage

- Arbitrage opportunity: A riskless trading strategy that costs nothing to implement but generates a positive profit (free money!)
- In well functioning financial markets, there should not be arbitrage opportunities. If arbitrage opportunities occur, then market prices adjust until the arbitrage opportunities disappear. This should happen quickly.

Example

- Suppose the 2 year spot rate is 0.05. That is, you can borrow and lend risklessly for 2 years at an annual rate of 0.05
- The no-arbitrage price of a 2 year ZCB with face value of $100 is
- Now suppose that the current market price of the 2 year zero is $85. Then there is an arbitrage opportunity.
- The arbitrage opportunity causes the demand for the underpriced 2 year zero to increase
- Prices will increase until the arbitrage opportunities disappear, that is, the price will increase until it equals the no-arbitrage price of $90.70

Arbitrage Strategy

Recall: Spot and Short Rates

- We used the concept of arbitrage to derive relationships between spot and short rates in a world of certainty.

→ n year spot rate is a geometric average of the 1 year short rates

Eg. in a world of certainty, the return from holding a 2 year bond should be exactly the same as the return from rolling over 2 one-year bonds.

Example

Suppose you know with *certainty* the following sequence of one year rates:

$$r_{0,1} = 7\%$$
$$r_{1,2} = 9\%$$

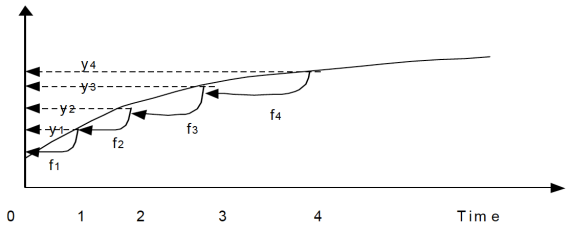
Calculate the no-arbitrage two year rate.

Forward Interest Rates

- In our previous example, our short rates are known with certainty
- In reality, we do not know with certainty any future interest rates
- We can use today's term structure, however, to infer something about future rates
- Future rates inferred from today's spot rates are called (implied) forward rates

Forward Interest Rates

- The forward rate is the "break-even" interest rate that equates the return on an n-period ZCB to that of an n-1 period ZCB rolled over into 1 year bond in year n



Forward Rates and the Term Structure

- Flat term structure
- Upward sloping term structure
- Downward sloping term structure

Examples

- What is the price of a 3 year 10% coupon bond?

T	Price	yield	forward
1	934.58	7%	7%
2	857.34	8%	9%
3	782.91	8.5%	9.5%

- Check whether using yields and forward rates give you the same price

Interest Rate Uncertainty and the Forward Rate

- The forward rate only equals the short rate when interest rates are certain.
- In general, the interest rate that will prevail in the future need not equal the forward rate, which is calculated from today's data
- It is not even necessary that the forward rate equals the expected value of the future short interest rate
- Why use the forward rate?
  - Used in a forward contract. A forward contract is a contract you enter into today to borrow/lend in the future at an interest rate that is specified today
  - Useful for understanding the theories of the term structure of interest rates

The forward rate as future borrowing rate

- A company will work on a short project that starts one year from today and ends 2 years from today. It will need a loan at the start of the project of $1000 and will get all the revenues to pay back the loan at the end of the project.
- The analysis of the project is based on current interest rates so you want to fix the future borrowing rate today.
- How would you use positions in the 1 and 2 year bonds to fix your borrowing rate today for borrowing $1000 between year 1 and 2 (disregard your default risk for now)?

t	price	yield	forward
1	952.38	5%	5%
2	898.45	5.5%	6.0
3	839.62	6%	7.12

Cash flow (year)	0	1	2
Buy 1 one-year zero	-952.38	1000	
Sell 1.06 (952.38/898.45) of 2.0 year zero	952.38	0	-1060
Net	0	1000	-1060

Interest rate = (1060-1000)/1000 = 6%

Theories of the Term Structure

- Recall that the future short rate is the interest rate that will prevail in a future period. In general it is not known for certain today
- When the future is uncertain what should be our best estimate of the future short rate?

→ The forward rate can be viewed as a forecast of the future short rate of interest.

→ How good is this forecast?

Empirical facts

Fact 1: Interest rates for different maturities tend to move together over time

Fact 2: Yields on short term bonds are more volatile than long term bonds

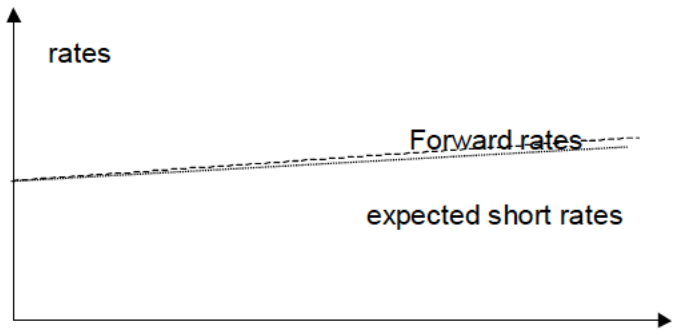
Fact 3: Long term bond yields tend to be higher than short term bond yields (upward sloping yield curve)

The Expectations Hypothesis

- Expectation Hypothesis
  - Term structure determined solely by expectations of future interest rates
  - The term structure rises or falls because the market expects interest rates in the future to be higher or lower
  - If the expectations hypothesis is true, then the forecast error has mean zero and is uncorrelated with the forward rate.
  - In other words, the forecast error is white noise or has no systematic component that can improve the forecast of the future spot rate

Implication of the Expectation Hypothesis

- If the Expectation Hypothesis is true, then investing in a succession of short term bonds is the same, on average, as investing in long term bonds.
- Empirically, Eugene Fama of the University of Chicago found that expectation hypothesis is not an exact depiction of the real world. ie. when forward rates exceeded the spot rates, future spot rates rise but by less than predicted by theory
- Implication: Investing in long term bond tends to give higher return than rolling over series of short term bonds
- The failure of the expectation hypothesis may be due to risk averse behavior of investors



Modifying the Expectations Hypothesis

- Liquidity Preference Theory
  - Expectation hypothesis omits fundamental notion that there is risk associated with longer term investments.
  - A long term treasury bond is more risky than a short term treasury bill – longer time over which interest rate changes can occur impacting price

- Liquidity preference suggests that investors require additional interest to account for additional risk to hold longer term investments. That is, the higher risk of longer term investments makes spot rates for longer maturities higher than spot rate for shorter maturities.
- The liquidity preference theory suggests an upward sloping term structure
- The difference between the higher forward rate and the spot rate is termed the liquidity premium

Interpreting the Term Structure

- Since the yield curve reflects expectations of future short rates, we can use the term structure to infer the expectations of other investors in the economy and use those expectations as benchmarks for our own analysis.
- Unfortunately, it is hard to extract information on interest rate expectations. Why?
  - The term structure also has information about liquidity premiums.
  - forecast of interest rate changes may be coming from changes in inflation expectations or the real interest rate, which has different implications for proper interpretation of the term structure
  - Changes in economic events can complicate analysis

Forward Inflation Rates

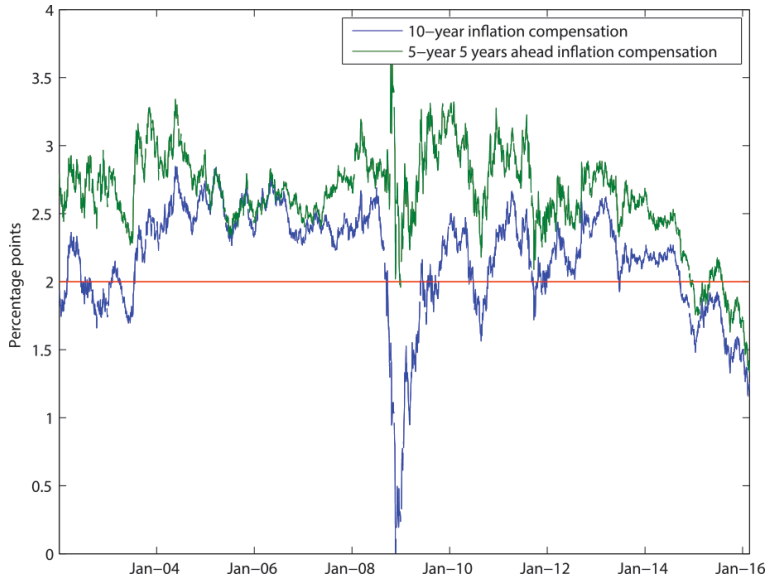
- Forward rates from bonds are nominal rates
- Using price-level indexed bonds such as Treasury inflation protected securities (TIPS) we can calculate the forward real interest rate and forward inflation rates
  - TIPS guarantees you a real, rather than a nominal return. The principal accrues with inflation throughout the lifetime and the coupon payment is based on a real rate of return

1st Year		*2nd Year*	
Principal	$1,000	Principal	$1,040
Inflation	4%	Inflation	5%
Inflation Accrual	$ 40	Inflation Accrual	$ 52
Principal at the end of the first year	$1,040	Principal at the end of the first year	$1,092
Real Rate of Return	3%	Real Rate of Return	3%
Interest ($1,040 X 3%)	$ 31	Interest ($1,092 X 3%)	$ 33
Total Return	7.1%	Total Return	8.2%
	$ 71		$ 85

- Does this mean that TIPs will outperform bonds?
  - The yield on a bond may be high enough to beat the yield on TIPS even with inflation because the price of TIPs depend on the market's expectations of inflation and whether those expectations are realized
  - TIPs usually carry interest rates lower than other government or corporate securities so they may be less optimal for investors especially when inflation is minimal or nonexistent

The Break-Even Inflation Rate

- The inflation rate that makes neither the nominal or real bond more attractive is called the break-even inflation rate
- The break-even inflation rate is a market based measure of expected inflation. It is the difference between the yield of a nominal bond and an inflation linked bond of the same maturity.

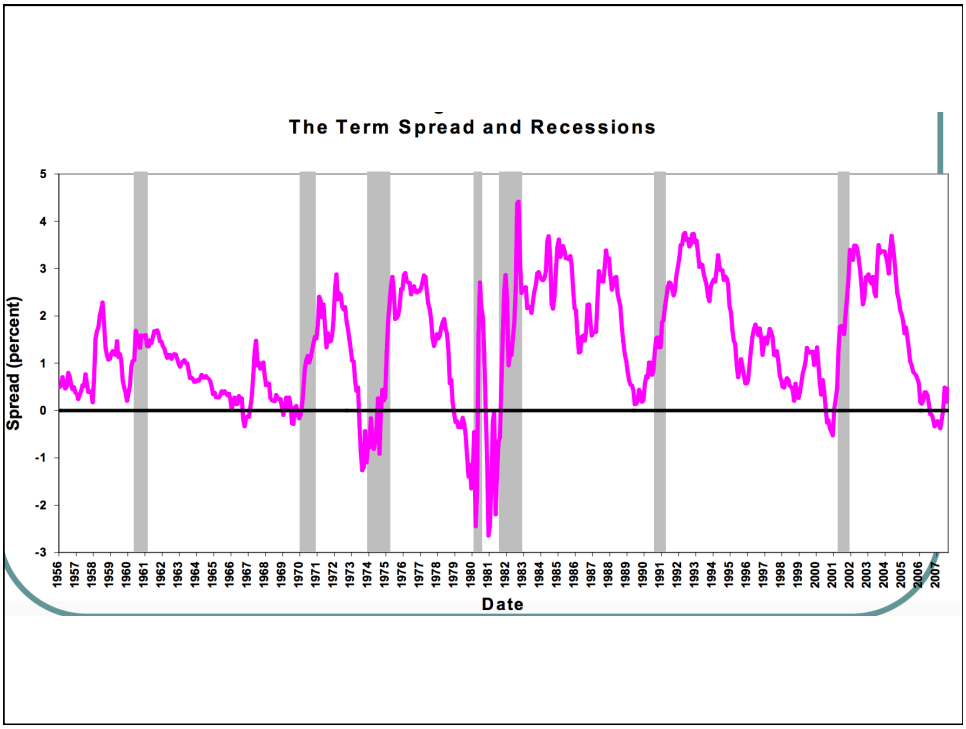


Caveat

- Inflation compensation = expected inflation + inflation risk premium + other factors
- So for example, we can have a falling break even rate even when inflation expectations are stable given that we have a large liquidity premium

Final words on the term structure

- The yield curve reflects expectations of future interest rates
- The forecasts of future rates however, are clouded by other factors, such as liquidity premium
- An upward sloping curve could indicate
  - Rates are expected to rise and/or
  - Investors require large liquidity premium to hold long term bonds
- The yield curve is a predictor of the business cycle
  - Long term rates tend to rise in anticipation of economic expansion
  - Inverted yield curve may indicate that interest rates are expected to fall and signal a recession



Homework

- Ch 15 # 1, 7, 8, 9, 11, 13, 18, 19