

Exercise II
solutions

067

①

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

(b) From U in part (a) we see that every column is a pivot column. The pivot columns from A are a basis for the column space: $(2, 2, 0)$, $(2, 5, 3)$, $(1, 0, 2)$. Since the rank is three, the column space is all of \mathbb{R}^3 , so another basis would be the standard basis $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. In fact, any three independent vectors in \mathbb{R}^3 will do.

(c) The rank is three because there are three pivots.

②

$$c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = 0$$

Perform elimination process

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$c \neq 0$$

\therefore These vectors are dependent.

$\&$ They do not span \mathbb{R}^4

③

Columns 2 & 3 are a basis for $\text{col } A$

④

Take any u in H - say $u = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ - and take any $c \neq 1$ - say $c = 2$

Then

$$cu = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$

If this is in H , then there is some s such that

$$\begin{bmatrix} 3s \\ 2+5s \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \end{bmatrix} \Rightarrow \text{that is } s = 2 \text{ and } s = \frac{12}{5} \text{ which is impossible}$$

Solve $AX=0$
 Yes, it is in $\text{Nul } A$

(6) \underline{w} is in $\text{Col } A \rightarrow$ by checking that $AX=w$ is consistent
 \underline{w} is in $\text{Nul } A \rightarrow$ by checking $AX=0$

vector x which is solution to

i.e. $x = c \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(7) Let $A = [\underline{v}_1 \ \underline{v}_2]$. Row operations show that

$$A = \begin{bmatrix} 1 & -2 \\ -2 & 7 \\ 3 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

not every row of A contains a pivot position. So the columns of A do not span \mathbb{R}^3 , by Hence $\{\underline{v}_1, \underline{v}_2\}$ is not a basis for \mathbb{R}^3 .

Since \underline{v}_1 and \underline{v}_2 are not in \mathbb{R}^2 , they cannot possibly be a basis for \mathbb{R}^2 . However, since \underline{v}_1 and \underline{v}_2 are obviously

linearly independent, they are a basis for a subspace of \mathbb{R}^3 , namely,

Span $\{\underline{v}_1, \underline{v}_2\}$

(8)
$$\begin{bmatrix} 1 & 6 & 2 & -4 \\ 3 & 2 & -2 & -8 \\ -4 & -1 & 3 & 9 \end{bmatrix} \xrightarrow{\text{sequence of row operations}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3/2 \\ 0 & 0 & 1 & 5/2 \end{bmatrix}$$

The Basis for $W = \left\{ \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} \right\}$