



B.E. International Program

Faculty of Economics, Thammasat University



EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Practice Problem 10 (Integrations and Its Applications)¹

1. In the manufacture of a product, the marginal cost of producing x units is $C'(x)$ and fixed cost are $C(0)$. Find the total cost function $C(x)$ when:

(a) $C'(x) = 3x + 4$, $C(0) = 40$.

(b) $C'(x) = ax + b$, $C(0) = C_0$.

2. (a) Find $F(x)$ if $F'(x) = \frac{1}{2}e^x - 2x$ and $F(0) = \frac{1}{2}$.

(b) Find $F(x)$ if $F'(x) = x(1 - x^2)$ and $F(0) = \frac{5}{12}$.

3. Compute the area A bounded by the graph of $f(x) = \frac{1}{x^3}$, the x -axis, and the two lines $x = -2$ and $x = -1$. Make a drawing. (Hint: $f(x) < 0$ in $[-2, -1]$.)

4. Compute the area A bounded by the graph of $f(x) = \frac{1}{2}(e^x - e^{-x})$, the x -axis, and the two lines $x = -1$ and $x = 1$.

¹ All questions are from Sydsaeter and Hammond, 2008.

5. (a) The profit of a firm as a function of its output x is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, \quad x > 0$$

Find the level of output that maximizes profit. Sketch the graph of f .

(b) The actual output varies between 1000 and 3000 units. Compute the average profit

$$I = \frac{1}{2000} \int_{1000}^{3000} f(x) dx.$$

6. Let $K(t)$ denote the capital stock of an economy at time t . Then net investment at time t , denoted by $I(t)$, is given by the rate of increase $\dot{K}(t)$ of $K(t)$. [Note: $\dot{K}(t) = \frac{dK}{dt}$.]

(a) If $I(t) = 3t^2 + 2t + 5, t \geq 0$, what is the total increase in the capital stock during the interval from $t = 0$ to $t = 5$?

(b) If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time $t = t_0$ to $t = T$ when the investment function $I(t)$ is as in part (a).

7. Suppose that the demand and supply curves are $P = f(Q) = 200 - 0.2Q$ and $P = g(Q) = 20 + 0.1Q$, respectively. Find the equilibrium quantity and compute the consumer and producer surplus.

8. Suppose that the demand and supply curves are $P = f(Q) = \frac{6000}{Q+50}$ and $P = g(Q) = Q + 10$. Find the equilibrium price, and compute the consumer and producer surplus.

9. Evaluate the following integrals by using integrations by parts ($r \neq 0$).

$$(a) \int_0^T bte^{-rt} dt$$

$$(b) \int_0^T (a + bt)e^{-rt} dt$$

10. Evaluate the following integrals by using integrations by substitution:

$$(a) \int_0^1 x\sqrt{1+x^2} dx$$

$$(b) \int_1^e \frac{\ln y}{y} dy$$