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CHAPTER 3

Static and Comparative Static Equilibrium Analysis

Topics: Static and Comparative Static Equilibrium Analysis

Outline:

- Individual and market demand
- Individual and market supply
- Excise tax and market equilibrium
- Y-DAE model
- IS-LM model
- Breakeven analysis
- Examples of nonlinear model

Market Demand and Market Supply

How can we derive market demand and market supply?

Market demand can be found by summing individual inverse demand function horizontally.

Suppose that:

Individual demand function for energy drink of individual 1 is: $q_1^d = a_1 - b_1 P$

Individual demand function for energy drink of individual 2 is: $q_2^d = a_2 - b_2 P$

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Individual demand function for energy drink of individual n is: $q_n^d = a_n - b_n P$

At each price p, we want to find total demand from every consumers in the market.

The total amount demanded is the horizontal sum of the individual demand curves.

This can be done as the following.

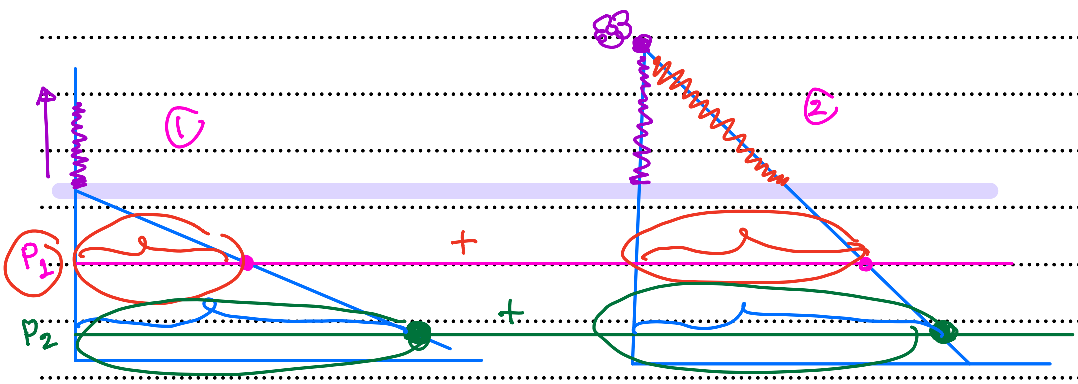
$Q^d = \sum_{i=1}^{n} q_i^d$

At each level of P

$\boxed{Q^d = q_1^d + q_2^d + \dots + q_n^d}$

$= (a_1 - b_1 P) + (a_2 - b_2 P) + \dots + (a_n - b_n P)$

$Q^d = (a_1 + a_2 + \dots + a_n) - (b_1 + b_2 + \dots + b_n) P$



Market supply can be found by summing individual inverse supply function horizontally.

Suppose that:

Individual supply function for energy drink of producer 1 is: $q_1^s = -c_1 + d_1 P$

Individual supply function for energy drink of producer 2 is: $q_2^s = -c_2 + d_2 P$

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Individual supply function for energy drink of producer n is: $q_m^s = -c_m + d_m P$

At each price p, we want to find total supply from every producers in the market. The total amount supplied is the horizontal sum of the individual supply curves. This can be done as the following.

At each price p, $Q^s = \sum_{i=1}^{m} q_i^s$

$= (-c_1 + d_1 p) + \ldots + (-c_m + d_m p)$

$Q^s = -(c_1 + c_2 + \ldots + c_m) + (d_1 + \ldots + d_m) p$

Example: Suppose that market A has three consumers, Melanie, Andre , and Wilford. Demand of each consumer is as follows:

Price of A	Demand of Melanie	Demand of Andre	Demand of Wilford	Mkt. Demand
1	6	10	16	32
2	4	8	13	25
3	2	6	10	
4	0	4	7	11
5	0	2	4	

Find individual demand function and market demand function

Melenie : $q_1^d = a_1 - b_1 p$

$q_1^d = 8 - 2p$

Andre : $q_2^d = a_2 - b_2 p$

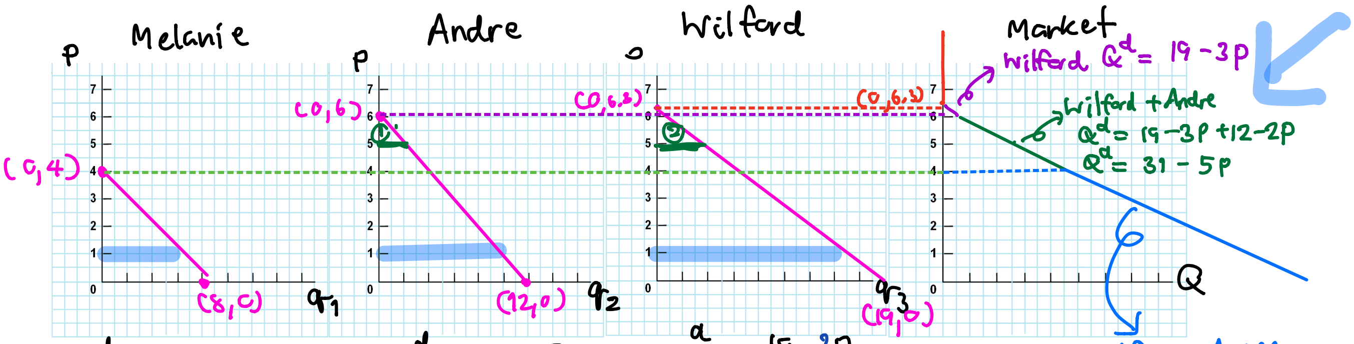
$= 12 - 2p$

Wilford : $q_3^d = a_3 - b_3 p$

$= 19 - 3p$

$$Q^d = \begin{cases} 19-3P, & 6 < P < 6.33 \\ 31-5P, & 4 < P < 6 \\ 39-7P, & 0 < P < 4 \end{cases}$$

$$P^d = \begin{cases} 6.33 - \frac{1}{3}Q, & 6 < P < 6.33 \\ 6.2 - \frac{1}{5}Q, & 4 < P < 6 \\ 5.6 - \frac{1}{7}Q, & 0 < P < 4 \end{cases}$$



$q_1^d = 8-2p$

$p^d = 4 - \frac{1}{2}q_1$

$q_2^d = 12-2P$

$p_2^d = 6 - \frac{1}{2}q$

$q_3^d = 19-3p$

$p_3^d = 6.3 - \frac{1}{3}q$

Wilford + A + M

$Q^d = 19-3P+12-2P+8-2P = 39-7P$

Example

Market demand for goods X is:

$$Q^D = 400 - 2P$$

Individual supply of each producer in the market is:

$$\boxed{q_i^S = -8 + P}$$

Please answer the following questions:

a.) If there are 10 producers, what are equilibrium market price and quantity?

$Q^S = \sum_{i=1}^{10} q_i^S = 10q_i^S = 10(-8+P) = -80+10P$

At mkt. eq. : $Q^D = Q^S$

$400 - 2P = -80 + 10P$

$P^* = \frac{480}{12} = 40$

$Q^* = 400 - 2(40) = 320$

b.) If government want to have equilibrium market price at 56 baht per unit, how should government implement each of below policies?

b.1) If there are 10 producers, how much tax or subsidy should government impose or give to each producer?

$P^*_{AT} = 56$ baht / unit

$P^*_{BT} = 40$ baht / unit

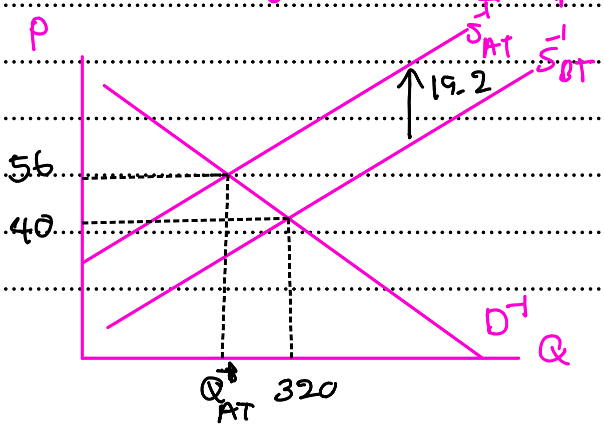
$Q^D = 400 - 2P$, $Q^S = -80 + 10P$

At mkt eq. $Q^D = Q^S$

$400 - 2P = -80 + 10(P - t)$

$400 - 2(56) = -80 + 10(56 - t)$

$t = 19.2$ ฿ / unit



b.2) If government doesn't want to use tax or subsidy, how many producers should government allow to be in the market? $P^*_{After} = 56$

At mkt eq. $Q^D = \boxed{Q^S}$

$400 - 2P = \boxed{m(-8 + P)}$; Supposing that there are m producers in the mkt.

$400 - 2(56) = m(48)$

$m^* = 6$

The Expenditure-Output (or Keynesian cross) model

Consider A full-version of open economy Keynesian cross model

$Y = C+I+G+X-M$

$DAE = C+I+G+X-M$ [Definitional equation]

output

$Y = DAE$ [Conditional equation]

$Y_d = Y - T$ [Definitional equation]

$T = tY$

$C = C_0 + bY_d$

$I = I_0$

$G = G_0$

$X = X_0$

$M = mY$

behavioral equation

Endogenous variables: Y, Y_d, T, C, M

Exogeneous variables: C_0, I_0, G_0, X_0

Parameters: b, t, m

We can solve for the equilibrium values of all endogenous variables as follows:

$Y = DAE$

$Y = C+I+G+X-M$

$Y = C_0 + b(Y - tY) + I_0 + G_0 + X_0 - mY$

⋮

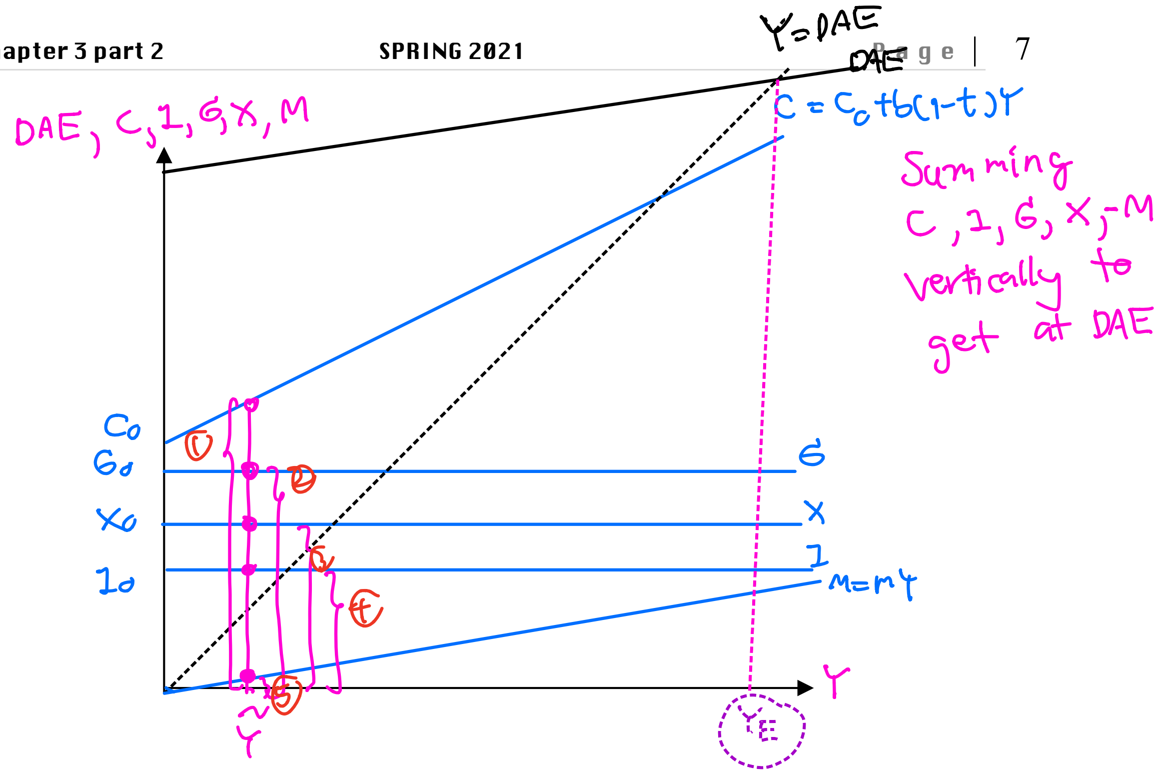
$Y_E = \dfrac{C_0 + I_0 + G_0 + X_0}{1 - b(1-t) + m}$ endo $= f($ exo, para $)$

$T_E = tY_E = t\left(\dfrac{C_0 + I_0 + G_0 + X_0}{1 - b(1-t) + m}\right)$

$Y_{d,E} = (1-t)Y_E$

$C_E = C_0 + b(1-t)Y_E$

$M_E = mY_E$



With comparative static analysis, the multipliers for autonomous expenditures, C_0, I_0, G_0, X_0 are:

$$\boxed{Y_E = \frac{C_0 + I_0 + G_0 + X_0}{1 - b(1-t) + m}} \quad (1.)$$

Suppose C_0 changes to C_0'

$$Y_E' = \frac{C_0' + I_0 + G_0 + X_0}{1 - b(1-t) + m} \quad (2.)$$

$$Y_E' - Y_E = \frac{C_0' - C_0}{1 - b(1-t) + m}$$

$$\Delta Y_E = \frac{1}{1 - b(1-t) + m} \Delta C_0$$

The multiplier $\frac{\Delta Y_E}{\Delta C_0} = \frac{1}{1 - b(1-t) + m} = \frac{\Delta Y}{\Delta I_0} = \frac{\Delta Y}{\Delta G_0} = \frac{\Delta Y}{\Delta X_0}$

If we are interested in how balance of trade, $X - M$, will change as a response of change in government expenditure, the analysis will be accordingly:

$B = X - M = X_0 - mY$

$\Delta G_0 \longrightarrow \Delta B$?

$\boxed{\Delta G_0} \longrightarrow \Delta Y_E = \frac{1}{1-b(1-t)+m}\Delta G_0 \longrightarrow \Delta M_E = m\Delta Y_E$

$= \frac{m}{1-b(1-t)+m}\Delta G_0$

$\longrightarrow \Delta B_E = \Delta(X - M_E)$

$= \Delta(X_0 - M_E)$

$= \cancel{\Delta X_0}^{=0} - \Delta M_E$

$= -\Delta M_E = \boxed{-\frac{m}{1-b(1-t)+m}\Delta G_0}$

IS-LM model

The Keynesian crossing model considers only equilibrium in a market for good. Now, we will add money market into our analysis.

The quantity of money supplied :M_0 is determined exogenously outside the model, while the quantity of money demanded is determined in the market : $M_D = fY - \beta r$, $f, \beta > 0$

The equilibrium in money market happens when the quantity of money supplied is equal to The quantity of money demanded.

That is, the conditional equation for equilibrium in money market is:

$M_D = M_S$

$fY - \beta r = M_0$ ←

When combining a full-version of open economy Keynesian cross model for goods market with money market, the equilibrium output and interest rate will be determined by the system of simultaneous equation:

Y = DAE

$$
\begin{aligned}
Y &= C + I + G + X - M \\
Y_d &= Y - T \\
T &= tY \\
C &= C_0 + bY_d \\
I &= I_0 - er \\
G &= G_0 \\
X &= X_0 \\
M &= mY \\
fY - \beta r &= M_0
\end{aligned}
$$

⇒ r & Y such that good mkt. is in equilibrium

IS equation

⇒ r & Y such that money mkt. is in equilibrium

Endogenous variables:

Y, T, Y_d, C, M, r, I

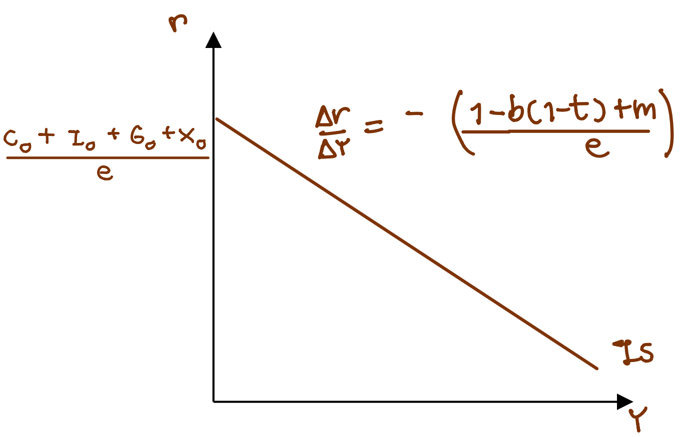
Exogenous variables:

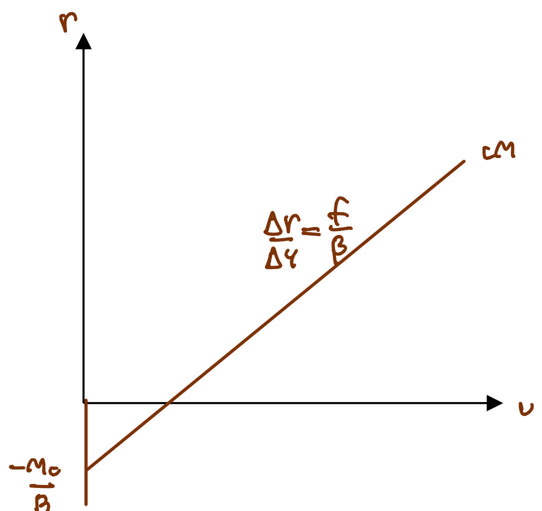
four levels of Autonomous spending:

the constant stock of money:

When we solve the goods market model, we will have ordered pair (Y, r) such that the goods market is in equilibrium. The graph of such ordered pair is called IS Curve. (To note, equilibrium in the good market requires that planned investment and planned saving be equal.)

When we solve the market market model, we will have ordered pair (Y, r) such that the money market is in equilibrium. The graph of such ordered pair is called LM Curve.





Equation for IS curve:

At equilibrium : $Y = OAE$

$Y = C_0 + b(1-t)Y + I_0 - er + G_0 + X_0 - mY$

(1.) IS : $\boxed{r = \frac{C_0 + I_0 + G_0 + X_0}{e} - \frac{(1-b(1-t)+m)}{e} Y}$

Equation for LM curve:

At equilibrium : $M_D = M_S$

$fY - \beta r = M_0$

(2.) LM $\boxed{r = \frac{-M_0}{\beta} + \frac{f}{\beta} Y}$

With just IS, LM equations : what are endogenous variables that we need to solve for ?

With both IS and LM equations, we can find the equilibrium output and interest rate, such that both goods and money market are in equilibrium.

$r_{\text{in the good mkt.}} = r_{\text{in the money mkt.}}$

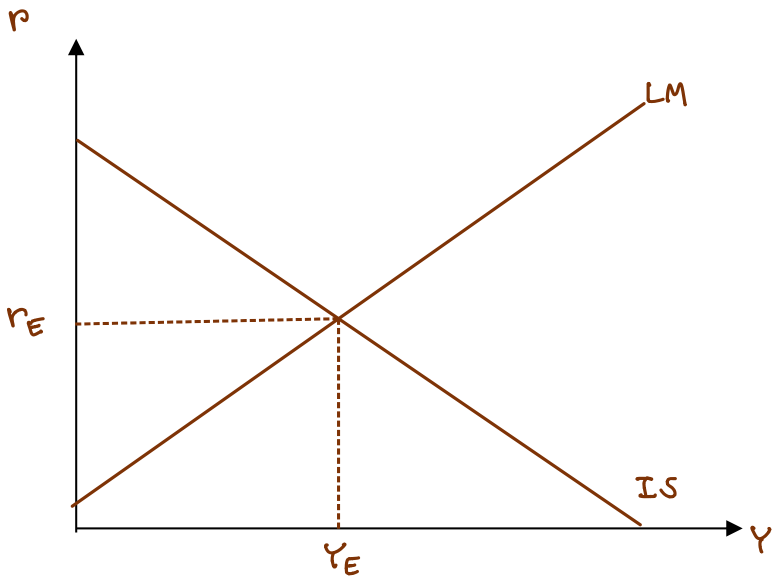
$$\frac{C_0 + I_0 + G_0 + X_0}{e} - \frac{(1-b(1-t)+m)}{e} Y = -\frac{M_0}{\beta} + \frac{f}{\beta} Y$$

$$\left[\frac{f}{\beta} + \frac{(1-b(1-t)+m)}{e}\right] Y = \frac{C_0 + I_0 + G_0 + X_0}{e} + \frac{M_0}{\beta}$$

$$Y_E = \left[\frac{\beta(C_0 + I_0 + G_0 + X_0) + eM_0}{\beta e}\right] \times \frac{\beta e}{fe + \beta(1-b(1-t)+m)}$$

$$= \frac{\beta(C_0 + I_0 + G_0 + X_0) + eM_0}{fe + \beta(1-b(1-t)+m)}$$

$$r_E = \frac{-M_0}{\beta} + \frac{f}{\beta}\left(\frac{\beta(C_0 + I_0 + G_0 + X_0) + eM_0}{fe + \beta(1-b(1-t)+m)}\right)$$



Break-Even Analysis

TFC = 300 ฿ AVC = 10 baht

Example: Let a producer have total fixed cost equal to 300 baht and average variable cost equal to 10 baht. Assume that total cost is linear.

a.) Write down definitional equation for total cost

$TC = TFC + TVC$

$= TFC + (AVC \times Q)$

$TC = 300 + 10Q$

$P = 2AVC = 2 \times 10 = 20$

b.) If the producer sets the price at the twice of average variable cost, how many output does the producer need to sell so that he is break-even? Q ?

$TR = PQ = 20Q$ $\pi = TR - TC$

At break-even point : $TR = TC$ or $\pi = 0$

$20Q = 300 + 10Q$

$Q^* = 30$

$P = 20$

c.) If the producer sets the price at the twice of average variable cost and needs to have profit at least 30% of total cost, how many output does the producer need to sell?

$\pi \geq 30\%$ of TC

$\pi = TR - TC$

$= 20Q - (300 + 10Q)$

$\pi = 10Q - 300$

$\pi = 10Q - 300 \geq 0.3(300 + 10Q)$

$Q \geq \frac{390}{7}$ #

Examples of nonlinear model

Suppose that market demand and market supply of one product take the forms:

$$Q^D = 4 - P^2$$
$$Q^S = -1 + 4P$$

What are the equilibrium price and quantity?

$Q^D = Q^S$

$4 - P^2 = -1 + 4P$

$P^2 + 4P - 5 = 0$

$(P + 5)(P - 1) = 0$

$P^* = 1, -5$ rule out

$Q^* = 3, -21$

The relationship between price elasticity of demand and total revenue

Using the definitional equation for Total Revenue(TR) and inverse demand function, we have that Total Revenue(TR) is equal to:

$TR = P \times Q$

If TR for a monopolist with $Q^D = a - bP$ $\Rightarrow P = \frac{a}{b} - \frac{1}{b}P$

$$TR = \overbrace{\left(\frac{a}{b} - \frac{1}{b}Q\right)}^{P \downarrow} Q$$

$TR = P_{mkt} Q$ — if firm is in competitive mkt.

$$\boxed{TR = \frac{a}{b}Q - \frac{1}{b}Q^2}$$

Intercept on x-axis

$TR = \frac{a}{b}Q - \frac{1}{b}Q^2 = 0$

$(a - Q)Q = 0$

$Q = 0, a$

$\frac{dTR}{dQ} = \frac{a}{b} - \frac{2}{b}Q = 0$

$Q_{max} = \frac{a}{2}$