Dynamic Oligopoly

- A model of Repeated Oligopoly

PV or V of the profit for firm i is :

$$V_i = \pi_{i0} + \delta\pi_{i1} + \delta^2\pi_{i2} + \dots + \delta^\infty \pi_{i\infty}$$
$$= \sum_{t=0}^{\infty} \delta^t \pi_{it}$$

where

π_{it} = profit in each period for firm i
so, could be different in
each period.

δ = discount factor $0 \leq \delta \leq 1$

t denotes time period

i denotes firm

- Now, suppose π_{it} can take 3 values

$\pi_{it} = \pi^*$ collude

$\pi_{it} = \pi^r$ defect

$\pi_{it} = \pi^c$ cournot (non-cooperating Nash)

$\pi^r > \pi^* > \pi^c$

- For the collusion to sustain

$$V_i^* > V_i^r$$

Condition to sustain collusion.

- PV from collude

$$V_i^* = \pi_i^* + \delta\pi_i^* + \delta^2\pi_i^* + \dots + \delta^\infty \pi_i^*$$
$$= \frac{\pi_i^*}{1-\delta}$$ ← use the perpetuity formula.

$$1 + \delta + \delta^2 + \dots + \delta^\infty = \frac{1}{1-\delta} \; ; \quad 0 \leq \delta \leq 1$$

proof :

Let $A = 1 + \delta + \delta^2 + \dots + \delta^\infty$ (1)

$\delta A = \delta + \delta^2 + \dots + \delta^\infty$ (2)

(1) − (2)

$A - \delta A = 1$

$A(1-\delta) = 1$

$A = \frac{1}{1-\delta}$ #

- PV from cheating

$$V_i^r = \pi_i^r + \delta\pi_i^c + \delta^2\pi_i^c + \dots + \delta^\infty \pi_i^c$$
$$= \pi_i^r + \delta(\pi_i^c + \delta\pi_i^c + \dots \delta^\infty \pi_i^c)$$
$$= \pi_i^r + \frac{\delta\pi_i^c}{1-\delta}$$

- for firm i not to cheat

$$V_i^{*} > V_i^{r}$$

Solve for "δ"

$$\frac{\pi_i^{*}}{1-\delta} > \pi_i^{r} + \frac{\delta \pi_i^{c}}{1-\delta}$$

$$\pi_i^{*} > (1-\delta)\pi_i^{r} + \delta\pi_i^{c}$$

$$\pi_i^{*} > \pi_i^{r} - \delta\pi_i^{r} + \delta\pi_i^{c}$$

$$\delta(\pi_i^{r} - \pi_i^{c}) > \pi_i^{r} - \pi_i^{*}$$

$$\delta > \frac{\pi_i^{r} - \pi_i^{*}}{\pi_i^{r} - \pi_i^{c}} \quad \#$$

Example: The exact value of δ

Suppose there are 5 firms in the market. MC = 10. find the value of δ that makes collusion sustain using the grim strategy.

from the previous, we have the condition $\delta > \frac{\pi_i^{r} - \pi_i^{*}}{\pi_i^{r} - \pi_i^{c}}$ which sustains collusion under the grim strategy.

So, need to find $\pi_i^{*}, \pi_i^{r}, \pi_i^{c}$

$$\pi_1^{*} = \frac{\pi^{monopoly}}{5} \quad \text{(identical firms)}$$

$$\pi^{mono} = TR - TC$$
$$= PQ - 10Q$$
$$= (P-10)Q$$
$$= (130 - Q - 10)Q$$
$$= 120Q - Q^2$$

FOC: $\frac{d\pi^{mono}}{dQ} = 0 = 120 - 2Q$

$$\boxed{Q = 60}$$

$$q_i = \frac{60}{5} = 12 \text{ units } \#$$

$$P = 130 - Q = 130 - 60 = 70$$

$$\therefore \pi_i^{*} = TR - TC = 70 \times 12 - 10 \times 12$$
$$= 60 \times 12 = \underline{\underline{720}}$$

π_i^{r} = 1 firm chooses q_i^{r} to maximize its own profit while all others choose $q_i = 12$

$$\therefore \pi_i^{r} = TR - TC$$
$$= P \times q_i^{r} - 10 \times q_i^{r}$$
$$= (P-10)q_i^{r}$$

$= (130 - Q - 10)\, q_i^r$

now, $Q = 12 \times 4 + q_i^r$ ← 1 firm defect.

$q_i = 12$ ↗ ↖ 4 firms collude

$= (130 - (12 \times 4 + q_i^r) - 10)\, q_i^r$

$= (72 - q_i^r)\, q_i^r$

$= 72 q_i^r - {q_i^r}^2$

FOC: $\frac{d\pi_i^r}{dq_i^r} = 0 = 72 - 2q_i^r$

$q_i^r = 36$ #

$P = 130 - Q$

$= 130 - (12 \times 4 + 36)$

$= 130 - 84$

$= 46$

$\pi_i^r = (46 - 10) \times 36$

$= 1296$

π_i^c = profit of 1 firm playing cournot.

$= TR - TC$

$= P q_i^c - 10 q_i^c = (P-10) q_i^c$

$= (130 - Q - 10)\, q_i^c$

$\underbrace{q_1 + q_2 + q_3 + q_4}_{} Q + q_5$ ← the cheater

$= (130 - (\sum_{j \neq i}^{4} q_j^c + q_i^c) - 10)\, q_i^c$

$= (120 - 4q_j^c - q_i^c)\, q_i^c$

FOC: $\frac{d\pi_i^c}{dq_i^c} = 0 = (120 - 4q_j^c - q_i^c) + q_i^c \times -1$

$- \neq j$

$\boxed{q_i^c = 60 - 2q_j^c}$

by symmetry

$q_j^c = 60 - 2q_i^c$ sub

$q_i^c = 60 - 2(60 - 2q_i^c)$

$= 60 - 120 + 4q_i^c$

$3q_i^c = 60$

$\boxed{q_i^c = 20}$

$P = 130 - Q$

$= 130 - (5 \times 20)$

$= 30$

$\pi_i^c = TR - TC = (P - mc)\, q_i^c$

$= (30 - 10) \times 20 = 400$ #

Thus, $\delta > \frac{\pi_i^r - \pi_i^*}{\ldots} \Rightarrow \delta > \frac{1296 - 720}{\ldots} \Rightarrow \boxed{\delta > 0.64}$