

Exercise 2 (Part 3)

1. Determine the truth value of each of these statements. Explain your answer.
 - (a) $\forall n \in \mathbb{Z}, n^3 > n$
 - (b) $\exists n \in \mathbb{Z}, n^2 + n = 1$
 - (c) $\exists n \in \mathbb{R}, n^2 + n = 1$
 - (d) $\forall n \in \mathbb{R}^+, 2n > \frac{1}{n}$
 - (e) $\forall n \in \mathbb{R}, (n > 0) \rightarrow (3n > n)$
2. Let $\mathbb{R}$ be the domain of x . Determine the **truth set** for each of these statements.
 - (a) $P(x) : "x < \frac{1}{x}"$
 - (b) $P(x) : "2x + 1 < 0 \text{ or } x \geq 1"$
3. Let the domain for variables x and y be the set of real numbers $\mathbb{R}$. Determine the truth values of the following statements. Explain your answer.
 - (a) $\exists y \forall x, xy = x$
 - (b) $\forall x \exists y, xy = x$
 - (c) $\forall y \exists x, y = x$
 - (d) $\exists x \forall y, y = x$
4. Let $Q(x, y, z)$ be the statement " $xy = z$." If the domain for variables x, y, z is the set of all integers, determine the truth values of the following statements. Explain your answer.
 - (a) $Q(1, 2, 2)$
 - (b) $Q(2, 0, 2)$
 - (c) $\exists y, Q(2, y, 1)$
 - (d) $\forall x \forall y \exists z, Q(x, y, z)$
 - (e) $\exists z \forall x \forall y, Q(x, y, z)$
5. Let $\mathbb{Z}^+$ be the domain of x . Let $P(x)$ and $Q(x)$ be the predicates " x is not divisible by 3," and " x is divisible by 12," respectively. Determine whether the following statements are true or false. Give a counterexample for each false statement.
 - (a) $Q(x) \Rightarrow P(x)$
 - (b) $P(x) \Rightarrow \sim Q(x)$
6. Write a negation for each statement without using the negation symbol " $\sim$."
 - (a) $\exists z \forall x \forall y, xy = z$
 - (b) $\forall x \forall y, (x < 0) \wedge (y \geq 0) \rightarrow (xy \leq 0)$
7. Show that each of the following arguments is valid by **universal modus ponens**, **universal modus tollens** and/or **universal transitivity**, or show that it is invalid from the **converse error** or the **inverse error**. In addition, use also the **diagram** to confirm that each argument is valid or invalid.
 - (a)

"Anyone who has a school email account has a school ID number."

"Kevin has a school ID number."

$\therefore$ "Kevin has a school email account. "
 - (b)

"Anyone who has a school email account has a school ID number."

"All students have school email accounts."

"Kim does not have a school ID number."

$\therefore$ "Kim is not a student."