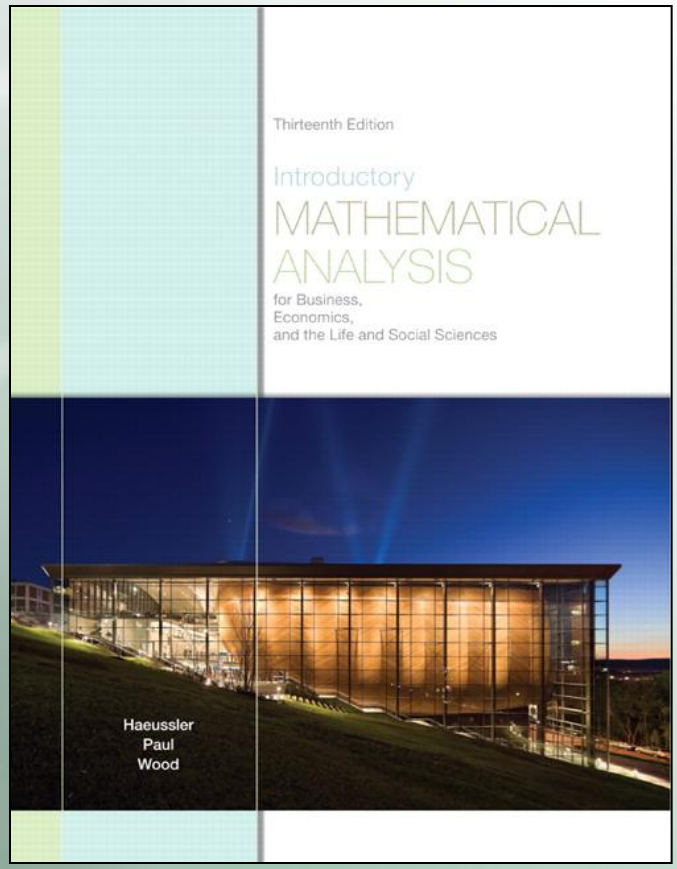
INTRODUCTORY MATHEMATICAL ANALYSIS

For Business, Economics, and the Life and Social Sciences

Chapter 4
Exponential and Logarithmic Functions



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Chapter Objectives

- To introduce exponential functions and their applications.
- To introduce logarithmic functions and their graphs.
- To study the basic properties of logarithmic functions.
- To develop techniques for solving logarithmic and exponential equations.

Chapter Outline

4.1 Exponential Functions

- The function f defined by $f(x) = b^x$

  where $b > 0$, $b \neq 1$, and the exponent x is any real number, is called an ***exponential function*** with base b[1].

Rules for Exponents

1. $b^x b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(bc)^x = b^x c^x$
5. $\left(\dfrac{b}{c}\right)^x = \dfrac{b^x}{c^x}$
6. $b^1 = b$
7. $b^0 = 1$
8. $b^{-x} = \dfrac{1}{b^x}$

Example 3 – Graphing Exponential Functions with $0 < b < 1$

Graph the exponential function $f(x) = (1/2)^x$.

Solution:

Properties of Exponential Functions

1. The domain of an exponential function consists of all real numbers.
   The range consists of all positive numbers.
2. The graph of $f(x) = b^x$ has y-intercept $(0, 1)$.
   There is no x-intercept.
3. If $b > 1$, the graph *rises* from left to right.
   If $0 < b < 1$, the graph *falls* from left to right.
4. If $b > 1$, the graph approaches the x-axis as x becomes more and more negative.
   If $0 < b < 1$, the graph approaches the x-axis as x becomes more and more positive.

4.2 Logarithmic Functions

- $y = \log_b x$ if and only if $b^y = x$.
- Fundamental equations are $\log_b b^x = x$ and $b^{\log_b x} = x$

Example 1 – Converting from Exponential to Logarithmic Form

	Exponential Form		*Logarithmic Form*
a. Since	$5^2 = 25$	it follows that	
b. Since		it follows that	$\log_3 81 = 4$
c. Since	$10^0 = 1$	it follows that	

Example 3 – Graph of a Logarithmic Function with $b > 1$

Sketch the graph of $y = \log_2 x$.

Solution:

Example 5 – Finding Logarithms

a. Find log 100.

b. Find ln 1.

c. Find log 0.1.

d. Find ln e^{-1}.

d. Find $\log_{36} 6$.

4.3 Properties of Logarithms

- Properties of logarithms are:

1. $\log_b(mn) = \log_b m + \log_b n$
2. $\log_b \frac{m}{n} = \log_b m - \log_b n$
3. $\log_b m^r = r \log_b m$
4. $\log_b \frac{1}{m} = -\log_b m$
5. $\log_b 1 = 0$
6. $\log_b b = 1$
7. $\log_b m = \frac{\log_a m}{\log_a b}$

Example 1 – Finding Logarithms

a. $\log 56 = \log(8 \cdot 7) = \log 8 + \log 7 \approx 0.9031 + 0.8451 = 1.7482$

b. $\log \frac{9}{2} = \log 9 - \log 2 \approx 0.9542 - 0.3010 = 0.6532$

c. $\log 64 = \log 8^2 = 2 \log 8 \approx 2(0.9031) = 1.8062$

d. $\log \sqrt{5} = \log 5^{1/2} = \frac{1}{2} \log 5 \approx \frac{1}{2}(0.6990) = 0.3495$

Example 3 – Writing Logarithms in Terms of Simpler Logarithms

a. $\ln \dfrac{x}{zw}$

b. $\ln \sqrt[3]{\dfrac{x^5(x-2)^8}{x-3}}$

Example 5 – Simplifying Logarithmic Expressions

a. $\ln e^{3x} =$

b. $\log 1 + \log 1000$

c. $\log_7 \sqrt[9]{7^8}$

d. $\log_3 \left(\frac{27}{81} \right)$

e. $\ln e + \log \frac{1}{10}$

Example 7 – Evaluating a Logarithm Base 5

Find $\log_5 2$.

Solution:

4.4 Logarithmic and Exponential Equations

- A **logarithmic equation** involves the logarithm of an expression containing an unknown.
- An **exponential equation** has the unknown appearing in an exponent.

Example 1 – Oxygen Composition

An experiment was conducted with a particular type of small animal. The logarithm of the amount of oxygen consumed per hour was determined for a number of the animals and was plotted against the logarithms of the weights of the animals. It was found that

$$\log y = \log 5.934 + 0.885 \log x$$

where y is the number of microliters of oxygen consumed per hour and x is the weight of the animal (in grams). Solve for y.

Example 3 – Using Logarithms to Solve an Exponential Equation

$Solve\, 5 + (3)4^{x-1} = 12.$

Solution: