Exercise of Asset Price Function

Lucas (1978): Pricing Arbitrary Assets

EX: Price a 1 period, pure-discount bond paying 1 unit tomorrow in all possible states

State: $z = (z_1, z_2)$ holdings of stock and bonds

$$(P1)\ v(z, y) = \max u(c) + \beta E[v(z_1', z_2', y')|y]\ s.t.$$

$$c + p_1(y)z_1' + p_2(y)z_2' \leq z_1(p_1(y) + y) + z_2\ and\ c \geq 0, z_1', z_2' \geq \underline{z}$$

Definition: A recursive equilibrium is $(c(z,y), s_1(z,y), s_2(z,y), p_1(y), p_2(y))$ s.t.

1. (optimization) $(c(z,y), s_1(z,y), s_2(z,y))$ solve P1.
2. (market clearing) $c(1,0,y) = y\ and\ s_1(1,0,y) = 1\ and\ s_2(1,0,y) = 0, \forall y \in Y$

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EX: Price a 1 period, pure-discount bond

FONC:

$$u'(c)p_2(y) = \beta E[v_2(z_1', z_2', y')|y]]$$

$$u'(c(z,y))p_2(y) = \beta E[u'(c(s_1(z,y), s_2(z,y), y'))|y]$$

FONC + equil condtions:

$$u'(y)p_2(y) = \beta E[u'(y')|y] \Rightarrow p_2(y) = E[\frac{\beta u'(y')}{u'(y)}|y]$$

Question: Within the Lucas model, when does the mean return to stock exceed the risk-free bond return?

Answer:

1. $1 = E[\frac{\beta u'(y')}{u'(y)} R^s(y, y')|y]$ *where* $R^s(y, y') = \frac{p_1(y')+y'}{p_1(y)}$
2. $1 = E[\frac{\beta u'(y')}{u'(y)} R^b(y, y')|y]$ *where* $R^b(y, y') = \frac{1}{p_2(y)}$
3. $0 = E[\frac{\beta u'(y')}{u'(y)} (R^s(y, y') - R^b(y, y'))|y]$
4. Use $E[ab] = E[a]E[b] + Cov(a, b)$ to get
   $0 = E[m|y]E[R^s - R^b|y] + Cov(m, R^s - R^b|y)$
5. $E[R^s - R^b|y] = \frac{-Cov(m,R^s-R^b|y)}{E[m|y]} = \frac{-Cov(m,R^s-R^b|y)}{p_2(y)}$
6. Need m to be random AND to covary negatively with R^s.
   Stock return needs to be high w/ high endowment shocks.