Extra Practice problem sets

Question 1: A firm's production function is given by

$$Q = [0.64K^{0.5} + 0.36L^{0.5}]^2$$

1.1. Determine the marginal rate of technical substitution (MRTS) of L for K.

1.2. Determine the values of the marginal products of K and L, when K = 256 and L = 324.

1.3. Suppose now that the capital input (K) and the labor input (L) change with time (t) according to the functions:

$K(t) = 232 + 10t - t^2$;

$L(t) = 292 + 12t - t^2$.

Use total derivatives to find the rate of change of the output (Q) with respect to time (t) at the time $t = 4$.

Question 2 A business conglomerate produces three different types of products, namely A, B and C. Each product has the market demand equation given by:

Product A: $P_A = 36 - Q_A^2$;

Product B: $P_B = 35 - 2Q_B^2 + Q_C$;

Product C: $P_C = 20 + Q_B - Q_C$;

Suppose that the cost function for producing the three types of products is given by:

$TC(Q_A, Q_B, Q_C) = 8 + 2Q_A^3 - 2Q_B^3 + 3Q_BQ_c + 30Q_B + 12Q_C + \frac{1}{2}Q_B^2$,

and the conglomerate is the sole supplier in each of the three markets.

Consider the following problems

2.1. Write down the profit function of this conglomerate firm.

2.2. Using the first-derivative test, solve for the profit-maximizing level of output for each of the three products.

2.3. Confirm your answer in (2.2) with the second-order derivative test.

2.4. Determine the level of maximized profit.

Question 3 Joe's utility function depends on consumption and leisure, and it is given by:

$$U(c, l) = 8\sqrt{c} + l,$$

where *c* is consumption *l* is leisure. Each day Joe works *n* hours and spends the rest of the time on leisure; that is, $l + n = 24$. Suppose further that Joe's total income is the sum of a fixed non-wage income (Y_0) and a wage income (Y_n). The wage income (Y_n) is equal to wn, where *w* is the hourly wage rate. If Joe spends all of his income on the consumption, his budget constraint becomes:

$$p_c c = Y_0 + wn = Y_0 + w(24 - l),$$

which can also be written as:

$$p_c c + wl = Y_0 + 24w.$$

3.1. Use Lagrange method to derive the optimal values of c^* and l^* in terms of the parameters p_c, w, and Y_0. State the condition under which the consumer demands some leisure (i.e., $l^* > 0$).

3.2. Based on your answer in (3.1), derive the impact of an increase in the hourly wage (*w*) on the demand for leisure (l^*). Interpret its economic meaning.

3.3. Suppose now that the hourly wage rate (w) is \$24, the price of consumption (p_c) is \$12, and the fixed non-wage income (Y_0) is 384. Use Lagrange method to determine the levels of c^* and l^* that maximizes Joe's utility subject to the budget constraint, and determine the level of constrained maximum utility.

3.4. Verify your answers in (3.3) by using the second-order sufficient condition.

Question 4 A monopoly is facing a non-linear inverse market demand given by $P = \frac{100}{\sqrt{Q}}$ where *P* is the price per unit of output, and Q is the quantity of output. Assume further that the cost function of this monopolist is given by $C = 20Q + 10$.

4.1. Calculate the level of profit maximizing output and price under non-discriminatory pricing regime, and compute the consumer surplus and producer surplus.

4.2. Calculate the deadweight loss from the monopoly. [Hint: *You need to compute the optimal quantity in the perfectly competitive market*.]

4.3. Suppose a \$10 subsidy is given to the monopolist. Calculate the change in social welfare, and discuss your result.

Question 5 Suppose that there are three groups of people who take Sky train to commute in Bangkok. The first group is students (s), the second group is senior citizens (old), and the third group is working-aged people. The demand for each group is given by the following equations:

Demand of students: $P_s = 8 - \left(\frac{1}{2}\right) Q_s$

Demand of senior citizens: $P_{old} = 16 - 2Q_{old}$

Demand of working-aged people: $P_w = 20 - Q_w$

The Sky train operator has a constant marginal cost at MC = \$4, and total cost at TC = 4Q + 10. Consider the following problems.

5.1) Determine the profit-maximizing level of output/price under third-degree price discrimination. Calculate the level of maximized profit.

5.2) Confirm your result in (2.1) with the second-order derivative test.

5.3) Calculate (i) consumer surplus for each of the three groups of consumers, and (ii) producer surplus.

5.4) Calculate the optimal level of total output if the Sky train operator can practice the first-degree price discrimination for each group of the consumers in the market.

Question 6 Suppose that the preference set of a household can be given by

$U(x, y) = x^{1/2} + y^{1/2}$.

6.1) Define an indifference curve (i) as an implicit function, and (ii) with y as function of x. Show that the indifference curve is a decreasing and convex function. Provide economic interpretation of indifference curve.

6.2) (4 points) For this utility function, find marginal utility. Derive the expression for the MRS and show that MRS is decreasing in x.

6.3) (4 marks) Show that the marginal rate of substitution equals the (absolute value of the) slope of the indifference curve.

Question 7 Suppose the production function of a competitive firm is given by:

$$Q = f(L) = A_0 K_0 \sqrt{L}$$

where A_0 is the exogenous level of technology, K_0 is the given level of capital installed in the firm, and L is the number of workers hired. Assume further that the each worker gets paid $\$w$, F is the level of fixed cost that has been advanced for the capital installation process.

Assume that each unit of output can be sold at the given market price equal to $\$P$. Set up the profit maximization problem and determine the optimal level of workers. Discuss the impact of an increase in the exogenous level of technology on the optimal level of workers.

Question 8 Georg is a stamp (x) collector, but he also likes fancy clothes (y). His utility function is given by $U(x,y) = x + 10y - \frac{1}{2}y^2$. Suppose that each stamp costs him p_x and a piece of his favorite clothing costs p_y. Consider the following problem.

8.1) Given that his income is equal to M, find his optimal choice of x^* and y^*. (Hint: you don't need to check the second-order derivative test.)

8.2) Calculate the indirect utility function. What happen to the indirect utility function if prices of both goods and income equally increase in the same proportion? Explain your result with some economic intuitions.

8.3) Calculate the optimal level of x^* and y^* when $p_x = \$1, p_y = \$2$ and M = \$10.

8.4) Confirm the answer as obtained in (3.3) using the second-order condition test.

8.5) Without redoing the optimization problem, can you make an educational guess on the new level of optimized utility if the level of income is now \$15?

Question 9 Suppose that a monopolist has its marginal cost function given by $MC = 16 + 6q^2$ where q is the amount of output produced. The monopolist faces the market demand function given by $P = 160 - 10q^2$ where P is the price per unit of output. Consider the following problem.

9.1) Suppose that fixed cost is equal to \$240. Calculate the total and the average cost when q = 9 units.

9.2) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the second derivative test.

9.3) Calculate the social welfare under the monopoly environment.

9.4) Calculate the social welfare loss under the monopoly environment.

Question 10 Find the solution for the Optimization and check that it is maximal *or* minimal point by Second order condition.

(a) $f(a,b) = a^2 + b^2 + ab + 10a + 10b$

(b) $f(x,y) = 2x^2 - y^2 - 4x + 8y$

(c) $f(x,y) = 3x^2 - 8xy + 8y^2$

(d) $f(x,y) = 6x^{\frac{1}{3}} - y^{\frac{1}{3}} - x - y \ : \ x > 0, y > 0$

(e) $f(x,y) = \ln(x-1) + \ln(y-2) - 2x - 3y$

(f) $f(x,y) = \ln(x+3) - 2y$

(g) $f(x,y) = x\left(1 - e^{-x/y}\right) + 8y \ : \ x > 0, y > 0$

(h) $x^2 + y^2 + 3z^2 + 3yz; sub: x + 2y + 3z = 1$

(i) $\ln(xy); sub: (x+1)(y+1) = 4$

(j) $\sum_{i=1}^{n} \ln(x_i - \beta_i); sub: \sum_{i=1}^{n} p_i x_i = y$ where n = 2.

(k) $4x + 2y - 8; sub: (x-2)^2 + (y-1)^2 = 80$

(l) $f(x,y) = (2x^2 + y^2)e^{-x^2-y^2}$

(m) $y = x_1^2 - x_2^2 - x_3^2 - 4x_1 - 2x_2 - 3x_3 + 12$