Assignment 12
Poisson – Negative Binomial – Zero Inflated Poisson
Guideline Solution

The model

The generalized linear regression model can be stated as:

$$I_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + u_i \quad (1)$$

and

$$\Pr(Y_i = y_i) = f(I_i)$$

where: I_i is index variables.
y_i is counted number 0, 1, 2,...
x_{ki} is independent variable k.
$f(.)$ is either Poisson or Negative Binomial probability distribution function.
u_i is disturbance term.

Requirements (using Data set file – `assign12.dta` **):**

1 Estimate models for y_i assuming that the model is traditional linear regression model. Interpret your estimated result.

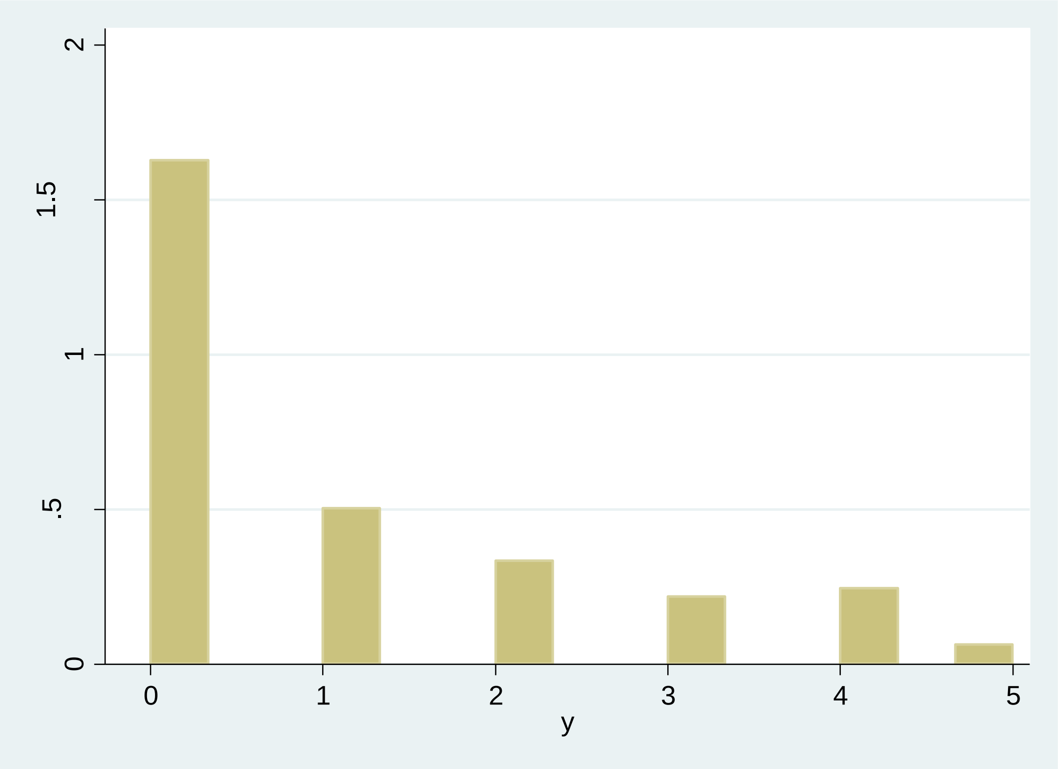
```
. reg y x1 x2 x3 x4

      Source |       SS           df       MS      Number of obs   =       232
-------------+----------------------------------   F(4, 227)       =      5.96
       Model |  44.7298499         4  11.1824625   Prob > F        =    0.0001
    Residual |  425.748598       227  1.87554449   R-squared       =    0.0951
-------------+----------------------------------   Adj R-squared   =    0.0791
       Total |  470.478448       231  2.03670324   Root MSE        =    1.3695

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .1016201   .0435073     2.34   0.020     .0158904    .1873499
          x2 |   .1345044   .0462142     2.91   0.004     .0434407     .225568
          x3 |  -.0748194   .0480457    -1.56   0.121    -.1694919    .0198531
          x4 |   .1684563   .0688243     2.45   0.015     .0328401    .3040725
       _cons |   .9568064    .107007     8.94   0.000     .7459523     1.16766
------------------------------------------------------------------------------

. est store linear
```

- ***According to the above estimated results, interpretation can be made concerning: sign and meaning of the estimated coefficient – correct sign and linear regression meaning, Overall test F-test – significant, R-squared 0.0951 – not quite fit the data, Individual test t-test – significant except x_3.***

By Tatre Jantarakolica

2 Create histogram for y_i. Determine whether there is limitation of dependent variable in this case. If yes, what type of limitation is it?

```
. histogram y
(bin=15, start=0, width=.33333333)
```



- ***According to Histogram, it looks like that the distribution of dependent variable follows Poisson distribution (the limitation), thus, Poisson regression model should be applied.***

3 Estimate models for y_i assuming that the probability functions follow Poisson probability distribution. Perform GOF test and determine whether Poisson is appropriated in this case. Interpret the estimated result (sign and meaning (in term of incidence-rate ratios), overall test, individual test, pseudo R^2, marginal effects).

```
. poisson y x1 x2 x3 x4, nolog

Poisson regression                                Number of obs   =        232
                                                  LR chi2(4)      =      43.33
                                                  Prob > chi2     =     0.0000
Log likelihood = -342.88107                       Pseudo R2       =     0.0594

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .0971474   .0306762     3.17   0.002     .0370231    .1572717
          x2 |   .1293024   .0330916     3.91   0.000      .064444    .1941607
          x3 |  -.0715533   .0342177    -2.09   0.037    -.1386187   -.0044879
          x4 |   .1734482   .0507707     3.42   0.001     .0739395    .2729569
       _cons |  -.1284876   .0849064    -1.51   0.130     -.294901    .0379259
------------------------------------------------------------------------------

. estat gof
```

```
Deviance goodness-of-fit =  409.4921
Prob > chi2(227)         =    0.0000

Pearson goodness-of-fit  =  423.3541
Prob > chi2(227)         =    0.0000
```

- ***According to GOF test, null hypothesis of the test was rejected, thus, Poisson regression model is inappropriated.***

```
. poisson y x1 x2 x3 x4, ir nolog

Poisson regression                                Number of obs     =        232
                                                  LR chi2(4)        =      43.33
                                                  Prob > chi2       =     0.0000
Log likelihood = -342.88107                       Pseudo R2         =     0.0594

------------------------------------------------------------------------------
           y |        IRR   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   1.102023   .0338059     3.17   0.002     1.037717    1.170314
          x2 |   1.138034   .0376594     3.91   0.000     1.066566    1.214291
          x3 |   .9309467   .0318548    -2.09   0.037     .8705599    .9955222
          x4 |   1.189399   .0603866     3.42   0.001     1.076742    1.313844
       _cons |   .8794245   .0746687    -1.51   0.130     .7446053    1.038654
------------------------------------------------------------------------------

. est store poisson

. mfx

Marginal effects after poisson
      y  = Predicted number of events (predict)
         =  .95621703
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
      x1 |    .092894      .02882    3.22   0.001   .036403  .149385  -.317697
      x2 |   .1236411       .0308    4.01   0.000   .063277  .184005   .812709
      x3 |  -.0684205      .03246   -2.11   0.035  -.13204 -.004801  -.818103
      x4 |   .1658541      .04736    3.50   0.000   .073022  .258686   -.28275
------------------------------------------------------------------------------
```

- ***According to the above estimated results, interpretation can be made concerning: sign and meaning of the estimated coefficient – correct sign (positive mfx sign and irr>1 for x1, x2, x4 and negative mfx sign and irr<1 for x3), Overall test LR-Chi-squared-test – significant, Pseudo R-squared 0.0594 – not quite fit the data, Individual test z-test – all significant.***

4 Estimate models for y_i assuming that the probability functions follow Negative Binomial probability distribution. Determine whether Negative Binomial regression model is appropriated in this case. Interpret your estimated result (sign and meaning (in term of incidence-rate ratios), overall test, individual test, pseudo R^2, marginal effects).

```
. nbreg y x1 x2 x3 x4, nolog

Negative binomial regression                      Number of obs     =        232
```

```
                                                  LR chi2(4)      =      21.24
Dispersion     = mean                             Prob > chi2     =     0.0003
Log likelihood = -317.49278                       Pseudo R2       =     0.0324

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .1285534   .0506934     2.54   0.011     .0291962    .2279106
          x2 |    .151011   .0506477     2.98   0.003     .0517434    .2502785
          x3 |  -.0672859   .0481376    -1.40   0.162    -.1616339    .0270621
          x4 |   .1726312   .0707035     2.44   0.015      .034055    .3112075
       _cons |  -.1435596   .1177204    -1.22   0.223    -.3742874    .0871682
-------------+----------------------------------------------------------------
    /lnalpha |   .0479945   .2389531                     -.4203449    .5163339
-------------+----------------------------------------------------------------
       alpha |   1.049165   .2507012                      .6568202    1.675872
------------------------------------------------------------------------------
LR test of alpha=0: chibar2(01) = 50.78                Prob >= chibar2 = 0.000
```

- ***According to LR test of alpha, null hypothesis of the test was rejected indicating that the distribution of dependent variable follows Negative Binomial distribution, thus, Negative Binomial regression model is more appropriated than Poisson regression model.***

```
. nbreg y x1 x2 x3 x4, ir nolog

Negative binomial regression                      Number of obs   =        232
                                                  LR chi2(4)      =      21.24
Dispersion     = mean                             Prob > chi2     =     0.0003
Log likelihood = -317.49278                       Pseudo R2       =     0.0324

------------------------------------------------------------------------------
           y |        IRR   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   1.137182   .0576476     2.54   0.011     1.029627    1.255973
          x2 |   1.163009   .0589037     2.98   0.003     1.053105    1.284383
          x3 |   .9349279   .0450052    -1.40   0.162     .8507526    1.027432
          x4 |   1.188428    .084026     2.44   0.015     1.034641    1.365072
       _cons |   .8662692   .1019776    -1.22   0.223     .6877792     1.09108
-------------+----------------------------------------------------------------
    /lnalpha |   .0479945   .2389531                     -.4203449    .5163339
-------------+----------------------------------------------------------------
       alpha |   1.049165   .2507012                      .6568202    1.675872
------------------------------------------------------------------------------
Likelihood-ratio test of alpha=0:  chibar2(01) =   50.78 Prob>=chibar2 = 0.000

. est store nb

. mfx

Marginal effects after nbreg
      y  = Predicted number of events (predict)
         =  .94607122
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
      x1 |   .1216207      .04796    2.54   0.011    .02763  .215611  -.317697
      x2 |   .1428671      .04796    2.98   0.003   .048862  .236872   .812709
      x3 |  -.0636573      .04556   -1.40   0.162  -.152956  .025641  -.818103
      x4 |   .1633214      .06686    2.44   0.015   .032269  .294374   -.28275
------------------------------------------------------------------------------
```

- ***According to the above estimated results, interpretation can be made concerning: sign and meaning of the estimated coefficient – correct sign (positive mfx sign and irr>1 for x1, x2, x4 and negative mfx sign and irr<1 for x3), Overall test LR-Chi-squared-test – significant, Pseudo R-squared 0.0324 – not quite fit the data, Individual test z-test – significant except x3.***

5 Estimate models for y_i assuming that the model is Zero Inflated Poisson (x_{1i}, x_{2i}, and x_{3i} are independent variables in Poisson model and x_{4i} is independent variable in Inflated (Logit) model). Interpret your estimated result. Determine which model (Linear regression model, Poisson, Negative Binomial, or ZIP) is the most appropriated model in this case? Why? (provide the tests)

```
. zip y x1 x2 x3, inflate(x4) vuong nolog

Zero-inflated Poisson regression                Number of obs     =        232
                                                Nonzero obs       =        106
                                                Zero obs          =        126

Inflation model = logit                         LR chi2(3)        =      10.35
Log likelihood  = -312.6158                     Prob > chi2       =     0.0158

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
          x1 |   .0805446   .0398159     2.02   0.043     .0025068    .1585824
          x2 |   .0857883   .0372107     2.31   0.021     .0128567    .1587199
          x3 |  -.0672468   .0357098    -1.88   0.060    -.1372367     .002743
       _cons |   .4589728   .1106031     4.15   0.000     .2421947    .6757508
-------------+----------------------------------------------------------------
inflate      |
          x4 |  -.2738532   .1212311    -2.26   0.024    -.5114618   -.0362446
       _cons |  -.3379298   .1908217    -1.77   0.077    -.7119334    .0360738
------------------------------------------------------------------------------
Vuong test of zip vs. standard Poisson:            z =     3.92  Pr>z = 0.0000
```

- ***According to Vuong test, null hypothesis of the test was rejected, thus, Zero Inflated Poisson regression model is more appropriated than Poisson regression model.***

```
. mfx

Marginal effects after zip
      y  = Predicted number of events (predict)
         =   .9868501
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
      x1 |   .0794855      .03893    2.04   0.041   .003192  .155779  -.317697
      x2 |   .0846602      .03606    2.35   0.019   .013992  .155328   .812709
      x3 |  -.0663626      .03507   -1.89   0.058  -.135102  .002377  -.818103
      x4 |   .1176249      .05326    2.21   0.027   .013231  .222019   -.28275
------------------------------------------------------------------------------
```

- ***According to the above estimated results, interpretation can be made concerning: sign and meaning of the estimated coefficient – correct sign (positive mfx sign for x1, x2, x4 and negative mfx sign for x3), Overall test LR-Chi-squared-test – significant, Individual test z-test –significant except x3.***

6 According to the above (1-5), determine the most appropriated model for this case. Give explanation why?

```
. est table linear poisson nb zip, star(.1 .05 .01) stat(N ll chi2 chi2_c vuong)

------------------------------------------------------------------------------
    Variable |    linear         poisson          nb              zip
-------------+----------------------------------------------------------------
_            |
          x1 |  .10162013**
          x2 |  .13450436***
          x3 | -.07481941
          x4 |  .16845626**
       _cons |   .9568064***
-------------+----------------------------------------------------------------
y            |
          x1 |                 .09714739***    .12855344**     .08054462**
          x2 |                 .12930238***    .15101096***    .08578827**
          x3 |                 -.0715533**    -.06728591      -.06724685*
          x4 |                 .17344816***    .17263123**
       _cons |                -.12848756      -.14355961       .45897275***
-------------+----------------------------------------------------------------
lnalpha      |
       _cons |                                 .0479945
-------------+----------------------------------------------------------------
inflate      |
          x4 |                                                -.27385319**
       _cons |                                                -.33792981*
-------------+----------------------------------------------------------------
Statistics   |
           N |        232             232             232             232
          ll | -399.61869      -342.88107      -317.49278       -312.6158
        chi2 |                   43.3288       21.240477       10.353862
      chi2_c |                                 50.776579
       vuong |                                                  3.9220694
------------------------------------------------------------------------------
                                                  legend: * p<.1; ** p<.05; *** p<.01
```

- ***According to GOF test, null hypothesis of the test was rejected, thus, Poisson regression model is more appropriated than linear regression model.***
- ***According to LR test of alpha, null hypothesis of the test was rejected indicating that the distribution of dependent variable follows Negative Binomial distribution, thus, Negative Binomial regression model is more appropriated than Poisson regression model.***
- ***According to Vuong test, null hypothesis of the test was rejected, thus, Zero Inflated Poisson regression model is more appropriated than Poisson regression model.***
- ***Also, Histogram illustrates zero inflated distribution of dependent variable, therefore, Zero Inflated Poisson regression model should be applied in this case.***