Chapter 4

Future Value, Present Value, and Interest Rates

Learning Objectives

1. Compare the value of monetary payments using present value and future value.
2. Apply present value to a stream of payments using internal rate of return and bond valuation.
3. Explain the difference between real and nominal interest rates and how each is calculated.

Introduction

- Credit is one of the critical mechanisms we have for allocating resources.
- Although interest has historically been unpopular, this comes from the failure to appreciate the opportunity cost of lending.
- Interest rates
  - Link the present to the future.
  - Tell the future reward for lending today.
  - Tell the cost of borrowing now and repaying later.

Valuing Monetary Payments Now and in the Future

- We must learn how to calculate and compare rates on different financial instruments.
- We need a set of tools:
  - Future value
  - Present value

Future Value and Compound Interest

- Future value is the value on some future date of an investment made today.
  - $100 invested today at 5% interest gives $105 in a year. So the future value of $100 today at 5% interest is $105 one year from now.
  - The $100 yields $5, which is why interest rates are sometimes called a yield.
  - This is the same as a simple loan of $100 for a year at 5% interest.

Future Value and Compound Interest

- If the present value is \$100 and the interest rate is 5%, then the *future value* one year from now is:

$$\$100 + \$100(0.05) = \$105$$

- This also shows that the higher the interest rate, the higher the future value.
- In general:

$$FV = PV + PV(i) = PV(1 + i)$$

Future Value and Compound Interest

- The higher the interest rate or the higher the amount invested, the higher the future value.
- Most financial instruments are not this simple, so what happens when time to repayment varies.
- When using one-year interest rates to compute the value repaid more than one year from now, we must consider compound interest.
  - Compound interest is the interest on the interest.

Future Value and Compound Interest

- What if you leave your \$100 in the bank for two years at 5% yearly interest rate?
- The future value is:

$$\$100 + \$100(0.05) + \$100(0.05) + \$5(0.05) = \$110.25$$

$$\$100(1.05)(1.05) = \$100(1.05)^2$$

- In general

$$FV_n = PV(1 + i)^n$$

Future Value and Compound Interest

- Table 4.1: Computing the future value of $100 at 5% annual interest

Years into Future	Computation	Future Value
1	$\$100(1.05)$	$105.00
2	$\$100(1.05)^2$	$110.25
3	$\$100(1.05)^3$	$115.76
4	$\$100(1.05)^4$	$121.55
5	$\$100(1.05)^5$	$127.63
10	$\$100(1.05)^{10}$	$162.89

Future Value and Compound Interest

- Converting *n* from years to months is easy, but converting the interest rate is harder.
  - If the annual interest rate is 5%, what is the monthly rate?
- Assume i^m is the one-month interest rate and n is the number of months, then a deposit made for one year will have a future value of $\$100(1 + i^m)^{12}$.

Future Value and Compound Interest

- We know that in one year the future value is \$100(1.05) so we can solve for i^m:

$$(1 + i^m)^{12} = (1.05)$$
$$(1 + i^m) = (1.05)^{1/12} = 1.0041$$

- These fractions of percentage points are called basis points.
  - A basis point is one one-hundredth of a percentage point, 0.01 percent.

YOUR FINANCIAL WORLD

How Long Does Your Investment Take to Double?

- Invest $100 at 5% annual interest
- How long until you have $200?
- The Rule of 72:
  - Divide the annual interest rate into 72
  - So 72/5=14.4 years.
  - $1.05^{14.4} = 2.02$

Present Value

- Financial instruments promise future cash payments so we need to know how to value those payments.
- Present value is the value today (in the present) of a payment that is promised to be made in the future.
- Or, present value is the amount that must be invested today in order to realize a specific amount on a given future date.

Present Value

- Solve the Future Value Formula for PV:

FV = PV × (1+i), so

$$PV = \frac{FV}{(1+i)}$$

- This is just the future value calculation inverted.

Present Value

- We can generalize the process as we did for future value.
- Present Value of payment received *n* years in the future:

$$PV = \frac{FV_n}{(1+i)^n}$$

Present Value

- From the previous equation, we can see that present value is higher:
  1. The higher future value of the payment, FV_n.
  2. The shorter time period until payment, *n*.
  3. The lower the interest rate, *i*.

LESSONS FROM THE CRISIS
RISK TAKING AND THE SEARCH FOR YIELD

- Risk requires compensation, but securing proper compensation means understanding the risks of what is purchased.
- If interest rates rise, losses on a long-term bond are greater than losses on a short-term bond.
  - Long term bonds are more sensitive to the risk that interest rates will change.

LESSONS FROM THE CRISIS
RISK TAKING AND THE SEARCH FOR YIELD

- Investors might misjudge or underestimate risk
- The search for yield can bid up prices of risky securities and depress the market compensation for risk below a sustainable level.
- When risk comes to fruition, like when defaults increase, the prices of riskier securities fall disproportionately, triggering financial losses.
  - During the 2007-2009 crisis, the plunge of corporate and mortgage security prices show how markets reprice risk when the search for yield has gone too far.

How Present Value Changes

- Doubling the future value of the payment, without changing the time of the payment or the interest rate, doubles the present value.
- The sooner a payment is to be made, the more it is worth.

Figure 4.1: Present Value of $100 at 5% Interest

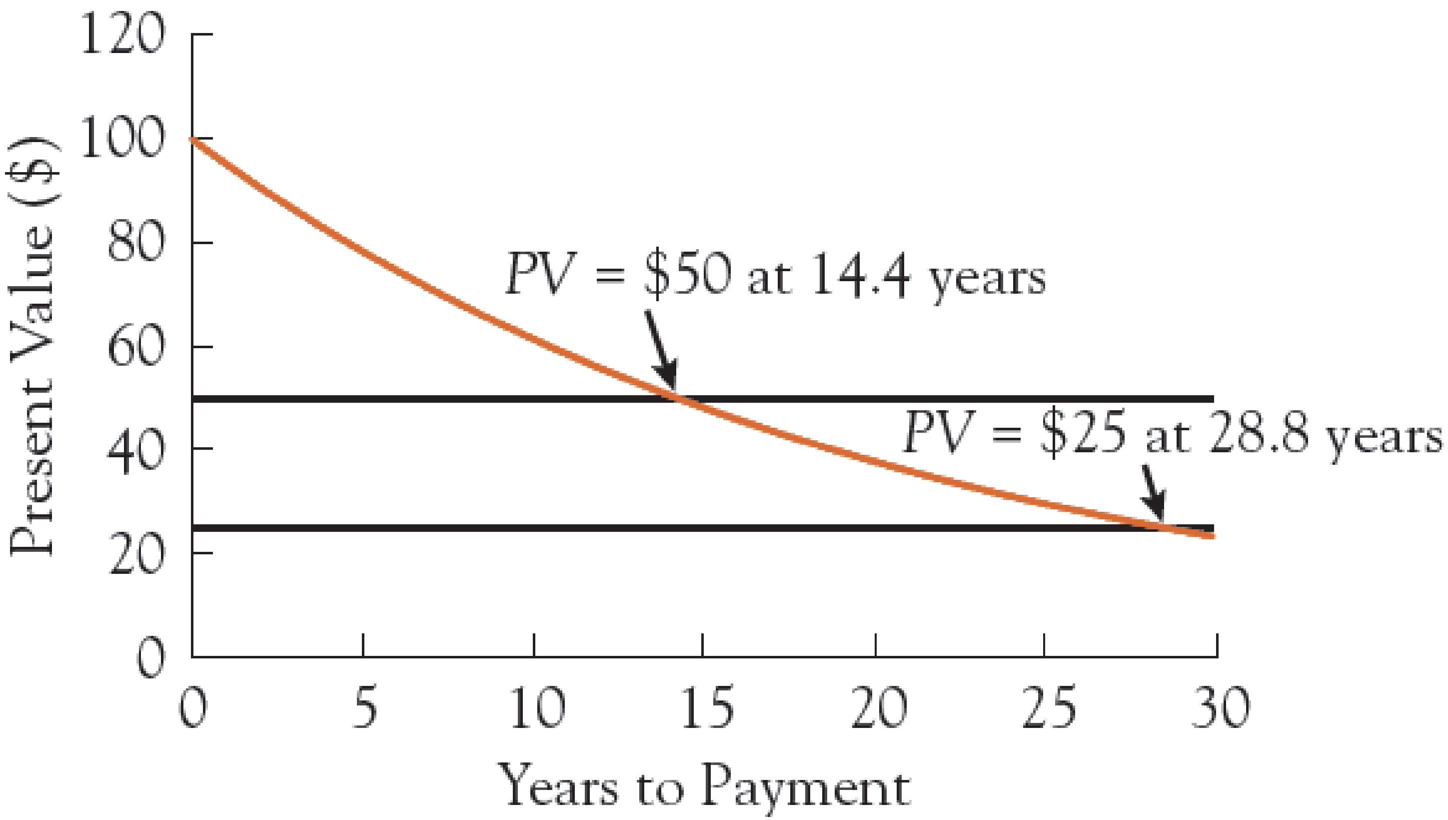


Table 4.2: Present Value of $100 Payment

Interest Rate	Payment due in			
	1 Year	5 Years	10 Years	20 Years
1%	$99.01	$95.15	$90.53	$81.95
2%	$98.04	$90.57	$82.03	$67.30
3%	$97.09	$86.26	$74.41	$55.37
4%	$96.15	$82.19	$67.56	$45.64
5%	$95.24	$78.35	$61.39	$37.69
6%	$94.34	$74.73	$55.84	$31.18
7%	$93.46	$71.30	$50.83	$25.84
8%	$92.59	$68.06	$46.32	$21.45
9%	$91.74	$64.99	$42.24	$17.84
10%	$90.91	$62.09	$38.55	$14.86
11%	$90.09	$59.35	$35.22	$12.40
12%	$89.29	$56.74	$32.20	$10.37
13%	$88.50	$54.28	$29.46	$ 8.68
14%	$87.72	$51.94	$26.97	$ 7.28
15%	$86.96	$49.72	$24.72	$ 6.11

- Higher interest rates are associated with lower present values, no matter what the size or timing of the payment.

- At any fixed interest rate, an increase in the time reduces its present value.

TOOLS OF THE TRADE

Computing Compound Annual Rates

- We can turn a monthly growth rate into a compound-annual rate using what we have learned in this chapter.
  - Investment grows 0.5% per month
  - What is the compound annual rate?

$FV_n = PV(1+i)^n$ = 100x(1.005)12 = 106.17

Compound annual rate = 6.17%

(Note: 6.17 > 12x0.05 = 6.0)

TOOLS OF THE TRADE
Computing Compound Annual Rates

- We can also use this to compute the percentage change per year when we know how much an investment has grown over a number of years.
  - An investment has increased 20 percent over five years: from 100 to 120.

$$FV_n = PV(1 + i)^n$$

$$120 = 100(1 + i)^5$$

$$i = 0.0371$$

Internal Rate of Return

- Imagine that you run a tennis racket company and that you are considering purchasing a new machine.
  - Machine costs $1 million and can produce 3000 rackets per year.
  - You sell the rackets for $50, generating $150,000 in revenue per year.
  - Assume the machine is only input, have certainty about the revenue, no maintenance and a 10 year lifespan.

Internal Rate of Return

- If you borrow $1 million, is the revenue enough to make the payments?
- We need to compare the internal rate of return to the cost of buying the machine.
  - The interest rate that equates the present value of an investment with its cost.

Internal Rate of Return

- Balance the cost of the machine against the revenue.
  - $1 million today versus $150,000 a year for ten years.
- To find the internal rate of return, we take the cost of the machine and equate it to the sum of the present value of each of the yearly revenues.
  - Solve for *i* - the internal rate of return.

Internal Rate of Return: Example

- Solving for i, i = 0.0814 or 8.14%

$$\$1,000,000 = \frac{\$150,000}{(1+i)^1} + \frac{\$150,000}{(1+i)^2} + \frac{\$150,000}{(1+i)^3} + \ldots\ldots + \frac{\$150,000}{(1+i)^{10}}$$

- So long as your interest rate at which you borrow the money is less than 8.14%, then you should buy the machine.

APPLYING THE CONCEPT
HOW MUCH IS OUR DISTANT FUTURE WORTH?

- What is the present value of the expected losses associated with a preventable future disaster?
  - Discount rates
  - Scale of future losses

Bond Basics

- A bond is a promise to make a series of payments on specific future dates.
- Bonds create obligations, and are therefore thought of as legal contracts that:
  – Require the borrower to make payments to the lender, and
  – Specify what happens if the borrower fails to do so.

Bond Basics

- The most common type of bond is a coupon bond.
  - Issuer is required to make annual payments, called coupon payments.
  - The annual interest the borrower pays (i_c), is the coupon rate.
  - The date on which the payments stop and the loan is repaid (n), is the maturity date or term to maturity.
  - The final payment is the principal, face value, or par value of the bond.

Coupon Bond

Called a coupon bond as buyer would receive a certificate with a number of dated coupons attached.

Coupons

IN THE BLOG
Investing in College

- On average as of 2016, a four-year school costs about $25,000 per year, or about $100,000 for a degree.
  - More than 1/3 the median 2016 house price
- Number of student loans is up significantly
  - Rising cost of schooling
  - The returns to a college education have risen
  - Expansion of for-profit colleges
  - Bankruptcy code changes in 2005

Valuing the Principal

- Assume a bond has a principle payment of $100 and its maturity date is *n* years in the future.
- The present value of the bond principal is:

$$P_{BP} = \frac{F}{(1+i)^n} = \frac{\$100}{(1+i)^n}$$

  – The higher the *n*, the lower the value of the payment.

Valuing the Coupon Payments

- These resemble loan payments.
- The longer the payments go, the higher their total value.
- The higher the interest rate, the lower the present value.
- The present value expression gives us a general formula for the string of yearly coupon payments made over *n* years.

$$P_{CP} = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots\ldots + \frac{C}{(1+i)^n}$$

Valuing the Coupon Payments Plus Principal

- We can just combine the previous two equations to get:

$$P_{CB} = P_{CP} + P_{BP} = \left[\frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots\ldots + \frac{C}{(1+i)^n}\right] + \frac{F}{(1+i)^n}$$

- The value of the coupon bond, P_{CB}, rises when
  - The yearly coupon payments, *C*, rise and
  - The interest rate, *i*, falls.

Bond Pricing

- The relationship between the bond price and interest rates is very important.
  - Bonds promise fixed payments on future dates, so the higher the interest rate, the lower their present value.
- *The value of a bond varies inversely with the interest rate used to calculate the present value of the promised payment.*

YOUR FINANCIAL WORLD

What Is *Your* Risk-Free Rate?

- Pay down your debt
  - The opportunity cost of investing in a retirement fund is the interest rate you are paying on your mortgage, credit card, loan, etc.
  - No riskless investment is likely to match the rate you receive when you reduce the size of your debt

Real and Nominal Interest Rates

- Borrowers care about the resources required to repay.
- Lenders care about the purchasing power of the payments they received.
- *Neither cares solely about the number of dollars, they care about what the dollars buy.*

Real and Nominal Interest Rates

- Nominal Interest Rates (i)
  - The interest rate expressed in current-dollar terms.
- Real Interest Rates (r)
  - The inflation adjusted interest rate.

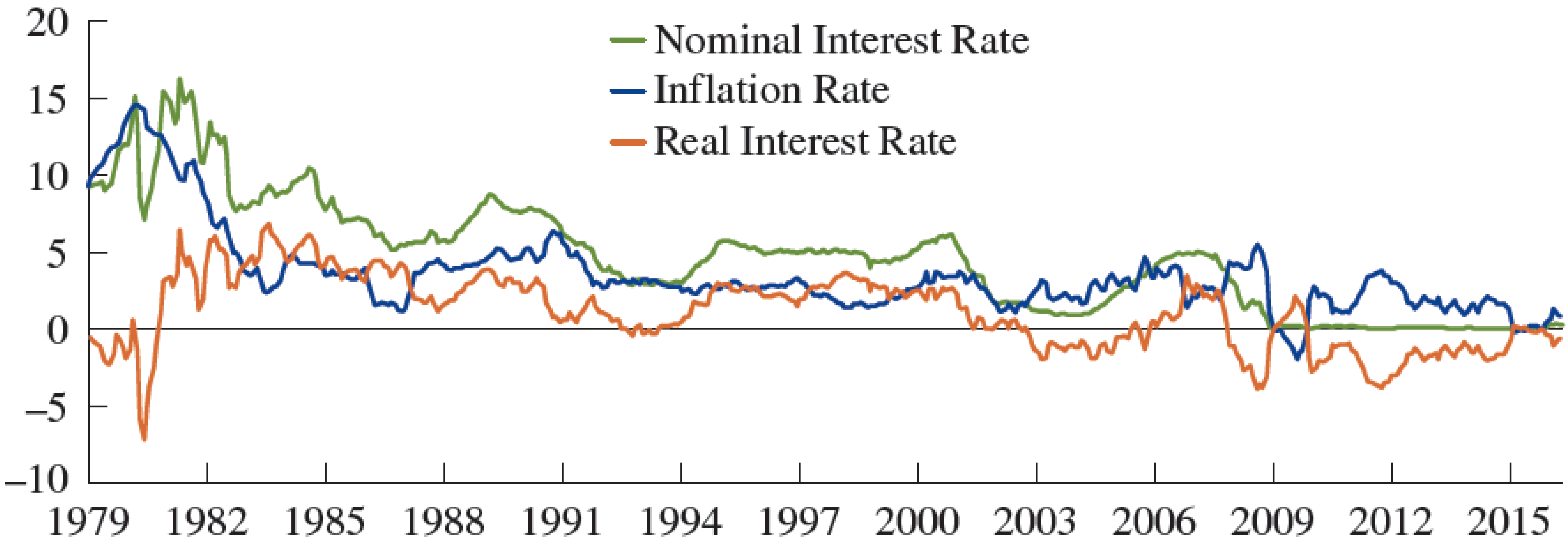
Real and Nominal Interest Rates

- The nominal interest rate you agree on (*i*) must be based on *expected inflation* (π^e) over the term of the loan plus the real interest rate you agree on (*r*).

$$i = r + \pi^e$$

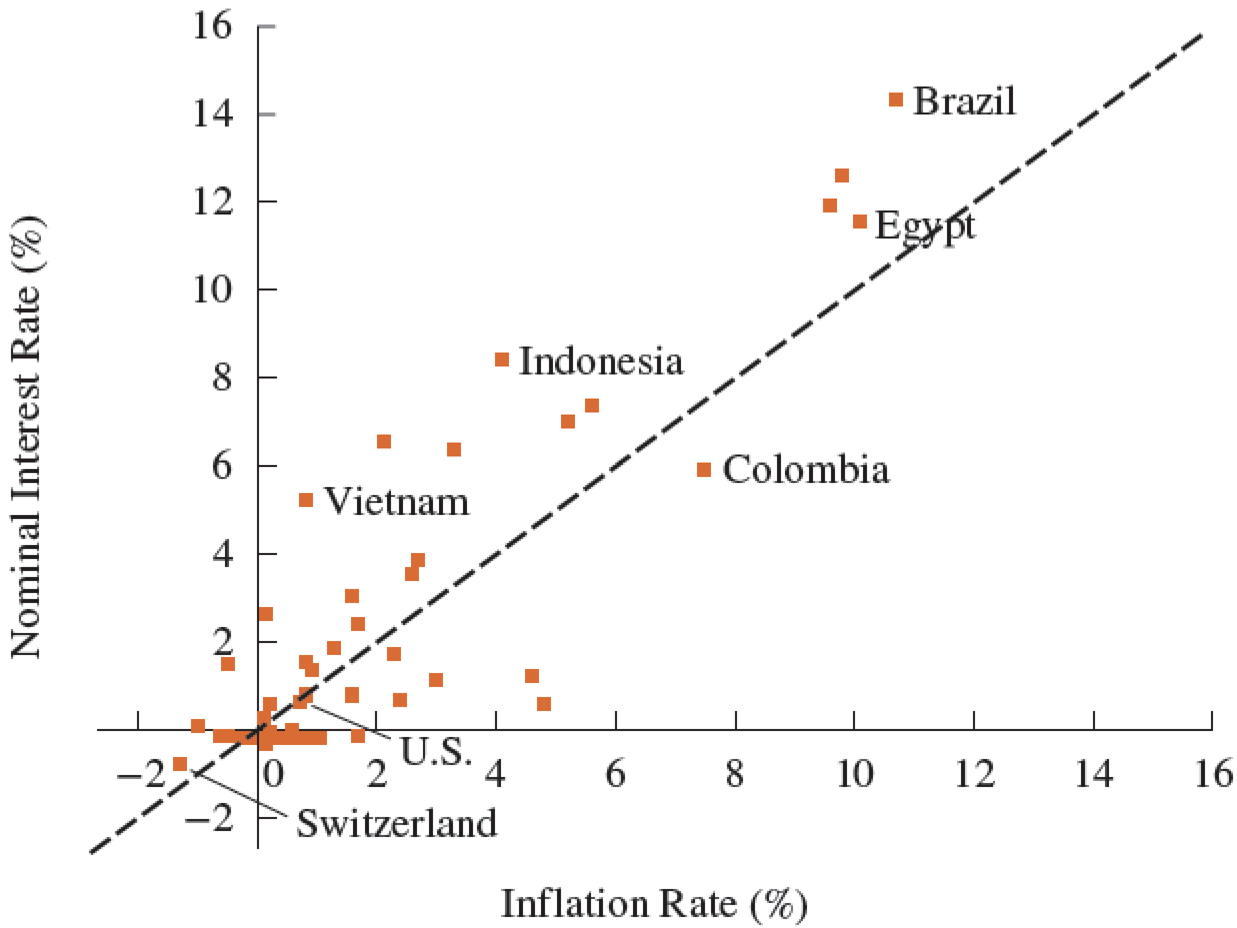
- This is called the *Fisher Equation.*
- The higher expected inflation, the higher the nominal interest rate.

Figure 4.2: Nominal Interest Rate, Inflation Rate and Real Interest Rate



APPLYING THE CONCEPT
HIGH INTEREST RATES, LOW INTEREST RATES

- This figure shows the nominal interest rate and the inflation rate in 50 countries and the euro area in early 2016.



Real and Nominal Interest Rates

- Financial markets quote *nominal interest rates*.
- When people use the term interest rate, they are referring to the *nominal rate*.
- We cannot directly observe the *real interest rate*; we have to estimate it.

$$r = i - \pi^e$$