Chapter 2

Mathematics and Economic Relations

Outline

make it more

Real World problem $\xrightarrow{\text{easy}}$ economic model → a mathematical model

to understand
Make it more abstract

"demand of the car depends on ..."

"Supply of the car depends on..."

$Q^D = f(P, Inc, \ldots)$

$Q^S = g(P, technology, \ldots)$

↓

economic insight

An economic model is a skeletal and simplified representation of the actual economy in which only primary factors and relationships relevant to our problem at hand is presented.

There are many benefits of using mathematics in the analysis within economics.

2.1 Ingredients of a Mathematical Model

If the economic model is mathematical, it will usually consist of a set of equations designed to describe the structure of the model.

By relating a number of variables to one another in certain ways, these equations give mathematical form to the set of analytical assumptions adopted.

Then, through application of the relevant mathematical operations to these equations, we may seek to derive a set of conclusions which logically follow from those assumptions.

In an economic model, it will consist of:

→ constant

→ variables

→ equations

→ Variables

A variable is something whose magnitude can change and take on different values (so we use symbol, instead of using a specific number).

e.g. price P, profit π, revenue TR, cost TC, etc.

Variables whose solution values we seek from the model is called endogenous variable.

→ its value is determined within the model (we solve for it)

Variables which are assumed to be determined by forces external to the model and whose magnitudes are accepted as given is called exogenous variable.

→ its value is determined OUTSIDE the model

Examples: Think of an economic model for market of rice.

Endogenous variables are Price, Qrice

Exogenous variables are tax, Price of other commodities, weather

→ substitutes

→ complement

(in the case of partial equilibrium)

→ Constant

A constant is a magnitude that does not change.

It can by numerical, e.g. 7, 8, 10, etc. a specific number

Or, it can be represented by a symbol, e.g. a, b, c, α, β, γ in case we haven't assigned the value to it yet.

If it is represented in a form of symbol, it will be called parametric constant or parameters.

Note: Parameters closely resemble exogenous variables in that they are to be treated as "givens" in a model.

Take constants & exogenous variables as given when solving a math model

→Equations

Equation relates variables to one another. There are three types of equations.

- A Definitional equation sets up an identity between two alternate expressions that have exactly the same meaning.

The regular equals sign $=$ or the identical-equality sign $\equiv$ "identically equal to" can be used interchangeably.

Example, $\pi \equiv TR - TC$.

$DAE \equiv C + I + G + X - M$

- A behavioral equation specifies the manner in which a variable behaves in response to changes in other variables.

Example,

if we define TC as the combination of cost from labor & capital : $TC = wL + rK$, "definitional equation"

$TC = 75 + 10Q$: TC depends linearly on quantity, $Q\uparrow$, $TC\uparrow$

$TC = 100 + Q^3$: " — " nonlinearly " — "

- A conditional equation states the requirement to be satisfied.

For example, ch 3

→ An equilibrium condition describes the prerequisite for the attainment of equilibrium.

Example,

$Y = DAE$, $S = I$

$Q^D = Q^S$

For example, Max U st budget const. → "optimization condition"

An optimization condition is derived from an optimization model.

Example,

$MC = MR$

Examples of economic models

An economic model for market equilibrium (equilibrium condition)

$\boxed{Q_d = Q_s}$ (1.) conditional equation

$Q_d = a - bP + W + Inc$ (2.) behavioral equation

$Q_s = -c + dP + Tech$ (3.) behavioral equation

endogenous variables (what you are solving for) : Q_d, Q_s, P

exogenous var. (what you take as given) : $W, Inc, Tech$

parameters (what you take as given) : a, b, c, d

A Keynesian crossing model : Answer : Which type of equation is for each equation?

$Y = DAE$ (1.) Conditional equation

$DAE = C + I + G$ (2.) definitional equation — what are endogenous variables?

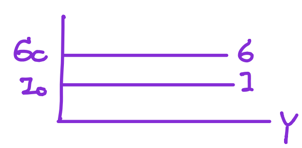
$C = a + bY_d$ (3.) behavioral equation $a > 0,\ b > 0$ — what are exogenous variable & parameters?

$Y_d = Y - T$ (4.) definitional equation

$T = T_0 + T_1 Y$ (5.) behavioral equation

$I = I_0 + 0Y$ (6.) behavioral equation

$G = G_0 + 0Y$ (7.)



2.2 Relation and Function

Set is a collection of items/objects. Each item or object in the set must share some properties/characteristic to the group/set where it belongs.

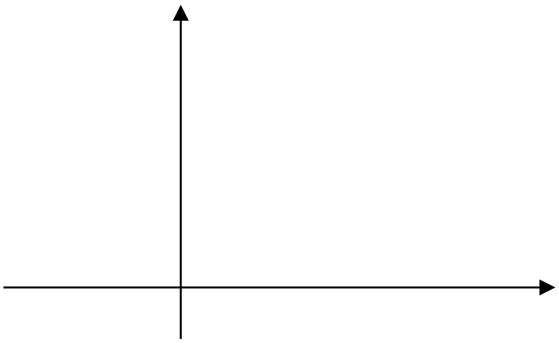
Element of one set (sets) can be associated with element of the other set, through a relation.

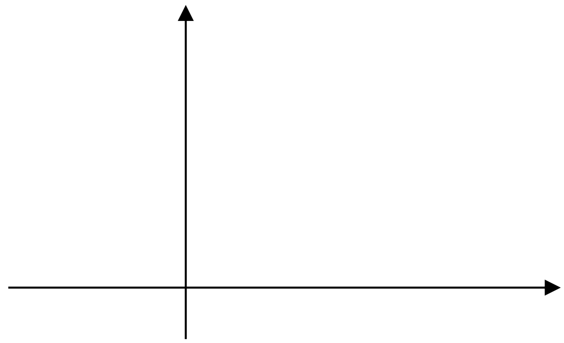
A **relation** represents a prescribed **mapping rule** between element in one set and element of the other set.

A **relation** between y and x is constituted of a collection of ordered pairs (x, y) that associates a y value with an x value.

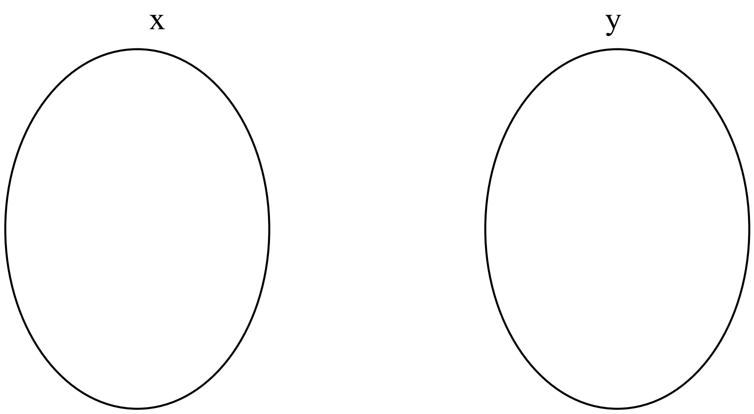
That is, given an x value, one or more y values will be specified by that relation.

$\{(x, y) | y = 2x\}$ $\{(x, y) | y \leq x\}$





In the case of $\{(x, y) | y = 2x\}$, we have that:



For each x value, there exists only one corresponding y value. This is "One-to-One Relationship" or "One-to-One Mapping"

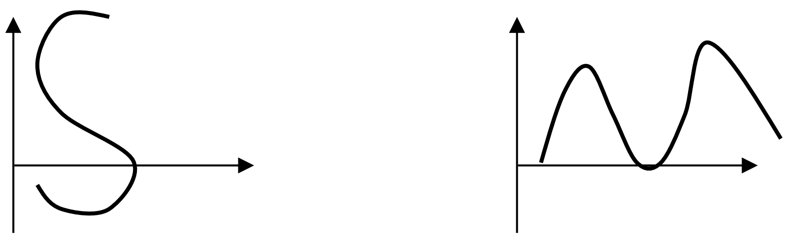
In this case, y is said to be a ________________ of x, and this is denoted by_________________ "y equals f of x".

We may write __________________ to show that the function f is a mapping rule from set x to set y.

In $y = f(x)$, x is referred to as the argument of the function or "____________Variable"
y is called the value of the function or "_________________Variable".

Test:

Is this following graph a function?



(a) (b)

Note: A function needs a unique y for each x, but the converse is not required. More than one x value may legitimately be associated with the same y value. That is, function is one-to-one or many-to-one relation, but not one-to-many relation.

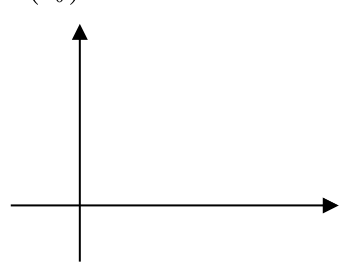
2.3 Types of Function

Different types of function represent different rules of mapping.

2.3.1 Constant Functions

A function whose range (the set of all values that the y variable can take) consists of only one element is called a constant function.

Example: In a national-income model, if we assume that investment is exogenously determined and equal to autonomous investment $\left(I_0\right)$ which takes value of 100 millionbaht, the graph is:



2.3.2 Polynomial Functions

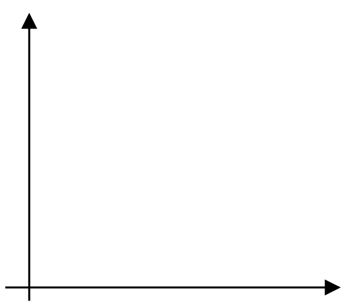
A polynomial(multi-term) function of a single variable x has the general form of

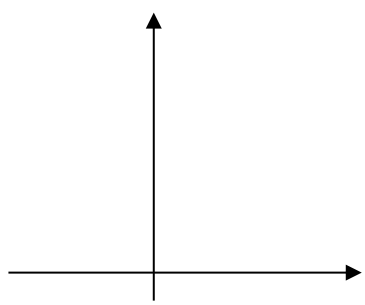
The highest power involved, the value of n is called the degree of the polynomial function.

Depending on the value of the integer n, subclasses of polynomial function are:

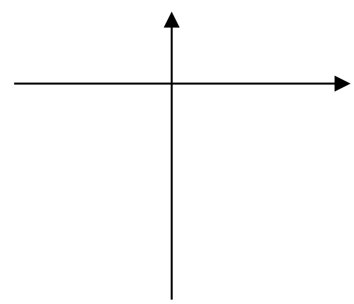
Degree n	Type of function	name
n=0	$y = a_0$	Constant
n=1	$y = a_0 + a_1 x$	Linear
n=2	$y = a_0 + a_1 x + a_2 x^2$	Quadratic
n=3	$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$	Cubic

n=1

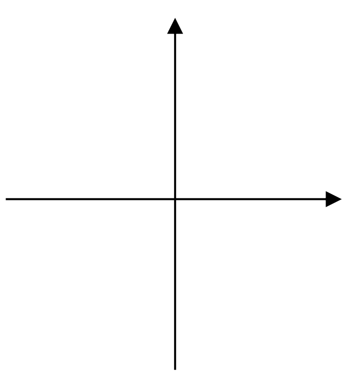


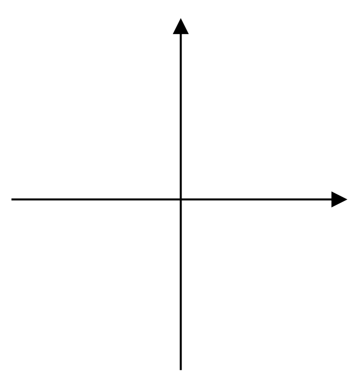


n=2



n=3





Note: The roots of a function are the x-intercepts. By definition, the y-coordinate of points lying on the x-axis is zero.

For $y = ax^2 + bx + c$, we can find the roots of a quadratic function, by using Quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example:

Suppose a research has a priori belief that the relationship between "q" and "p" is linear. From the survey, the research only knows that when p is equal to 1, q is equal to 4. Meanwhile, when p is equal to 5, q is then equal to 0. Find the equation of the line that represents the p-q relationship? What does the relationship represent?

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Note:

the "demand function" is in the form of …………………………………………………………

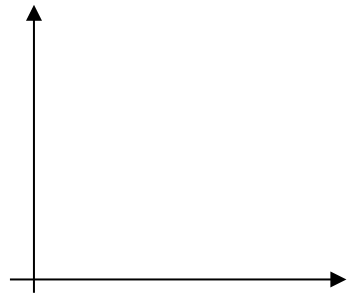
the "inverse demand function" is in the form of ………………………………………………

Example:

Consider Keynesian aggregate consumption, $C = a + bY_d$.

Does it make sense to assume that each individual household has a linear consumption function?

What if SES panel survey shows that the rich tend to consume less out of each additional unit of income they have earned? How should the graph for consumption function be?



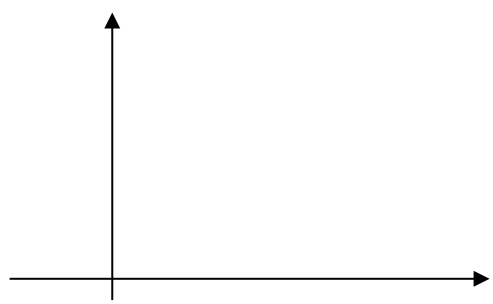
2.3.3 Exponential and Logarithmic Function

Exponential Function

Example: draw the following graphs

$y = g(t) = b^t$

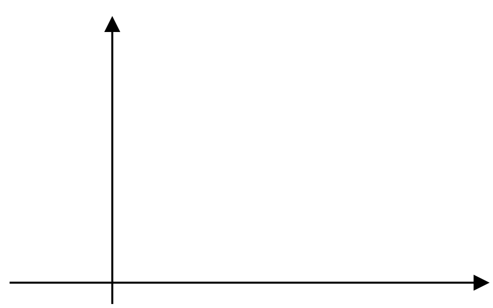
$y = f(t) = b^{2t}$



Example: draw the following graphs

$y = g(t) = b^t$

$y = f(t) = 2b^t$



If the base is the natural number e, we call it Natural exponential function:

$$y = e^t$$

, where $e = 2.71828...$

Common applications of exponential function are such as:

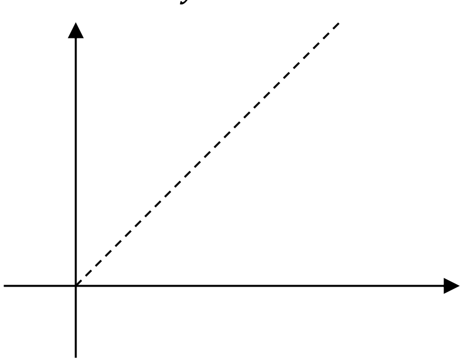
1. Growth calculation
$X_t = (1+g)X_{t-1}$, where g is growth rate, X_t is output at the t.
If we know output at the beginning period and growth rate, can we calculate output at time t?

2. Present value

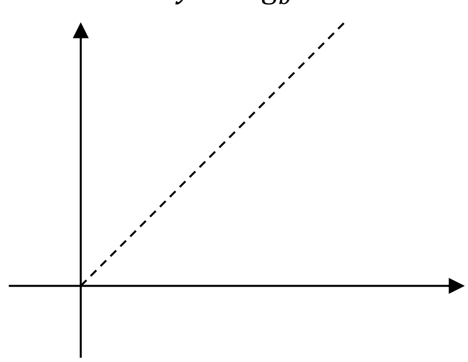
Logarithmic Function

Logarithmic Function is the inverse function of exponential function.

$y = b^x$

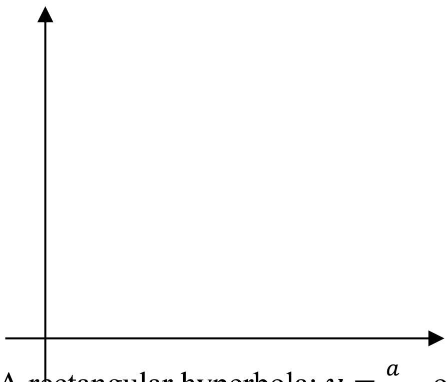


$y = \log_b x$



2.3.4 Other functions

Power Function: $y = Ax^b$



A rectangular hyperbola: $y = \frac{a}{x}$, or $xy = a$

Example:………………………………