

# **B.E. International Program**

Faculty of Economics, Thammasat University

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EE 320 Introductory Mathematical Economics

Semester 1/2015

## **Practice Problem 3 (Differential Calculus)**

1. Exercise 7.2 questions 1, 2, 4, 7, and 10 in Chiang and Wainwright (2005)
2. Find the marginal and the average functions for each of the following total functions.

a)  $TC = 2Q^2 + 5Q + 11$

b)  $\pi = Q^2 - 10Q + 68$

3. Differentiate the following functions:

a)  $y = \frac{x^2 - 9x + 20}{x - 5}$

b)  $y = \sqrt{x + \sqrt{x}}$

c)  $y = \ln(e^x + x)$

d)  $y = x^4 e^x$

4. Given the total-product function:

$$Q = 3L + 2L^2 - L^3$$

- a) Find the average product (AP) function.
- b) Find the marginal product (MP) function.
- c) Determine the slopes of the AP and MP functions. What can you conclude about their relative slopes?

5. A monopolist faces the demand curve given by  $P = kQ^{-b}$ ,  $k, b > 0$ . Answer the following questions.

- a. Find the absolute value of elasticity of demand. Does the value decrease as the quantity of output increases? Interpret your result.
- b. Find the expression for the revenue function
- c. What is the average revenue function? Describe the property of average revenue as the quantity of output increases.
- d. Derive the marginal revenue function. How does the value of “b” determine the property of marginal revenue function?
- e. Based on the functional form of market demand equation, prove/disprove the following argument. The statement is “*Raising up the price always increases the revenue of firm, since an increase in price would always make-up with the loss in the quantity sold.*” (Hint: your proof has to make use of some mathematical results regarding the marginal revenue function.)

6. Let the production function be  $Q = -3(7 - L)^5 + k$ , where  $k$  is a very large value constant. determine the level (interval) of  $L$  that the production function exhibits the law of diminishing returns.

7. Suppose that the consumption function can be given by,  $C = 5 \left( \frac{2\sqrt{Y^3+3}}{Y+10} \right)$ , find the value of MPC and MPS when  $Y = 100$

8. Suppose that total output of a firm can be given by  $Q = 10 \frac{L^2}{\sqrt{L^2+19}}$ , where is  $L$  is the number of workers hired. Answer the following questions

- Find the marginal product of labor (worker).
- Suppose this firm can only charge for a constant fixed price equal to “P”. Find the expression for the revenue function (TR) in term of  $L$ .
- Using the function derived above, Find the marginal revenue product of labor or  $\frac{dTR}{dL}$ . (Hint: you may just treat “P” as a constant term in your equation.)

Continue with the same production function given above, but replace the assumption that the firm can only charge for a constant fixed price equal to “P” with that the firm can charge price based on the quantity of output that it sells. Assume the the pricing equation is given by  $P = \frac{900}{Q+9}$ . Consider the following questions.

- Find the express of the revenue function (TR) in term of  $L$ .
- Applying the chain rule and derive the expression of marginal revenue product of labor or  $\frac{dTR}{dL}$ . Then find the value of marginal revenue product of labor when  $L = 9$ .