BACHELOR of ECONOMICS
International Program

Thammasat University | Thailand

FN 312 Investments

Capital Asset Pricing Model (CAPM) and other Index models

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Road map/Key ideas

- Conceptual derivation of the CAPM
- Expected return of individual securities
- Applications of the CAPM
- The CAPM in practice
- Other factor or index models

Conceptual derivation of the CAPM

The CAPM World

1 There are many small investors who take prices as given (i.e., their trading does not cause large moves in prices)
2 All investors have a common holding period
3 Homogeneous expectations
4 Investments are limited to publicly traded assets
5 Rational, mean-variance optimizing agents

Conceptual derivation of the CAPM

6. No taxes, no transaction costs

7. There is a risk free asset (same borrowing and lending rate)

NOTE: Investors may have different levels of risk aversion and different endowments

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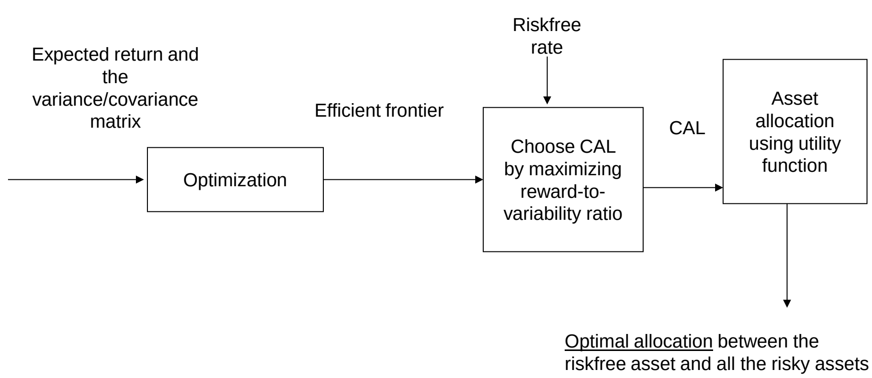
Conceptual derivation of the CAPM

Markowitz's portfolio selection



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Conceptual derivation of the CAPM

- Investors solve the Markowitz's portfolio selection problem
- Investors have the same input to the optimization problem; therefore, they obtain the same efficient frontier and optimal CAL
- Investors hold the same optimal risky portfolio
- The proportion of wealth each investor invests in the optimal risky portfolio depends on his/her risk aversion level

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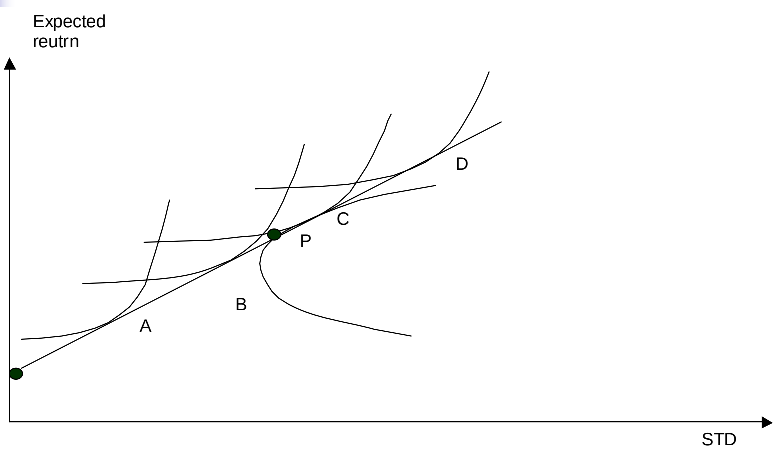
Conceptual derivation of the CAPM



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Conceptual derivation of the CAPM

Optimal risky portfolio P

w1	w2	w3	w4
0.2	0.2	0.3	0.3

Investor's Wealth	**Risk Aversion**	**Proportion of wealth in riskfree**	**Proportion of risky assets**	**Investor' $ investment in each risky asset w1**	**w2**	**w3**	**w4**
A: 1000	highest	0.8 ($800)	0.2 ($200)	40	40	60	60
B:10000	medium	0.04 (400)	0.96 ($9600)	1920	1920	2880	2880
C:10000	low	-0.1($-1000)	1.1 ($11000)	2200	2200	3300	3300
D: 1000	very low	-0.2 ($-200)	1.2 ($1200)	240	240	360	360
22000		**0**	**22000**	**4400**	**4400**	**6600**	**6600**

- Proportions of each security in the market portfolio = proportions of each asset in P

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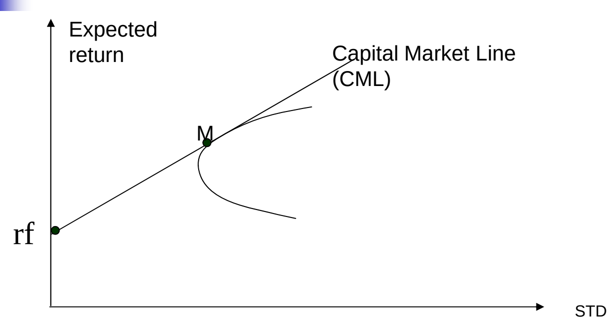
Conceptual derivation of the CAPM: Implications



1. All investors hold the market portfolio
2. The optimal CAL is the Capital Market Line

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Implications of the CAPM

3. The risk premium of the market portfolio is proportional to the average risk aversion of investors in the market and the variance of the market portfolio

How? For the portfolio which holds only the market portfolio y*=1

$$E[\tilde{r}_M] - r_f / A \bullet \sigma_M^2 = 1$$

We have that

$$E[\tilde{r}_M] - r_f = A \bullet \sigma_M^2$$

Expected return of individual securities

4. <u>The expected return of individual securities depends only on its correlation with the market portfolio (beta risk)</u>

- The risk of an individual security is its contribution to the variance of the portfolio that the investor holds
- Investors hold the market portfolio, M

Change in the variance of the market portfolio when we add security i

$$= w^2{}_i \sigma_i^2 + \sum_{j=1}^{N} w_i w_j \, \rho_{ij} \sigma_i \sigma_j \approx \sum_{j=1}^{N} w_i w_j \, \rho_{ij} \sigma_i \sigma_j = Cov(r_i, r_m)$$

Implications of the CAPM

- Firm specific variation is diversified away. The contribution of asset i to the variance of the market portfolio is

$$\frac{Cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m} = \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j = Cov(\tilde{r}_i, \tilde{r}_m)$$

- Thus the risk of an individual security is measured by (per STD of the market)
- The payoff to holding risk of the market portfolio (per STD) is its reward-to-variability ratio: $\frac{E[\tilde{r}_m] - r_f}{\sigma_m}$

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Implications of the CAPM cont.

- Putting it all together we have the CAPM:

$$E[\tilde{r}_i] = r_f + \frac{Cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m} * \frac{[E[\tilde{r}_m] - r_f]}{\sigma_m} = r_f + \frac{Cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}[E[\tilde{r}_m] - r_f]$$

- Define beta as: $\beta_i = \frac{Cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}$
- The expected return of asset i in the market is

$$E[\tilde{r}_i] = r_f + \beta_i [E[\tilde{r}_m] - r_f]$$

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Concept check

Which of the following two companies do you think has a higher return volatility (or earnings volatility)?

1. An oil exploration company that gets contracts to drill for oil but does not sell or own the oil field. It gets paid a fee and a large bonus when oil is found
2. A consumer goods retail company

G) Oil exploration

Y) Retail company

R) Both equal

Concept check

What do you think should be the expected eturn of the following two companies?

1. An oil exploration company
2. A consumer goods retail company

G) Expected return of the two are the same

Y) Expected return of oil exploration company is higher

R) Expected return of consumer retail company is higher.

Expected return of individual securities: Beta

How to get beta?

- Use 5 years of monthly historical returns and estimate the time series regression

$$\tilde{r}_{it} = \gamma_0 + \gamma_1 \tilde{r}_{mt} + \tilde{\varepsilon}$$

where $\tilde{r}_{it}, \tilde{r}_{mt}, \text{and}, \tilde{\varepsilon}_{it}$ are past security returns, market portfolio returns, and random noise respectively.

$$\gamma_1 = \frac{Cov(\tilde{r}_{it}\tilde{r}_{mt})}{Var(\tilde{r}_{mt})} = \beta_i$$

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Expected return of individual securities: Beta

- Use beta of a firm that has similar characteristics (i.e., same industry, same size, similar growth projections)
  - Note: must be careful in an oligopoly industry
- Beta of a portfolio C that invests w_A in stock A and w_B in stock B is

$$\beta_c = w_A \beta_A + w_B \beta_B$$

where the weights are value weights

$$W_A = \frac{MKT\_Cap_A}{MKT\_Cap_A + MKT\_Cap_B}$$

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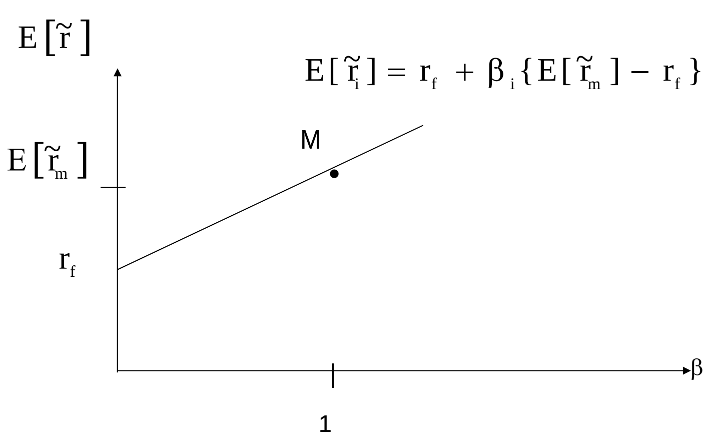
Concept check

- Suppose that the risk premium on the market portfolio is estimated at 8% with a standard deviation of 22%. What is the risk premium on a portfolio invested 25% in GM and 75% in Ford, if they have betas of 1.1 and 1.25, respectively?

G) 9.7%
Y) 9.1%
R) 10.7%

Expected return of individual securities: SML

The Security Market Line (SML)



Expected return of individual securities: SML

- The SML shows the expected return of efficiently priced assets given their beta risks
- The SML passes through r_f at $\beta = 0$ and $E[\tilde{r}_m]$ at $\beta = 1$
- The slope of the SML = risk premium of the market portfolio

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Some Applications of the CAPM

- Portfolio performance measurement
- Identify inefficiently priced assets
- Discount uncertain cash flow for pricing projects, IPO, non-traded assets

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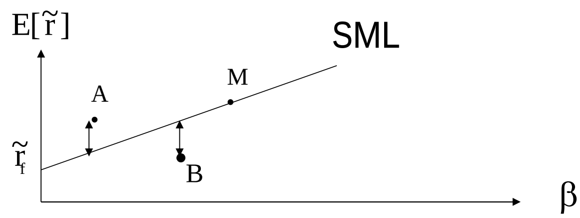
1. Performance measurement

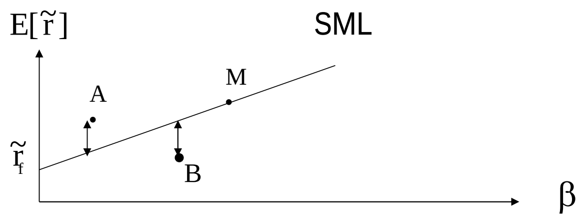
- Compare the performance of mutual funds and fund managers
- Example: $E[\tilde{r}_m]=15\%$ and $r_f=5\%$. The average returns of fund A and B are 16% and 18%. Their betas are 1 and 1.4 Which fund manager has better information than the market?
- Jensen's alpha measures the performance of a portfolio or security relative to the market
- Jensen's alpha $\alpha_i = E[\tilde{r}_i] - [r_f + \beta_i\{E[\tilde{r}_m] - r_f\}]$
- Fund A: α_B = 16% -[5%+1*(15%-5%)]= 1%
- Fund B: = 18% -[5%+1.4*(15%-5%)]= -1%

2. Identify inefficient securities



- All efficiently priced assets fall on the SML
- Which security should we buy/sell A or B?

2. Identify inefficient securities



- All efficiently priced assets fall on the SML
- Which security should we buy/sell A or B?
  - Buy A sell B

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2. Identify inefficient securities

Example: What should be the expected return of a stock with a beta of 1.2 when the expected return on the market = 12% and the risk-free rate = 4%?

$$E[\tilde{r}_i] = r_f + \beta_i \{E[\tilde{r}_m] - r_f\}$$

$$E[\tilde{r}_i] = 4\% + 1.2\{12\% - 4\%\} = 13.6\%$$

If the expected return of this stock in the stock market is > 13.6% then it is under priced.

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3. Compute NPV of uncertain Cash flows

Project or IPO

Example: $E[\tilde{r}_m]=15\%$ and $r_p=5\%$. Project A has a beta of 1.2 the following cash flows:

	CF_1	CF_2		CF_3
YEAR	1	2	3	

NPV of A=

$$\frac{CF_1}{(1+r_1+\beta_i(E[\tilde{r}_m]-r_1)}+\frac{CF_2}{(1+r_2+\beta_i(E[\tilde{r}_m]-r_2))^2}+\frac{CF_3}{(1+r_3+\beta_i(E[\tilde{r}_m]-r_3))^3}$$

r_1, r_2, and r_3 are the interest rates at 1, 2, and 3 years.

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Single Factor Model

$$r_i = E(r_i)+\beta_i m+e_i$$

$ß_i$ = index of a securities' particular return to the factor

m = Unanticipated movement related to security returns

e_i = Assumption: a broad market index like the S&P 500 is the common factor.

Single-Index Model

- Regression Equation:

$$R_t(t) = \alpha_i + \beta_t R_M(t) + e_i(t)$$

- Expected return-beta relationship:

$$E(R_i) = \alpha_i + \beta_i E(R_M)$$

Single-Index Model Continued

Risk and covariance:

- Total risk = Systematic risk + Firm-specific risk: $\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$
- Covariance = product of betas x market index risk: $Cov(r_i, r_j) = \beta_i \beta_j \sigma_M^2$
- Correlation = product of correlations with the market index

$$Corr(r_i, r_j) = \frac{\beta_i \beta_j \sigma_M^2}{\sigma_i \sigma_j} = \frac{\beta_i \sigma_M^2 \beta_j \sigma_M^2}{\sigma_i \sigma_M \sigma_j \sigma_M} = Corr(r_i, r_M) x Corr(r_j, r_M)$$

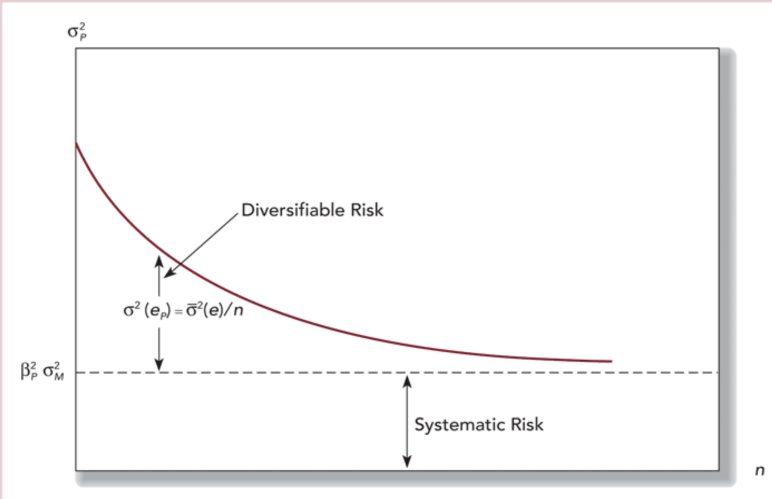

FIGURE 8.1 The variance of an equally weighted portfolio with risk coefficient β_P in the single-factor economy

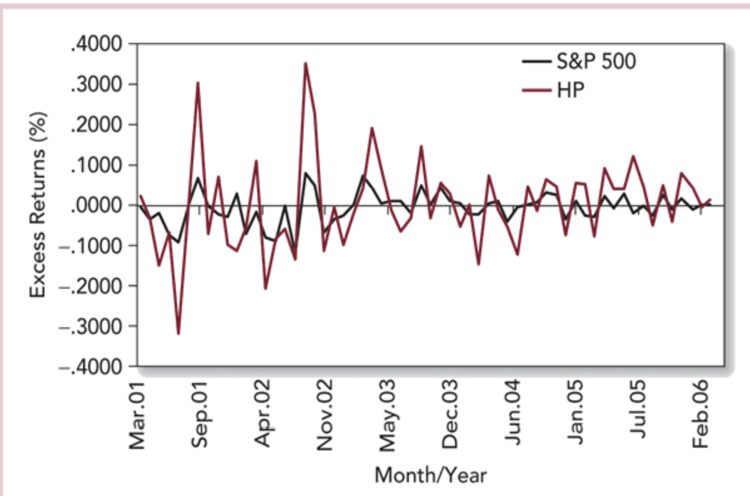

FIGURE 8.2 Excess returns on HP and S&P 500 for April 2001 to March 2006

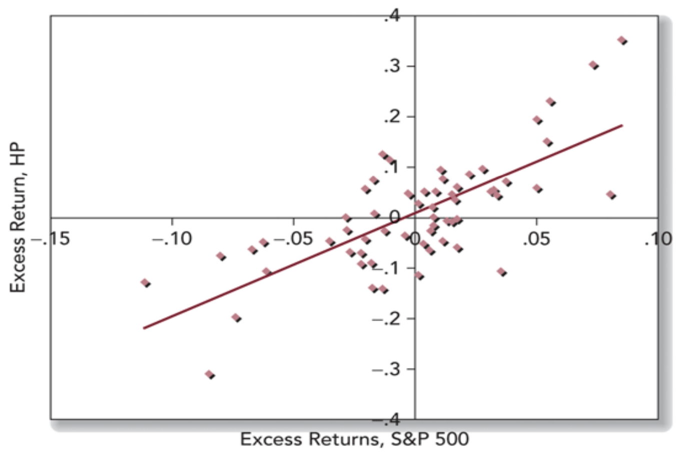

FIGURE 8.3 Scatter diagram of HP, the S&P 500, and the security characteristic line (SCL) for HP

Regression Statistics for the SCL of Hewlett-Packard

TABLE 8.1

Excel output: Regression statistics for the SCL of Hewlett-Packard

Regression Statistics				
Multiple *R*	.7238			
R-square	.5239			
Adjusted *R*-square	.5157			
Standard error	.0767			
Observations	60			
ANOVA				
	df	**SS**	**MS**	
Regression	1	.3752	.3752	
Residual	58	.3410	.0059	
Total	59	.7162		
	Coefficients	**Standard Error**	***t-Stat***	***p-Value***
Intercept	0.0086	.0099	0.8719	.3868
S&P500	2.0348	.2547	7.9888	.0000

Single-Index Model Input List

- Risk premium on the S&P 500 portfolio
- Estimate of the SD of the S&P 500 portfolio
- *n* sets of estimates of
  - Beta coefficient
  - Stock residual variances
  - Alpha values

Is CAPM a good representative of security returns in the market?

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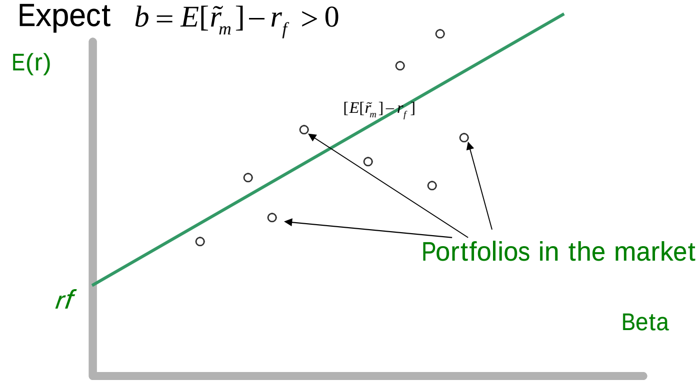
CAPM in Practice

- Testing the CAPM: estimate $E[\tilde{r}_i] = a + b\beta + \tilde{\varepsilon}$
- Expect $b = E[\tilde{r}_m] - r_f > 0$



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CAPM in Practice

Current State of the CAPM

- Overall, CAPM is a good first pass approximation
- CAPM is very intuitive
- CAPM does poorly with small firms (the 5% smallest firms in the intersection of NYSE, Nasdaq, and AMEX

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CAPM in Practice

From Fama and French (1992, 1993) and Cochrane (2000)

- CAPM is a good predictor of average return of securities from 1900 to the late 1960s
- CAPM does not seem to have statistical power to predict average returns of securities from 1960s to 1990s
- Size and book-to-market seem to have the power to explain average returns of securities from 1960s to 1990s

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CAPM in Practice

- What does this mean?

1. Size and book-to-market proxy for risk factors not captured by the market portfolio
2. Size and book-to-market indicate inefficiencies in the market

- The first view is more widely accepted

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Extensions of the CAPM

- The market portfolio may not capture all systematic risk
- Use multi-factor models that include other variable such as other economic risks

$$E[\tilde{r}_i] = r_f + \gamma_1(E[\tilde{r}_m] - r_f) + \gamma_2(E[f_2] - r_f) + \gamma_3(E[f_3] - r_f)$$

f_1 and f_2 are factors that capture the nondiversifiable risk that is not captured by the market portfolio

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Extensions of the CAPM cont.

- Economically driven factors

  - The difference between the long-term and short-term bond yield

  - Unexpected inflation

  - Return of human capital

- Empirically driven factors (These factors can be obtained from French's website at MIT)

  - Size

  - Book-to-market

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Extensions of the CAPM cont. (aside)

BOX 1

The CAPM and multifactor models

The CAPM uses a *time-series* regression to measure beta, β, which quantifies an asset's or portfolio's tendency to move with the market as a whole,

$$R_t^i - R_t^f = a_i + \beta_{im}(R_t^m - R_t^f) + \varepsilon_t^i; \quad t = 1, 2 \ldots T \text{ for each asset } i.$$

Then, the CAPM predicts that the expected excess return should be proportional to beta,

$$E(R_t^i - R_t^f) = \beta_{im}\lambda_m \text{ for each } i.$$

λ_m gives the "price of beta risk" or "market risk premium"—the amount by which expected returns must rise to compensate investors for higher beta. Since the model applies to the market return as well, we can measure λ_m via

$$\lambda_m = E(R_t^m - R_t^f).$$

Multifactor models extend this theory in a straightforward way. They use a time-series *multiple* regression to quantify an asset's tendency to move with multiple risk factors F^A, F^B, etc.

3) $$R_t^i - R_t^f = a_i + \beta_{im}(R_t^m - R_t^f) + \beta_{iA}F_t^A + \beta_{iB}F_t^B + \ldots + \varepsilon_t^i; \quad t = 1,2 \ldots T \text{ for each asset } i.$$

Then, the multifactor model predicts that the expected excess return is proportional to the betas

4) $$E(R_t^i - R_t^f) = \beta_{im}\lambda_m + \beta_{iA}\lambda_A + \beta_{iB}\lambda_B + \ldots \text{ for each } i.$$

The residual or unexplained average return in either case is called an alpha,

$$\alpha_i \equiv E(R_t^i - R_t^f) - (\beta_{im}\lambda_m + \beta_{iA}\lambda_A + \beta_{iB}\lambda_B + \ldots).$$

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Summary

- Conceptual derivation of the CAPM
- Expected return of individual securities
- Applications of the CAPM
- Index models
- The CAPM in practice

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