CH10. A Real Intertemporal Model with Investment

EE312 (for Section 046402)

Read: Williamson Ch. 11

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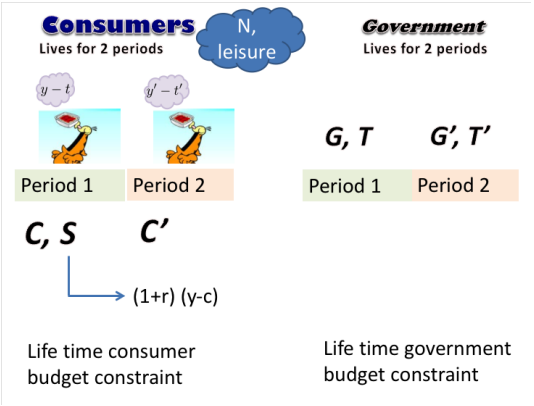
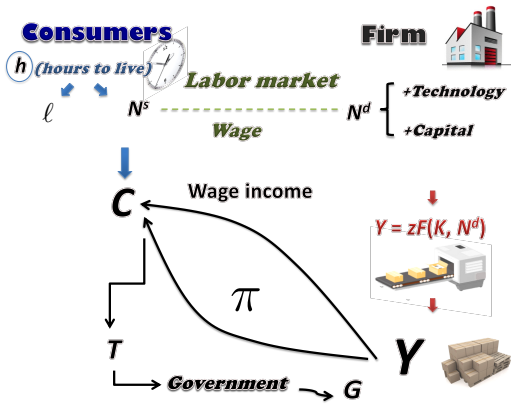
2 Introduction





- **The real model (no money)** with three sectors as the basis for the analysis of short-term economic fluctuations.

  1. **Representative consumers** (consumption, labor supply and saving);
  2. **Representative firms** (production, labor demand and investment);
  3. **Government** (spending, taxes and borrowing).

- **Two-market model**

  - **The labor market:** the firm's demand and the consumer's supply of labor.
    $\Rightarrow$ The real wage rate.
  - **The output market:** the firm's supply and the consumer's demand for output.
    $\Rightarrow$The real interest rate. Analysis of real macroeconomic shocks.
  - **Changes in government spending, capital stock, total factor productivity.**

- **Investment**

  - Expenditure on plants, equipment and new housing.
  - Goods currently produced for future production of goods and services.
  - Increases in future productive capacity.
  - **The consumer's tradeoff** between current and future consumptions (saving).
  - **The firm's tradeoff** between current profits and higher future capital stock (and future profits).

3 Consumer's optimal decisions

- Work-leisure in current and future periods.
- Consumption-savings in the current period.
  - h = total time available;
  - w and w' = current and future real wages;
  - r = the real interest rate;
  - T and T' = current and future lump-sum taxes;
  - C and C' = current and future consumptions;
  - ℓ and ℓ' = current and future leisure time;
  - S^p = private savings.

3.1 Budget Constraint

3.1.1 Current budget constraint

- The consumer is a price-taker (w, w', r and T are given).
- $w(h-\ell)$ = real-wage income;
- π = dividend income from the firm;
- T = lump-sum taxes paid to the government.
- Then, disposable income is:

$$C + S^P = w(h-\ell) + \pi - T$$

3.1.2 Future budget constraint

- The consumer still receives real wages, dividend income, and pays future taxes.
- Receives the principal and interest on savings.
- No bequests; all wealth is consumed.

$$C' = w'(h-\ell') + \pi' - T' + (1+r)S^p$$

3.1.3 Lifetime budget constraint

$$C + \frac{C'}{1+r} = w(h-\ell) + \pi - T + \frac{w'(h-\ell') + \pi' - T'}{1+r}$$

- The PV of lifetime consumption equals the PV of lifetime disposable income.
- Decision on the optimal bundles of C, C', ℓ and ℓ' subject to the lifetime budget constraint.

3.2 Optimal Condition

3.2.1 Current period optimal condition

$$MRS_{\ell,C} = w$$

- The consumer chooses the optimal bundle of current leisure and consumption:
  - The marginal rate of substitution of current leisure for current consumption is equal to the real wage.
  - w is the relative price of leisure in terms of consumption goods.

3.2.2 Future period optimal condition

$$MRS_{\ell',C'} = w'$$

- The consumer chooses the optimal bundle of future leisure and future consumption:
- The marginal rate of substitution of future leisure for future consumption is equal to the future real wage.

3.2.3 Intertemporal optimal condition

$$MRS_{C,C'} = 1 + r$$

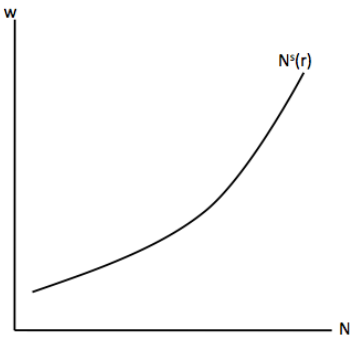
- The consumer chooses the optimal bundle of current and future consumption (savings):
- The marginal rate of substitution of current consumption for future consumption is equal to the real interest rate.
- (1 + r) is the relative price of current consumption in terms of future consumption.

3.3 Current labor supply

3.3.1 Current Labor Supply curve

- The consumer provides labor supply to the firm through work-leisure decision.
- Factors which determine current labor supply:
  - The current real wage;
  - The real interest rate;
  - Lifetime wealth.

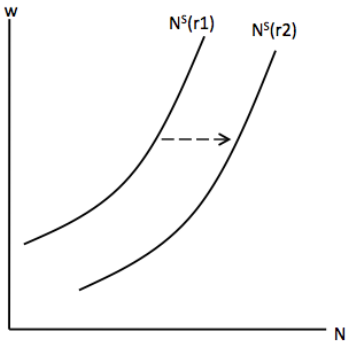
- **Current Labor Supply Curve**
  - Current labor supply increases with the real wage, given r (assuming the dominant substitution effect).



3.3.2 Effect of An increase in the real interest rate on Labor Supply Curve

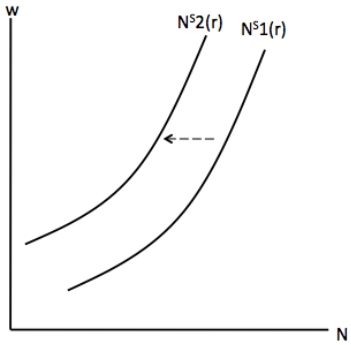
- Current labor supply increases as the real interest rate increases.
- $\frac{w(1+r)}{w'}$ is the relative price of current leisure in terms of future leisure.
- Given w and w', a higher r means the higher price of current leisure in terms of future leisure.
- Less current leisure, and more current supply of labor, assuming the dominant substitution effect.

- **Labor supply increases with r**
  - Given w, labor supply increases with the rising real interest rate (r2 > r1), assuming the dominant substitution effect.



3.3.3 Effect of An increase in lifetime wealth on Labor Supply Curve

- Current leisure increases and current labor supply decreases with rising lifetime wealth.
- Current and future consumption also increase.

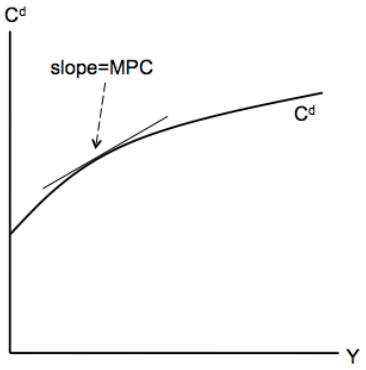


3.4 Demand for current consumption goods

3.4.1 Derivation of Demand for current consumption

- The individual demand for current consumption goods (C^d) is a function of current income (Y), given r.
- The marginal propensity to consume (MPC) < 1 .
- A higher real interest rate (r) causes the demand to fall, assuming:
- The substitution effect dominates the income effect; the consumer is a lender.

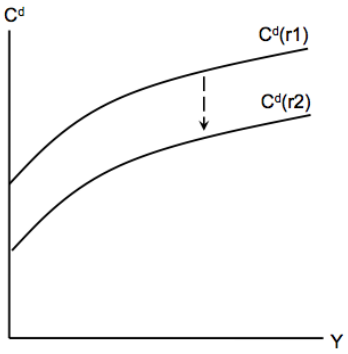
- $C^d = f(Y, r)$
- $MPC = \frac{\partial C^d}{\partial Y} < 1$;
- $\frac{\partial C^d}{\partial r} < 0; \frac{\partial C^d}{\partial we} > 0$



3.4.2 Effect of an increase in interest rate on demand for current consumption

A higher r reduces C^d

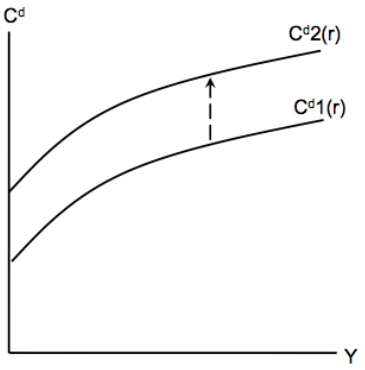
- r2 > r1.
- The consumer reduces current consumption, assuming stronger substitution effect and a lender.



3.4.3 Effect of an increase in we on demand for current consumption

An increase in we raises C^d

- An increase in lifetime wealth raises current consumption.



4 The Representative firm

- Optimal decisions on:
  - Maximized present value of profits;
  - The level of current labor inputs.
  - **Optimal investment level**: tradeoff between current profits and future capital stock (and future profits).

4.1 Production function

- **Current production function**

$$Y = zF(K, N)$$

- Y = current output;
- z = current total factor productivity;
- K = current capital stock;
- N = current labour input.
- And t**he future production function**:

$$Y' = z'F(K', N')$$

4.2 Change in capital stock

- The firm's investment is foregone current profits (consumption) for future profits:
  - d = the rate of depreciation;
  - I = current investment.
- Future capital stock is current capital stock net of depreciation plus investment.

$$K' = (1 - d)K + I$$

4.3 The Firm's profit

4.3.1 The firm's current profits

- Maximization of the present value of current and future profits.
- π = current profits;
- I = current investment = foregone current consumption goods.

$$\pi = Y - wN - I$$

4.3.2 The firm's future profits

- The leftover capital stock in the future period can be sold off as junk value.
- $(1 - d)K'$ = capital stock remaining as junk at the end of the future period.

$$\pi' = Y' - w'N' + (1 - d)K'$$

4.4 Maximized PV of profits

- The firm maximizes the PV of profits.
  - The same as maximized PV of dividend income for the consumer.
  - V = max. present value of profits through optimal choice on N, N' and I:

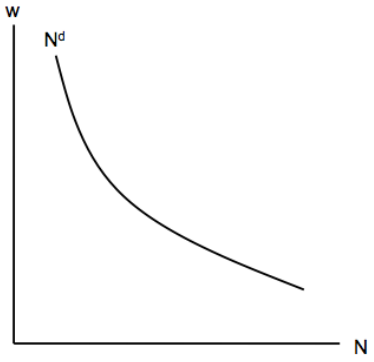
$$V = \pi + \frac{\pi'}{1+r}$$

4.5 Current labor demand

- The firm's choice of current labor demand (N^d) affects only current profits (π).
- The firm hires current labor until the current marginal product of labor equals the current real wage (MPN= w).
- Thus the firm's MP_N curve is also the firm's current labor demand curve.
- An increase in current z or K raises MP_N and current labor demand.

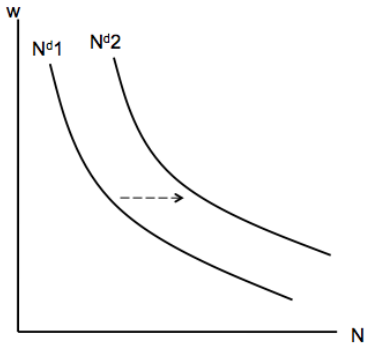
4.5.1 Current labor demand curve

- The current labor demand: MP_N = w.
- MP_N is falling as the labor input increases.



4.5.2 Labor demand with rising z or K

- An increase in current z or K shifts the current labor demand curve to the right.



4.6 The firm's investment decision

- The firm invests to the point where the marginal benefit from investment equals marginal cost.
- MC(I) = marginal cost of investment = PV of profits (V) given up for one unit of capital.
- One unit of investment reduces current π (and V) by one unit.

$$MC(I) = 1$$

- MB(I) = marginal benefit of investment = additional units of V (PV of profits) received from one extra unit of current investment.
- MPK' = additional output from one extra unit of K' .
- Quantity of capital left from depreciation at the end of the future period (1 - d) for liquidation.
- I's future profits is $(MP'_K + 1 - d) = \pi'$.

$$MB(I) = \frac{MP'_K + 1 - d}{1 + r}.$$

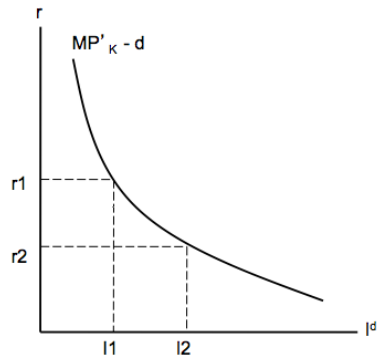
4.6.1 Optimal investment decision

$$\frac{MP'_K + 1 - d}{1 + r} = 1.$$

$$MP'_K - d = r.$$

- The firm invests until the net future marginal product of capital equals the real interest rate.
- **r = the opportunity cost of more capital** = the rate of return on the alternative asset (bonds) otherwise earned by the consumer who owns the firm.

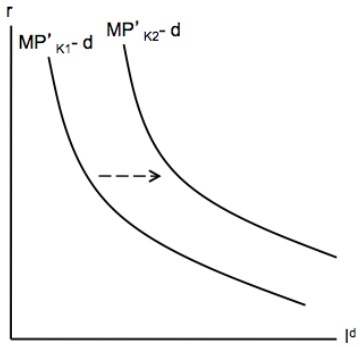
- $I^d = (MP'_K - d)$ gives the level of I required for the net MP'K to be equal to the real interest rate, given K.



4.6.2 Changes in z' and K

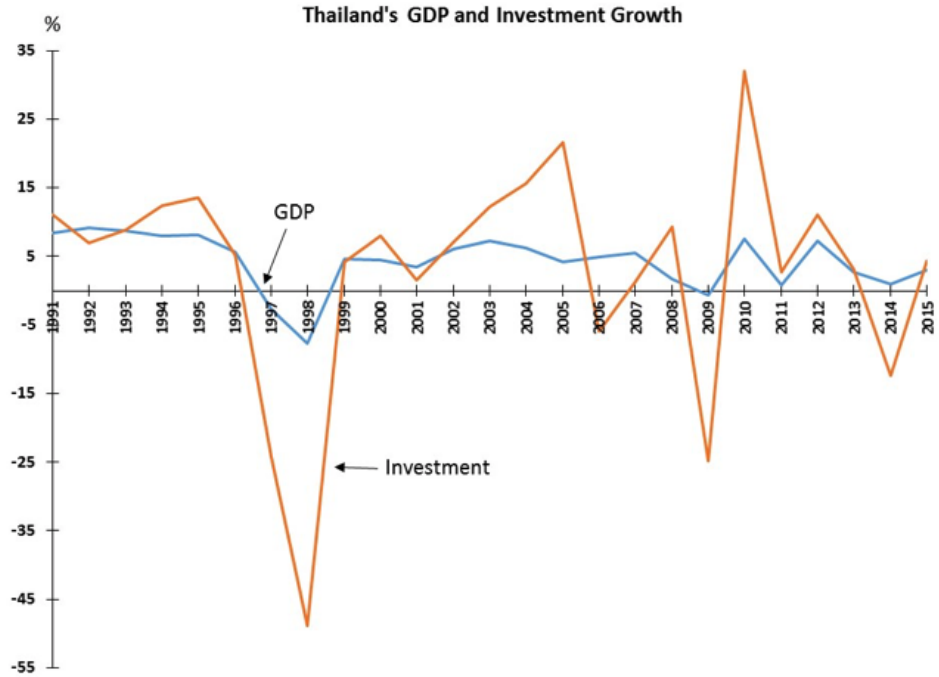
- Factors affecting future marginal product of capital shift the optimal investment curve.
- Higher **future total factor productivity** (z') increases future MP'_K and current optimal investment.
- The optimal investment curve shifts to the right.
- Higher **current capital stock** results in larger future net capital stock and lower MP'_K .
- The optimal investment curve shift to the left.

- A higher z' or a lower K increases MP'_K. The optimal investment curve shifts to the right.



4.6.3 Volatile investment and GDP

- Aggregate consumption is less variable than income due to consumption smoothing.
- Investment is much more volatile — short-run economic fluctuations.
  - Investment responds to perceived marginal rates of return to investment.
  - Changes in the real interest rate cause movements along the investment curve.
  - Changes in future total factor productivity shift the investment curve.



5 Government sector

- Government purchases of consumption goods (G and G') are exogenously determined.
- Government financing:
  - Current lump-sum taxes and bond sale;
  - Future lump-sum taxes and payments of the principal and interest.

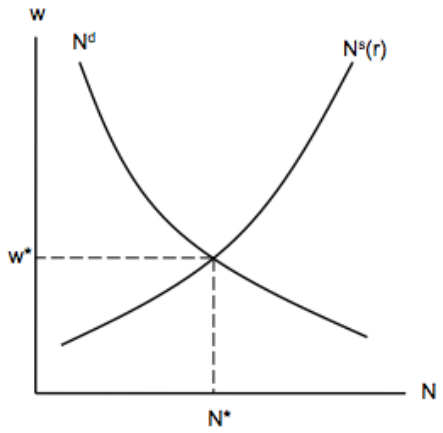
$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}.$$

6 Competitive equilibrium

- The labor market:
  - The consumer supplies labor service.
  - The firm demands labor service.
  - The real wage and the level of employment.
- The output market:
  - The consumer, the firm and government purchase output.
  - The firm supplies the goods.
  - The real interest rate and the level of aggregate output.

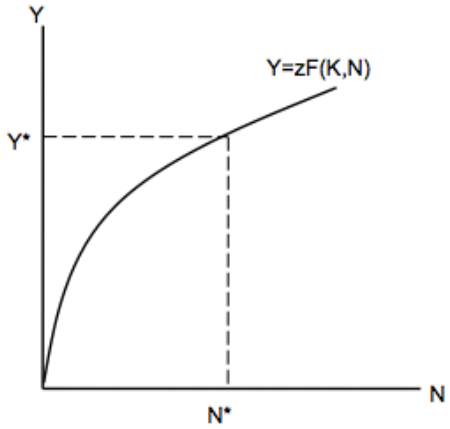
6.1 Equilibrium in the labor market

- N^s is sloped upwards with dominant substitution effect, given r.
- N^d is MPN for the firm.
- N*= equilibrium employment.



- **Aggregate output supplied**

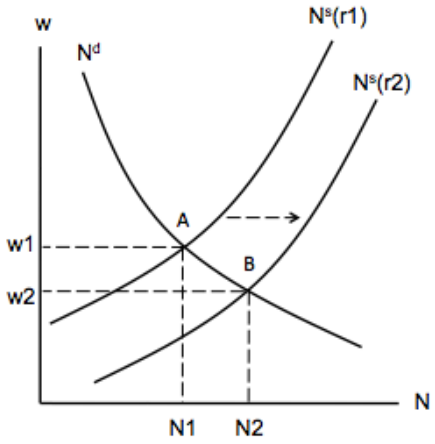
  - With N* input, Y* is the quantity of aggregate output supplied, given z and K.

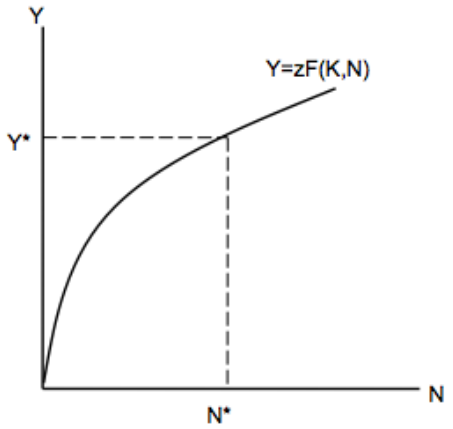


6.2 Output supply

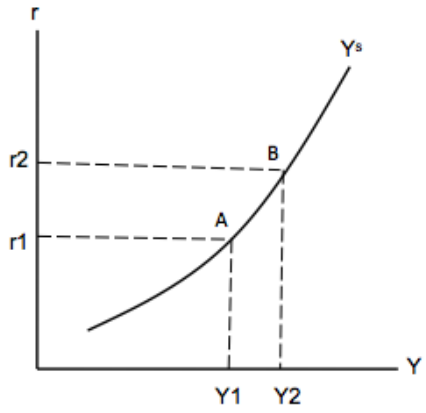
6.2.1 Output supply and real interest rate

- The relationship between the real interest rate and the level of aggregate output.
- An increase in the real interest rate causes a reduction in current consumption and leisure.
  - **The labor market:** current labor supply increases.
  - **The production function**: current aggregate output increases.
- The output supply curve is sloped upwards.
- Employment and output





- **Output supply curve**
  - The higher r causes more labor supply, employment and output.
  - The labor market is in equilibrium at each level of r.



6.2.2 Output supply shifts

- Changes in **exogenous variables** shift the output supply curve.
  - Lifetime wealth (labor supply shifts);
  - Current total factor productivity or current capital stock (labor demand and production function shifts).
- Changes in the real interest rate move along the output supply curve.

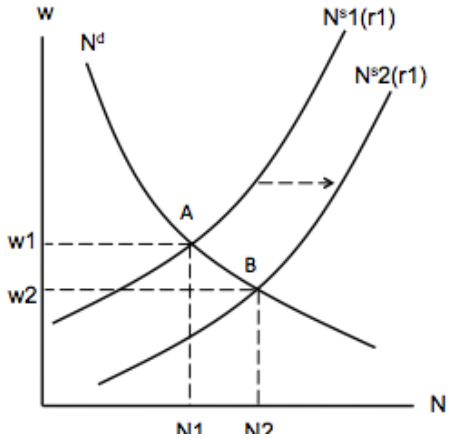
6.2.2.1 Changes in lifetime wealth

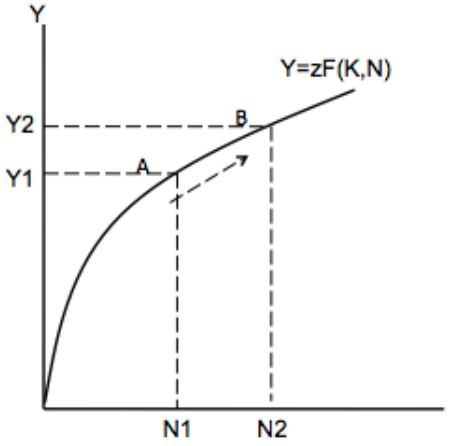
- A decrease in lifetime wealth reduces leisure (income effect) and increases labor supply, given the real wage.
  - Increases in current or future government spending reduce lifetime wealth.
  - The government PV budget constraint implies increases in the PV of taxes.

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

- **The labor market:**
  - The labor supply curve shifts to the right.
  - The real wage falls while employment increases.
- **The production function**:
  - More labor input increases production.
  - Output increases, given r.
- The output supply curve shifts to the right.

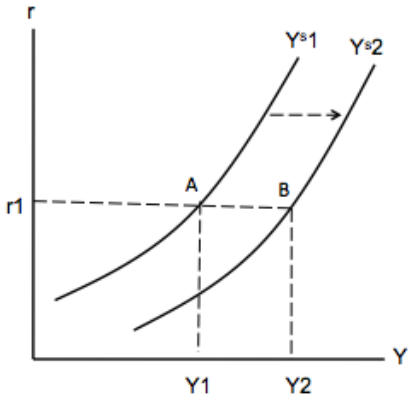
A decrease in lifetime wealth





Rising output supply for lower we

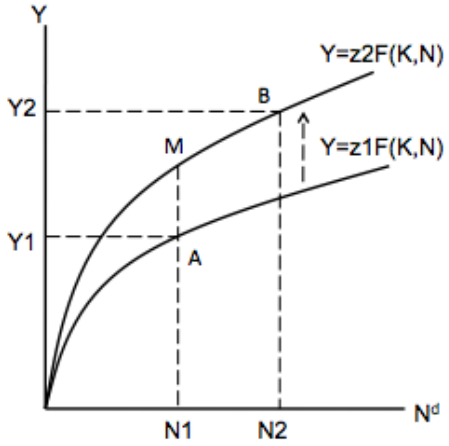
- Output supply increases, given r, when lifetime wealth decreases and labor supply increases.

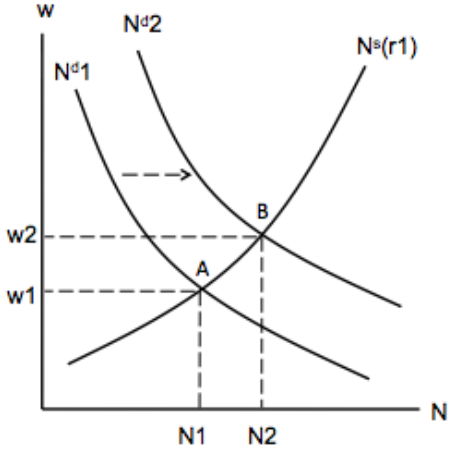


6.2.2.2 Changes in z or current K

- An increase in total factor productivity or current capital stock.
  - Marginal product of labor (MPN) increases.
  - The production function shifts up;
  - The labor demand curve shifts to the right.
  - Employment increases with the real wage.
  - Output increases, given the real interest rate.
- The output supply curve shifts to the right.

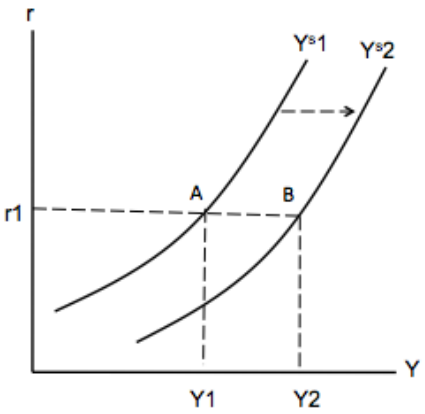
An increase in z or K





Rising output supply for higher z, K

- Higher z or K raises labour demand and the real wage.
- Rising employment and output supply, given r.

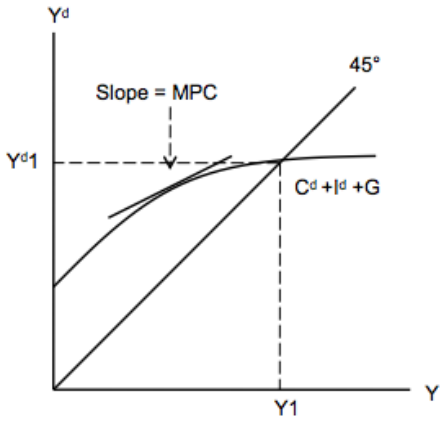


6.3 Output Demand

- Total current demand for goods (Y^d) equals the sum of:
  - The consumer's demand for consumption goods (C^d);
  - The firm's demand for investment goods (I^d);
  - The government purchases of current goods (G).
- C^d and I^d are negatively related to the real interest rate.

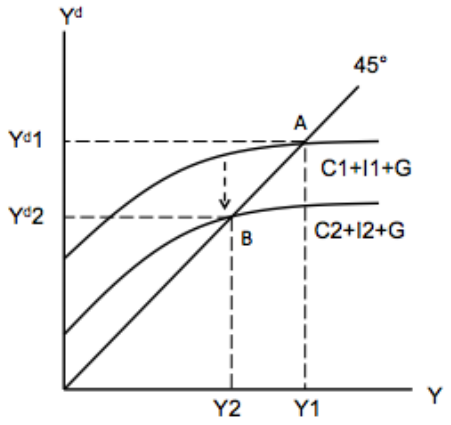
$$Y^d = C^d(r) + I^d(r) + G$$

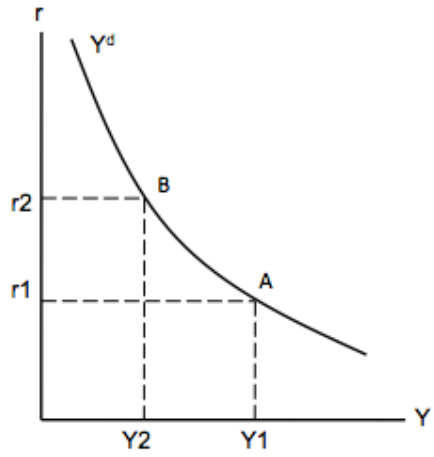
- I and G are not related to Y.
- The slope = MPC.
- Equilibrium Y^d = Y1.



6.3.1 Output demand and real interest rate

- An increase in the real interest rate causes a reduction in demand for current output.
  - Shifts towards future consumption: falling demand for current consumption goods.
  - Lower optimal investment: higher opportunity cost of capital.
- The output demand curve is sloped downwards.
- **Output Demand Curve**

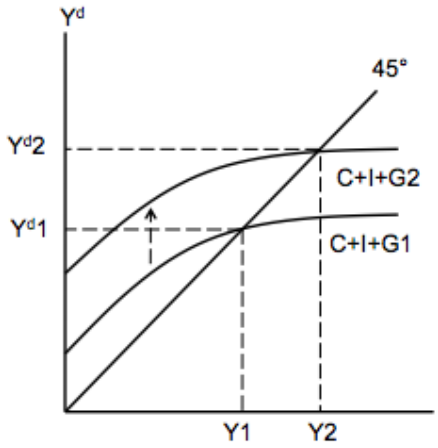


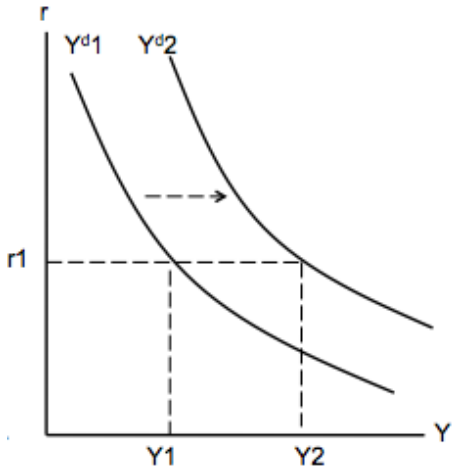


6.3.2 Rightward shifts in output demand curve

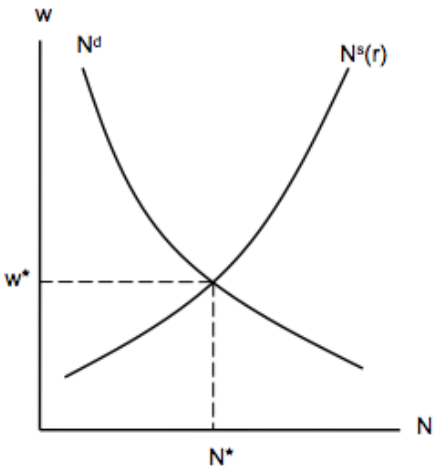
- An increase in current government purchases (G);
- A decrease in the PV of taxes (T or T');
- An increase in future income (Y');
- An increase in future total factor productivity (z') or a decrease in current capital stock (K).
- Higher future MP'_K and rising current I^d

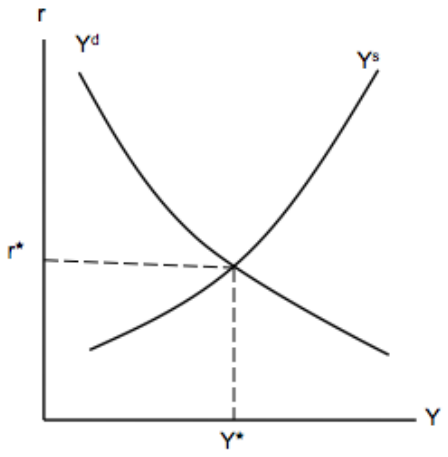
An increase in government spending





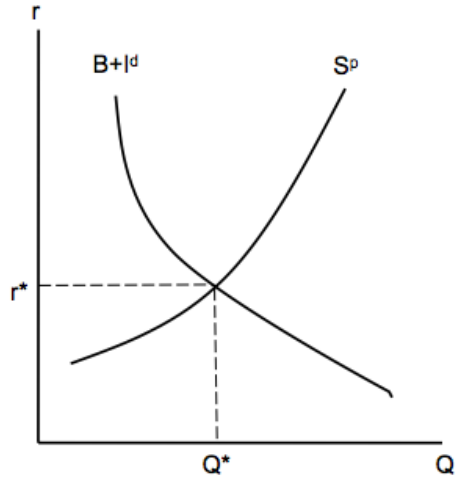
6.4 Complete real intertemporal model





7 Credit Market

- The supply of credit is the consumer's private saving.
- The demand for credit is government's borrowing and the firm's investment demand (I^d).



8 Shocks Experiments

- Exogenous shocks in the model
  - A shock in the model occurs when one of exogenous variable changes, causing endogenous variables to change accordingly.
  - The macro effect depends on whether it is temporary or permanent.
  - An expected shock in the future has effects in the current period.
- Shocks Experiments
  1. Current government purchases increase temporarily (G);
  2. Current capital stock decreases due to a natural disaster or war (K);
  3. A temporary increase in current total factor productivity (z);
  4. An increase in future total factor productivity (z').

8.1 A temporary increase in G

- Assume an increase in G with G' unchanged.

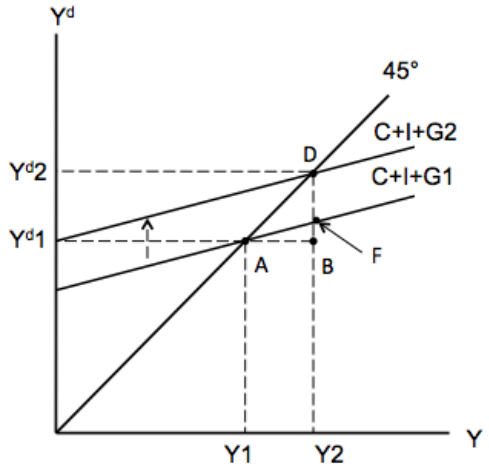
8.1.1 Keynesian (EE212) analysis:

- A higher G causes the demand for goods to increase.
  - Output and income increases.
  - Part of the increase in income is spent on consumption goods — more demand for output.
  - Direct and indirect increases in the demand for output — **the multiplier effect.**
- The Keynesian Y^d multiplier

$$\begin{aligned} \Delta Y^d &= \Delta G \\ \Delta C &= MPC \times \Delta Y^d \quad \text{,where } 0 < MPC < 1 \\ \Delta Y^d &= \Delta G + \left(MPC \times \Delta Y^d\right) \\ \Delta Y^d &= \frac{1}{1 - MPC}\Delta G \end{aligned}$$

- The larger is MPC, the larger the Yd multiplier, and the more powerful ΔG!
- The Keynesian Yd multiplier > 1

- Assume constant MPC.
- ΔG=DF
- $\Delta Y = \Delta Y^d = AB = DB.$
- But DB > DF
- $\frac{\Delta Y}{\Delta G} > 1$



- Keynesian assumptions
  - The increase in G has no negative effect on lifetime wealth and consumption spending.
    - But PV of taxes must rise and lifetime wealth falls.
  - Total income or output (Y) increases by the same amount as the demand for goods (Y^d).
  - The effect on the real interest rate?
  - Increases in C and Y come as a free lunch!
- Demand multiplier = 1
  - The increase in total current demand for goods $\left(Y^d\right)$.
  - The increase in government spending (ΔG).
  - The multiplier effect = $MPC\Delta G$.
  - Lifetime wealth drops = PV of taxes = ΔG. ; so current consumption **falls** by $MPC\Delta G$.

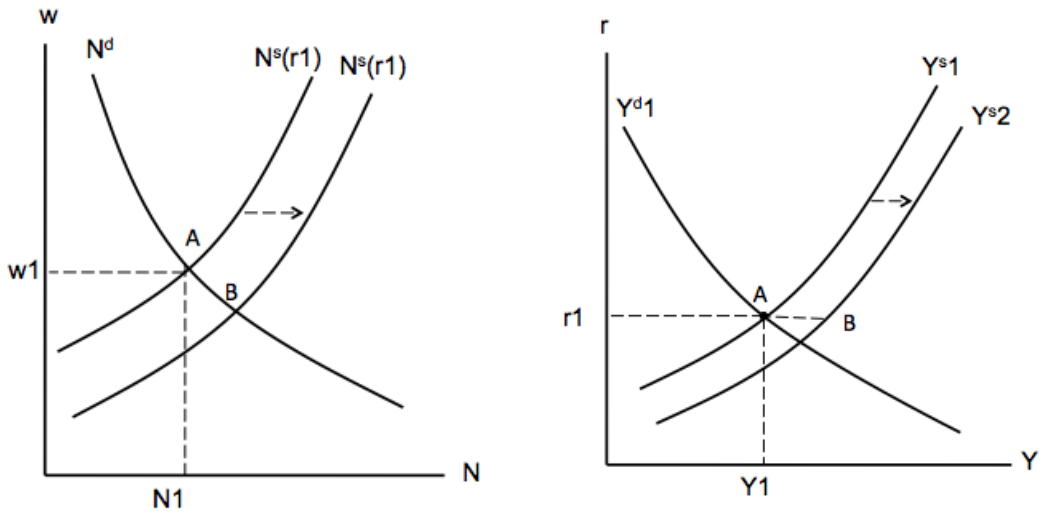
$$\begin{aligned} \Delta Y^d &= \Delta G + MPC\Delta G - MPC\Delta G \\ \Delta Y^d &= \Delta G \end{aligned}$$

8.1.2 Intertemporal model analysis

8.1.2.1 Step 1: direct effect of ΔG,

Effects on Y^s:

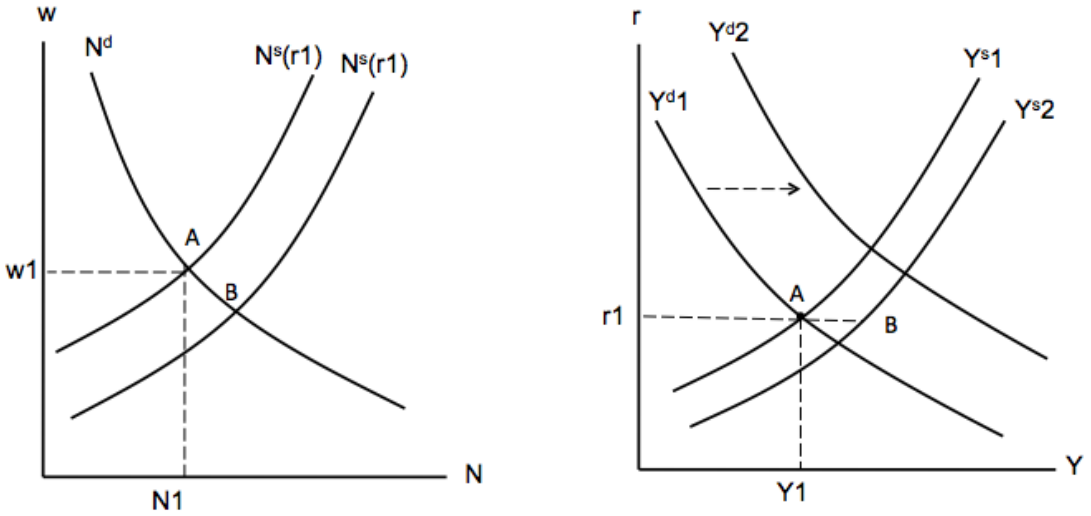
- The PV of taxes rises; the consumer's lifetime wealth falls.
- Leisure decreases and labor supply increases, given the real wage.
- The output supply curve shifts rightwards.



Effects on Y^d

- Government's demand for output (G) increases.
- Falling lifetime wealth reduces the consumer's demand for current consumption goods (C^d).
- Current demand for goods increases by the amount of $\Delta Y^d = \Delta G$; the Y^d multiplier = 1 .
- Y^d shifts rightwards by the amount of ΔG.

Both Y^s and Y^d shift to the right; what happens to the real interest rate?

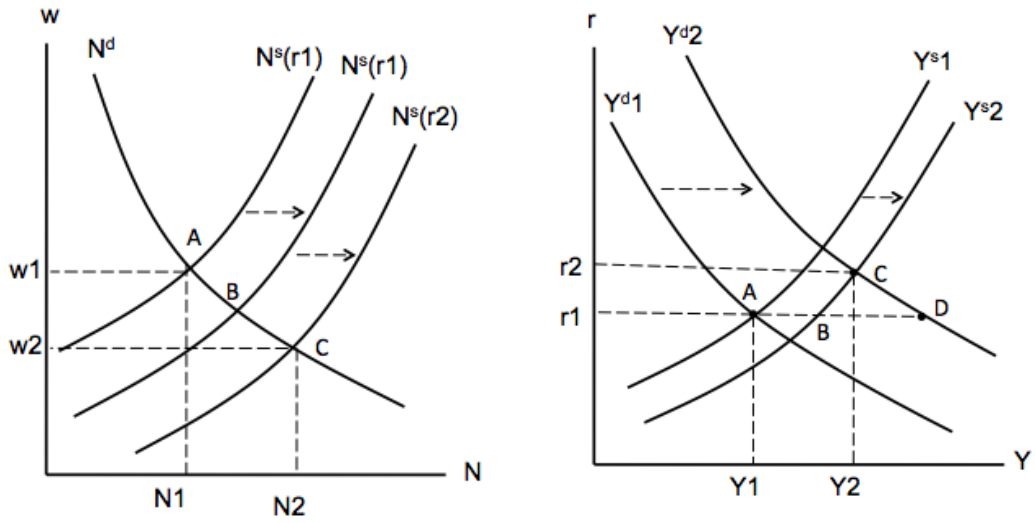


- The real interest rate increases as Y^d **shifts more than** Y^s.
  - ΔG is temporary and has a small negative effect on lifetime wealth.
  - A small decrease in leisure, and small increases in labor supply and output supply (small Y^s shift).
  - A small decrease in current consumption while the increase in G remains large (larger Y^d shift).

8.1.2.2 Step 2: effect of the rising r.

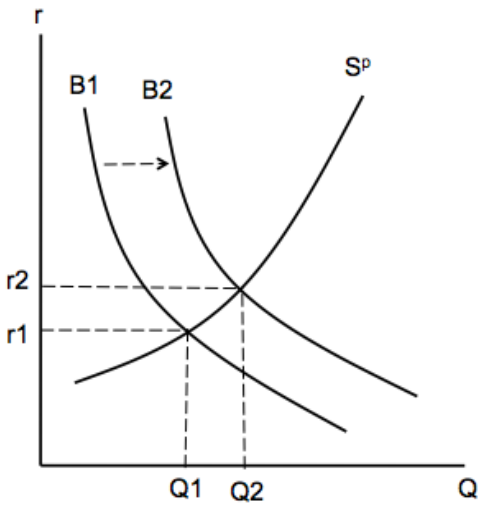
- A higher r reduces leisure, current consumption and investment.
- Leisure falls and labor supply increases again.
  - The real wage falls further; employment and output increase — a movement along the Ys.
- Investment decreases due to the higher real interest rate.
- Current consumption falls:
  - Falling lifetime wealth reduces current consumption while higher income raises it — small net effect.
  - The higher *r* also reduces it — dominant effect.

Step 2 An increase in G: rising r



Rising r and the credit market

- The government increases current borrowing (bond sale).
- The real interest rate increases.



8.1.2.3 Overall effect of an increase in a temporary government spending

- The government increases borrowing in the current period.
  - The increased bond sale raises the real interest rate.
- A temporary increase in G **crowds out** both current consumption and investment by raising the real interest rate.
  - The consumer works more for a lower real wage and consumes less.
  - Lower investment means lower future capital stock and future productive capacity.
- The total expenditure multiplier is less than 1.
  - $\Delta Y^d = \Delta G = AD > Y1Y2$.
  - So $\frac{\Delta Y}{\Delta G} < 1$.
  - The higher real interest rate results in the crowding-out effect on private spending (C^d and I^d).
- Higher government spending and larger output come at a cost — no free lunch!

8.2 A decrease in current capital stock

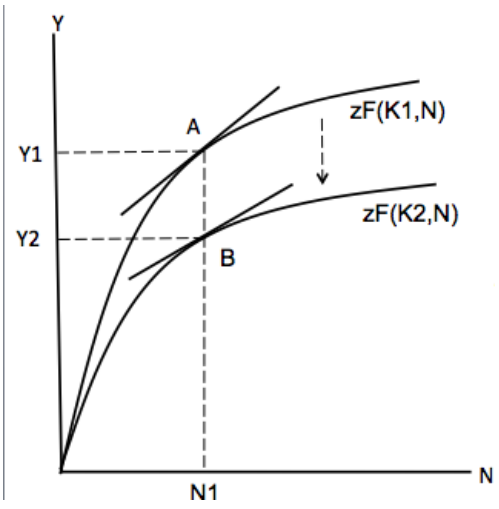
- Reduction in current capital stock (K) due to a natural disaster, war, etc.

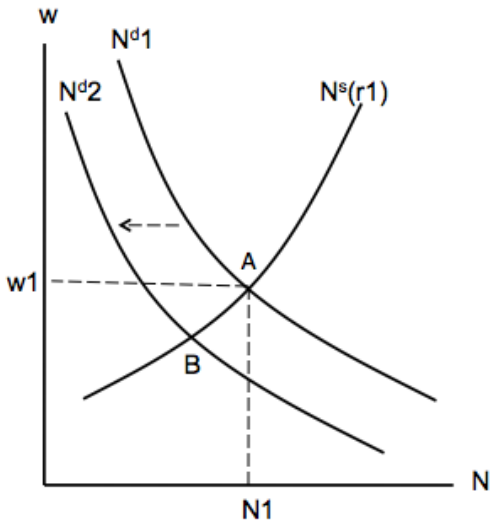
8.2.1 Step 1: Direct Effect

8.2.1.1 Effect on Y^s

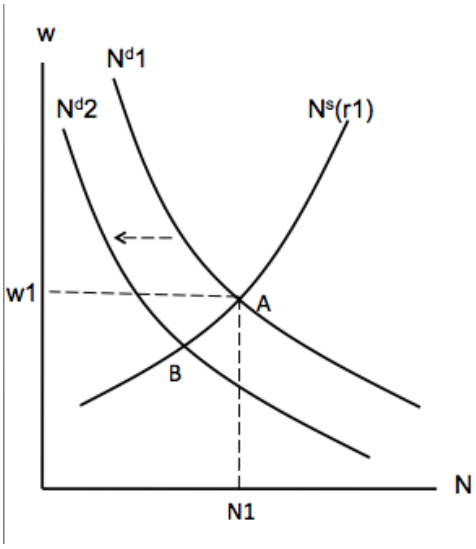
- A smaller K with the same N, current MP_N drops.
- The firm reduces its demand for labor.
- The labor demand curve shifts left (given w).
- the output supply curve (Y^s) shifts left.

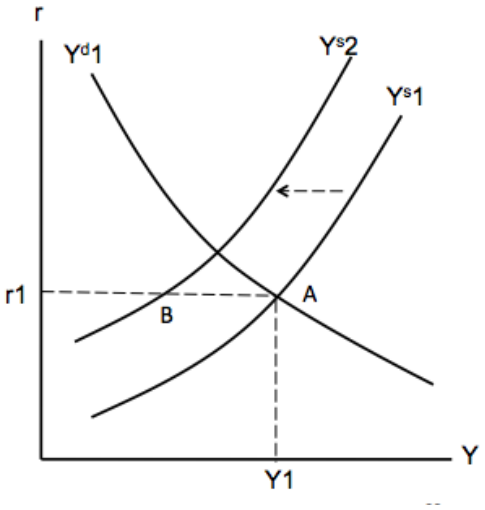
Step 1 A lower K reduces MPN





Step 1 Lower Nd: Ys shifts left

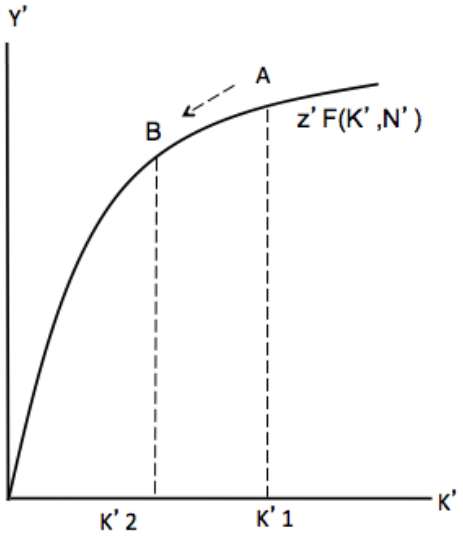


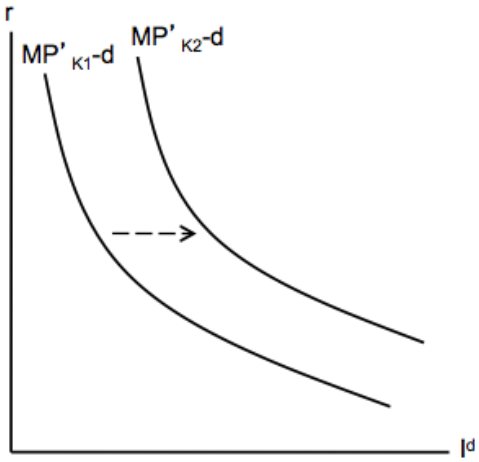


8.2.1.2 Effect on Y^d:

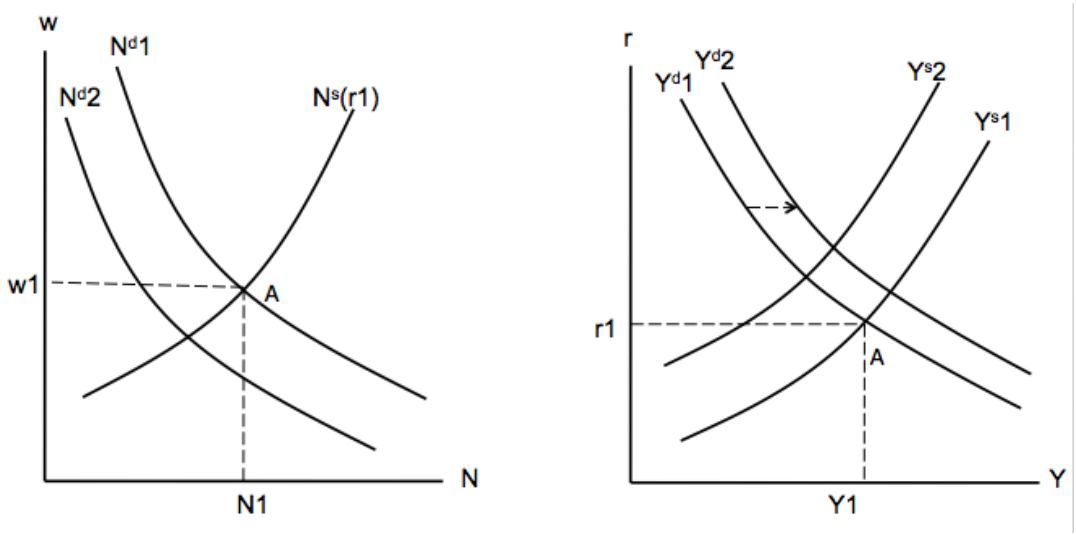
- A smaller current K means a smaller future K'.
- Future MP'_K rises; investment increases, given r.
- The optimal investment curve (I^d) shifts right.
- The output demand curve (Y^d) shifts right.
- The real interest rate must rise.

Step 1 : Higher MP'K and rising Id



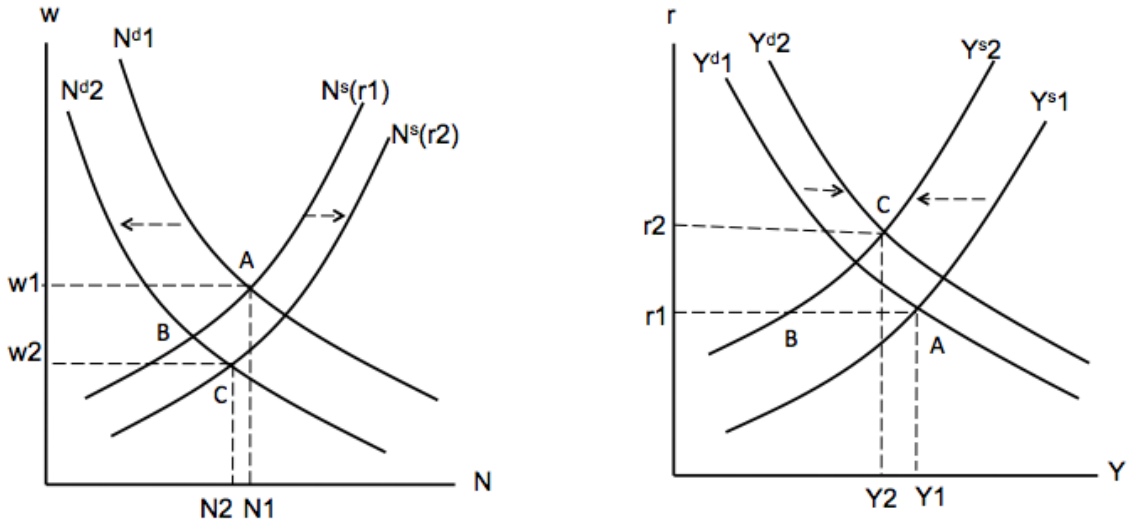


Step 1: A rising Id shifts Yd right.



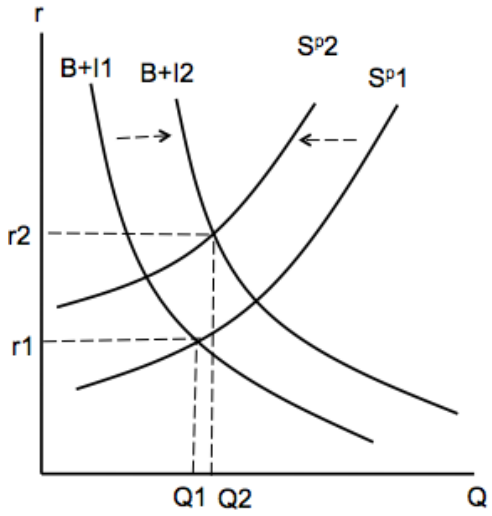
- Leisure decreases and labor supply increases.
  - The labor supply curve shifts to the right.
  - The real wage drops further.
  - A movement on the Ys curve.
- Investment increases to make up for the decline in the capital stock:
  - The higher real interest rate depresses investment, but higher MP'_K raise it.
  - If investment finally decreases, current K will be falling indefinitely — impossible.

8.2.2 Step 2 A decrease in current K: rising r



The credit market

- Lower current income reduces consumption and private saving.
- Investment increases (net effect).
- The real interest rate increases, given B.



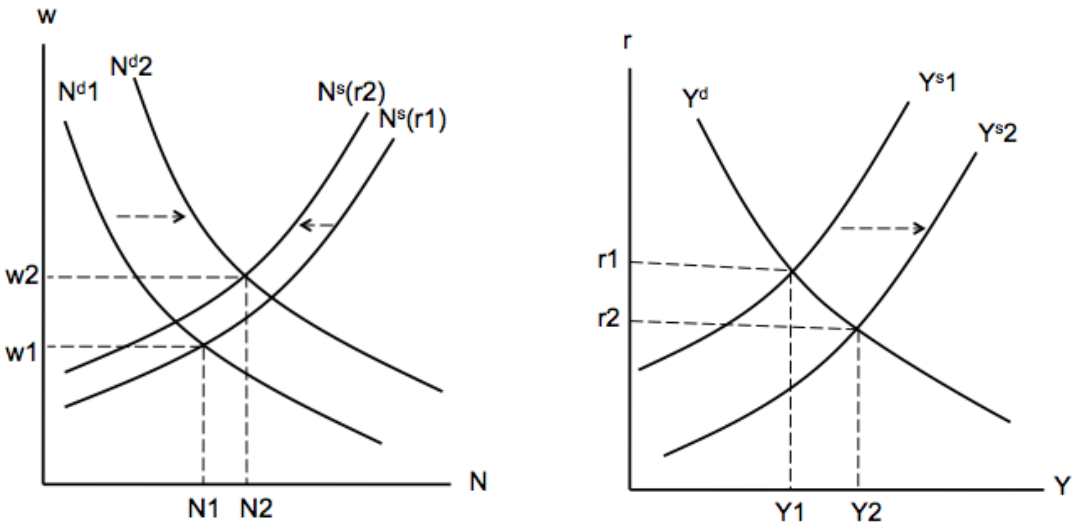
8.2.3 Overall effect of a drop in K

- A decrease in current K raises the real interest rate but may increase or reduce output.
  - Current consumption and leisure decrease.
  - Investment increases.
  - The real wage decreases.
  - Employment and output may increase or decrease.
- Destruction of K tends to reduce output; but higher investment increases output.

8.3 A temporary increase in z

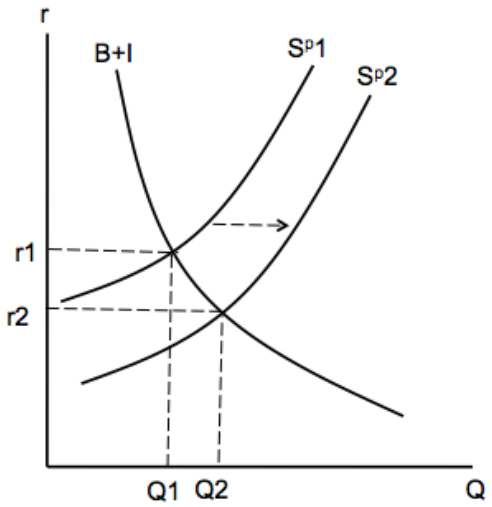
- Step 1: An increase in current total factor productivity (z) raises MPN.
  - Labor demand and output supply shift right.
  - The real interest rate decreases.
- Step 2: the lower r raises current consumption, investment and leisure.
  - Labor supply decreases; the labor supply curve shifts left.
  - Employment, output and the real wage increase.

An increase in z



The credit market

- Higher current income raises both consumption and private saving.
- The real interest rate decreases, given B.



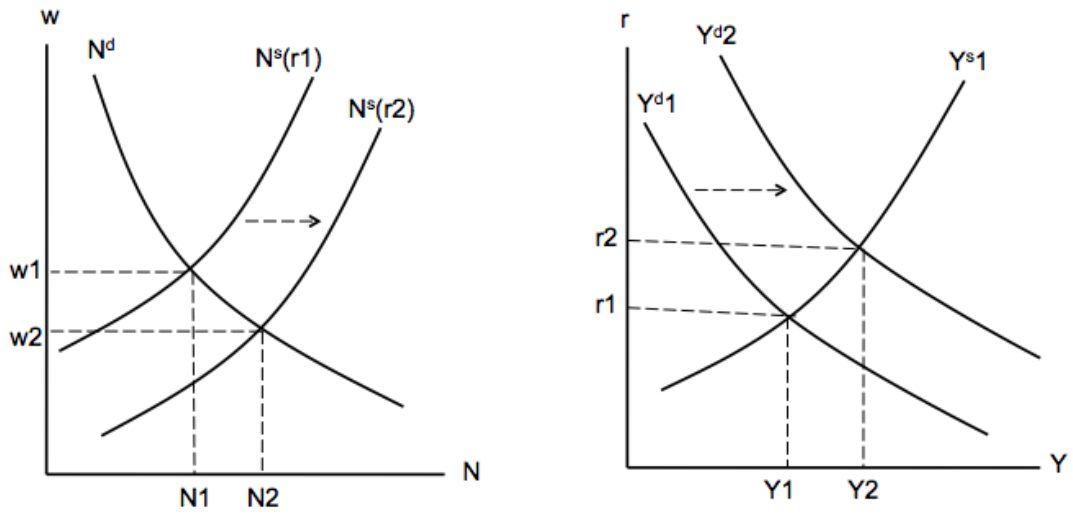
Overall effect of Δ z

- An increase in the current z reduces the real interest rate but increases output.
- Employment and output increase.
- The real wage increases.
- This is partly offset by the increase in leisure (with lower r and higher current income).
- Investment increases (with lower r).
- Current consumption increases with lower r and larger Y.

8.4 An increase in future z'

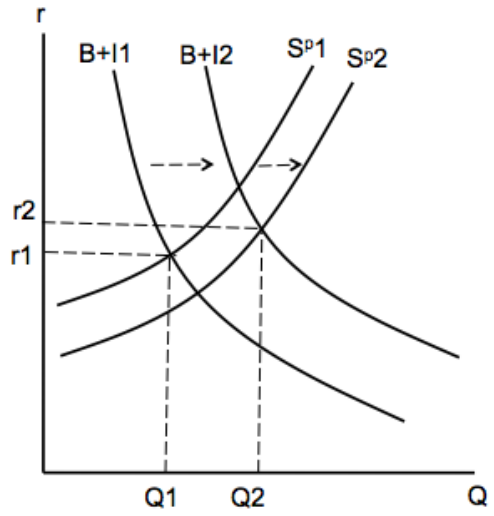
- Step 1: Future z' is expected to rise; future MP'_K increases.
- Investment (I^d) increases; output demand shifts right.
  - The real interest rate increases.
- Step 2: the higher r reduces consumption, investment and leisure.
  - The labor supply curve shifts right; the real wage drops.
  - Employment and output increase.
  - Higher current income raises consumption, but the higher real interest rate depresses it.
  - The higher investment from higher MP'_K is partially offset by the higher real interest rate.
  - Investment increases as the effect of MP'_K is stronger .

An expected increase in z'



The credit market

- Higher current income raises both consumption and private saving.
- Investment also increases.
- The real interest rate increases.



Overall effect of $\Delta z'$

- Investment increases with higher expected MP'_K, partly offset by the higher r.
  - A larger future capital stock due to higher expected z'.
- Both real interest rate and output increase.
- Current consumption may rise or fall due to higher income but higher real interest rate.
- Employment increases with falling real wage.