

Bond Pricing
&
Other Applications of PV

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

A Sketch of the Bond

Face value: \$1000
Coupon Rate: 10% per year
Maturity Date: 2028

- We cannot change anything specified on the bond.
- However, market price of the bond can be changed.

Cash flows from the bond

	Year	Payments	PV of the Payments
0	2018	0	
1	2019	100	$100/(1+r)$
2	2020	100	$100/(1+r)^2$
3	2021	100	$100/(1+r)^3$
4	2022	100	$100/(1+r)^4$
5	2023	100	$100/(1+r)^5$
6	2024	100	$100/(1+r)^6$
7	2025	100	$100/(1+r)^7$
8	2026	100	$100/(1+r)^8$
9	2027	100	$100/(1+r)^9$
10	2028	1,000+100	$1,100/(1+r)^{10}$

Calculation

- Suppose the market price is at \$1,200.
- We want to calculate r such that:

$$1,200 = \frac{100}{(1+r)} + \frac{100}{(1+r)^2} + \frac{100}{(1+r)^3} + \frac{100}{(1+r)^4} + \frac{100}{(1+r)^5} + \frac{100}{(1+r)^6} \\ + \frac{100}{(1+r)^7} + \frac{100}{(1+r)^8} + \frac{100}{(1+r)^9} + \frac{1,100}{(1+r)^{10}}$$

- It is not possible to solve this equation with pencil and paper!

Calculation

- We know that if r goes up, the value of each term goes down.
- We also know that:

$$1,000 = \frac{100}{(1 + 0.1)} + \frac{100}{(1 + 0.1)^2} + \frac{100}{(1 + 0.1)^3} + \frac{100}{(1 + 0.1)^4} + \frac{100}{(1 + 0.1)^5} + \frac{100}{(1 + 0.1)^6} \\ + \frac{100}{(1 + 0.1)^7} + \frac{100}{(1 + 0.1)^8} + \frac{100}{(1 + 0.1)^9} + \frac{1,100}{(1 + 0.1)^{10}}$$

- Then, we can conclude that r must be smaller than 10%.
- We can try the process trial and error.
- Or we can switch to some computer programs.

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year* (\$)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

A Sketch of the Bond

Face value: \$1000
Coupon Rate: 10% per year
Maturity Date: 2028

- We cannot change anything specified on the bond.
- Suppose we hold the bond and receive one interest payment.
- What is the price of this bond after that?

Cash flows from the bond

	Year	Payments	PV of the Payments
0	2019	0	
1	2020	100	$100/(1+r)^1$
2	2021	100	$100/(1+r)^2$
3	2022	100	$100/(1+r)^3$
4	2023	100	$100/(1+r)^4$
5	2024	100	$100/(1+r)^5$
6	2025	100	$100/(1+r)^6$
7	2026	100	$100/(1+r)^7$
8	2027	100	$100/(1+r)^8$
9	2028	1,000+100	$1,100/(1+r)^9$

Calculation

- Suppose the market interest rate is 20%.
- This means people can find alternative investment with 20% rate of return.
- Here, r must be 20% to induce people to buy this bond:

$$P = \frac{100}{(1+0.2)} + \frac{100}{(1+0.2)^2} + \frac{100}{(1+0.2)^3} + \frac{100}{(1+0.2)^4} + \frac{100}{(1+0.2)^5} + \frac{100}{(1+0.2)^6} \\ + \frac{100}{(1+0.2)^7} + \frac{100}{(1+0.2)^8} + \frac{1,100}{(1+0.2)^9}$$

$$P = 596.90$$

Retirement Plan

- I plan to retire at the age of 60.
- Suppose I plan to live to the age of 80.
- I plan to have my expense 20,000 baht/month (240,000 baht/year)
- The market interest rate is 3%.
- How much should I have at the age of 60?

Required cash flows

Year		Payments	PV of the Payments
0	Age = 60	0	
1	61	240,000	$240,000/(1+r)^1$
2	62	240,000	$240,000/(1+r)^2$
3	63	240,000	$240,000/(1+r)^3$
4	64	240,000	$240,000/(1+r)^4$
5	65	240,000	$240,000/(1+r)^5$
6	66	240,000	$240,000/(1+r)^6$
7	67	240,000	$240,000/(1+r)^7$
...
20	80	240,000	$240,000/(1+r)^{20}$

Calculation

- It turns out that I need a saving of 3.6 million baht at the age of 60!
- This is the calculation without the effect of inflation.

- The average inflation rate of Thailand for the last 20 years is 2%.
- With my expenses grow at the rate 2% per year, I need 4.3 million baht at the age of 60!