Topic 6

Cost Concepts for Decision Making (Chapter 7)

Explicit Costs and Implicit Costs

Explicit Costs:

Costs that involve a direct monetary outlay.

Implicit Costs:

Costs that do not involve outlays of cash.

Example: an airline company

Explicit Costs: expenditures on fuel and salaries

Implicit Costs: forgone income from leasing its jets

Economic Costs and Accounting Costs

The relevant concept of cost is **opportunity cost**: the value of the next best forgone alternative.

Economic Costs: Sum of a firm's explicit costs and implicit Costs, which is the same as "**opportunity cost**"

Accounting Costs: Total of a firm's explicit costs

Opportunity Cost – Example

Suppose you own a business:
The business pays $100 to hire workers and $80 to purchase supplies. You work X hrs/year.

Next Best Alternative:
You work X hrs in a company which pays $75 per year.

Opportunity Cost of continuing your own business

= Explicit Costs + Implicit Costs

= 180 + 75 = $255

Sunk Costs

Sunk Costs are costs that have been incurred and cannot be recovered. These costs are not part of opportunity costs.

Non-Sunk Costs are costs that are incurred only if a particular decision is made.

Example: Bowling Ball Factory

- It costs $5M to build and has no alternative uses
- $5M is not sunk cost for the decision of whether or not to build the factory
- $5M is sunk cost for the decision of whether to operate or shut down the factory

Cost Minimization

Cost minimization problem: Finding the input combination that minimizes a firm's total cost of producing a particular level of output.

Cost minimization firm: A firm that seeks to minimize the cost of producing a given amount of output.

Long run (LR): A period of time when the quantities of all of the firm's input can vary.

Short run (SR): A period of time when at least one of its inputs' quantities is fixed.

Long-Run Cost Minimization

In SR, a firm chooses ONE input, L, with K being fixed.
In LR, a firm chooses TWO inputs, K and L, to minimize its costs, subject to the firm producing a given output.

Cost to the Firm: $TC = wL + rK$

TC:	Total Cost
w:	Price of Labor (wage rate)
L:	Quantity of Labor
r:	Price of Capital (interest rate)
K:	Quantity of Capital

Isocost Lines

Isocost Line: The set of combinations of labor and capital that yield the same total cost for the firm.

Isocost Equation:

$TC = wL + rK$

$rK = TC – wL$

$K = TC/r – (w/r)L$

Isocost Slope

$= – (w/r)$

Example

w = \$10/hour

r = \$20/hour

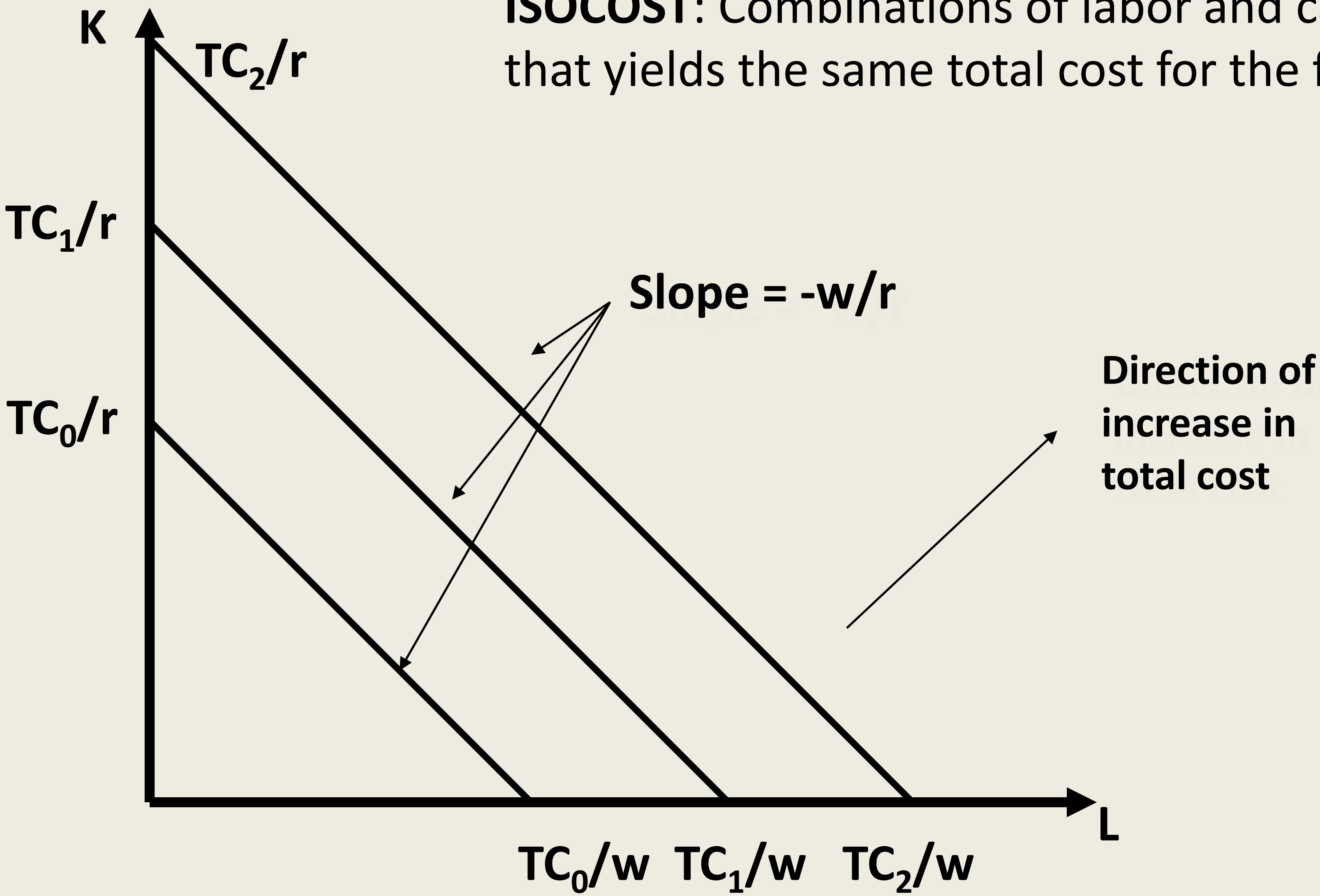
TC = \$1M

$\Rightarrow$ 1M = 10L + 20K

$\Rightarrow$ K = 1 mil/20 – (10/20)L

Isocost Lines

ISOCOST: Combinations of labor and capital that yields the same total cost for the firm.



Long-Run Cost Minimization

Suppose that a firm wishes to minimize costs, subject to a given output. Let such output be Q_0.

Production Function: $Q = f(L,K)$

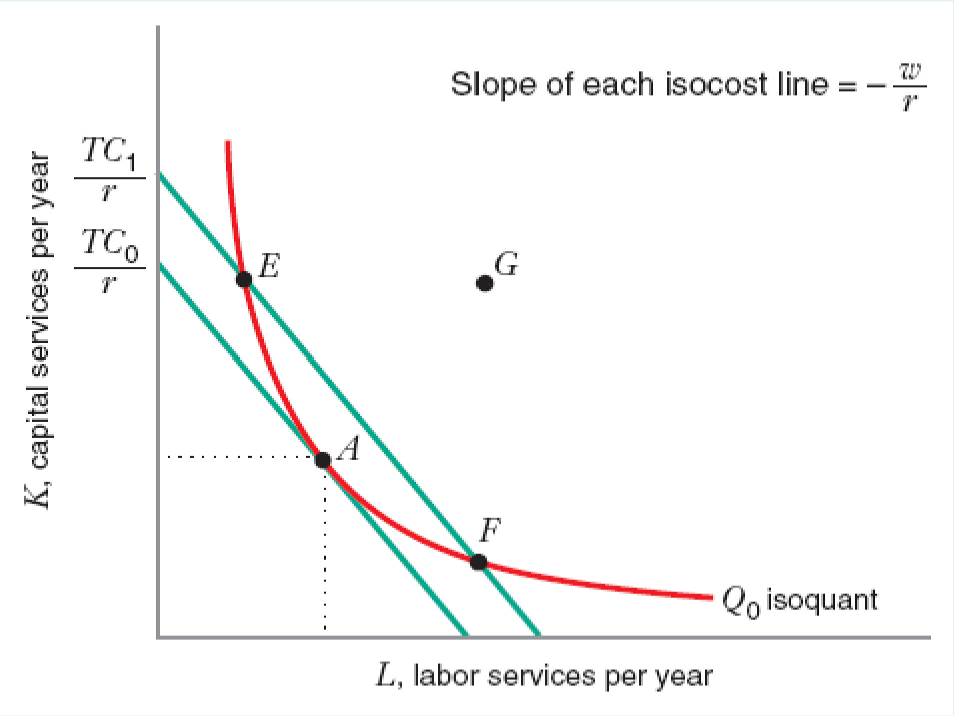
Cost Minimization Problem:

$$\min_{(K,L)} \; TC = rK + wL \quad \text{subject to } Q_0 = f(L,K)$$

That is, we are looking for K and L on the lowest isocost, such that Q_0 can be produced.

Long-Run Cost Minimization

Cost Minimization: looking for K and L on the lowest isocost, such that Q_0 can be produced.

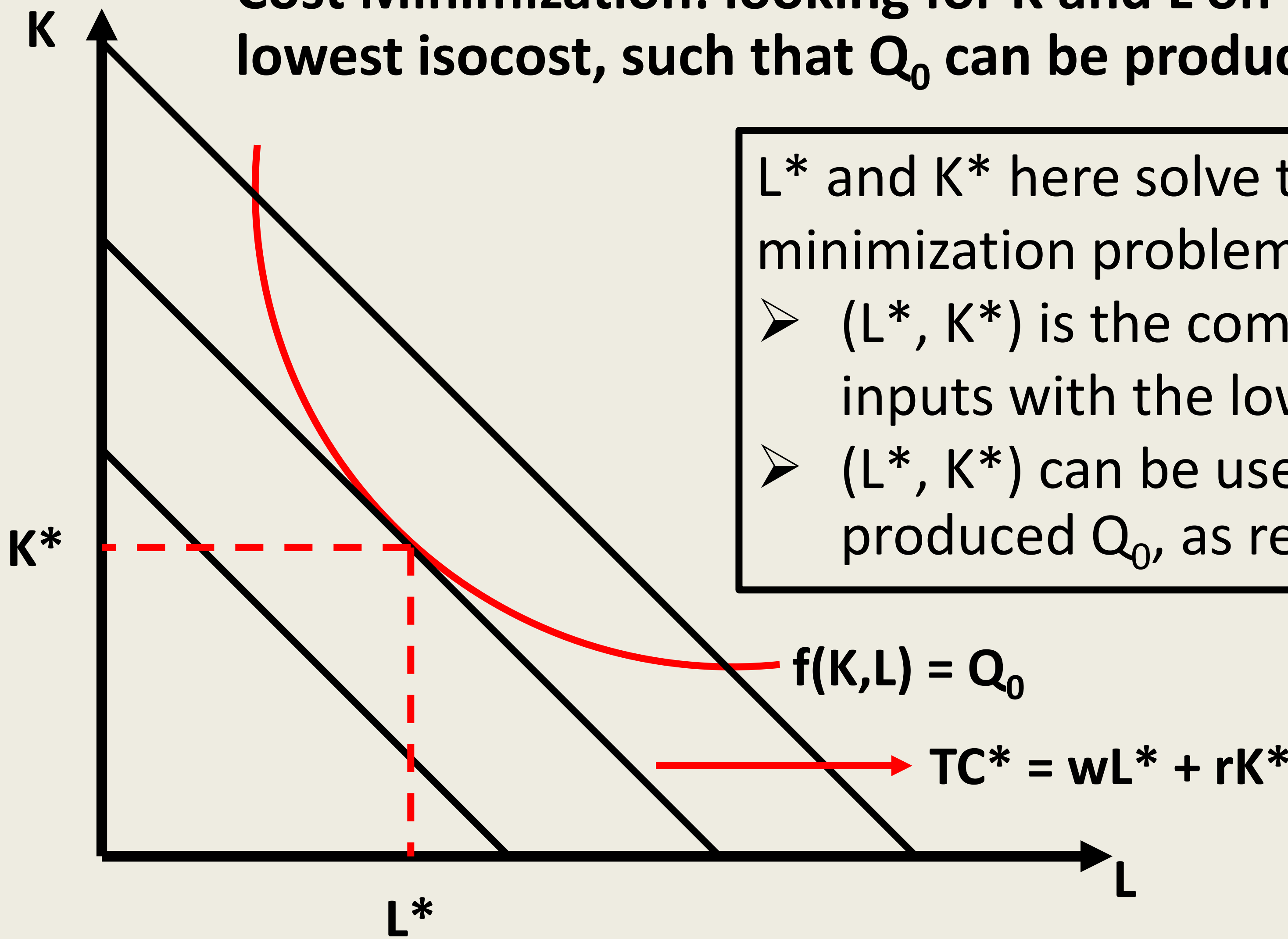


G is Technically Inefficient because less inputs can be used to produce Q_0.

E & F are Technically Efficient because they can be used to produce Q_0; however, they do not minimize cost.

Long-Run Cost Minimization

Cost Minimization: looking for K and L on the lowest isocost, such that Q_0 can be produced.



L* and K* here solve the cost minimization problem.

- (L*, K*) is the combination of inputs with the lowest cost.
- (L*, K*) can be used to produced Q_0, as required.

Long-Run Cost Minimization

Solution to Cost Minimization

- Slope of isoquant = Slope of isocost line

i.e. Ratio of marginal products = Ratio of input prices

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

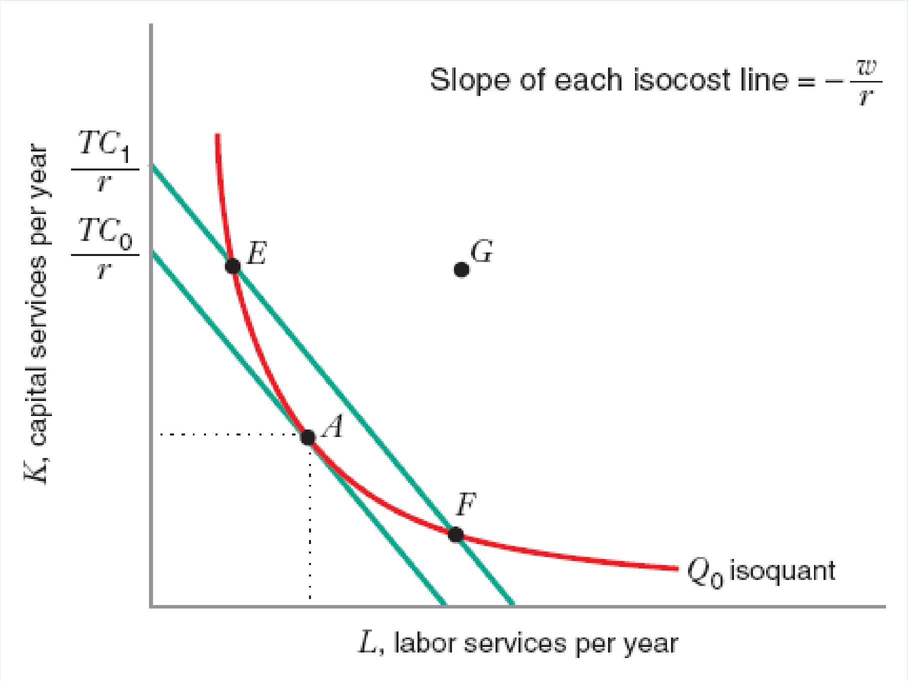
$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

- This means **“one $ spent on K gives the same number of marginal products as one $ spent on L”**.

Long-Run Cost Minimization

- At point E, Isoquant is **steeper** than Isocost:

$$\frac{MP_L}{MP_K} > \frac{w}{r} \; (or) \; \frac{MP_L}{w} > \frac{MP_K}{r}$$



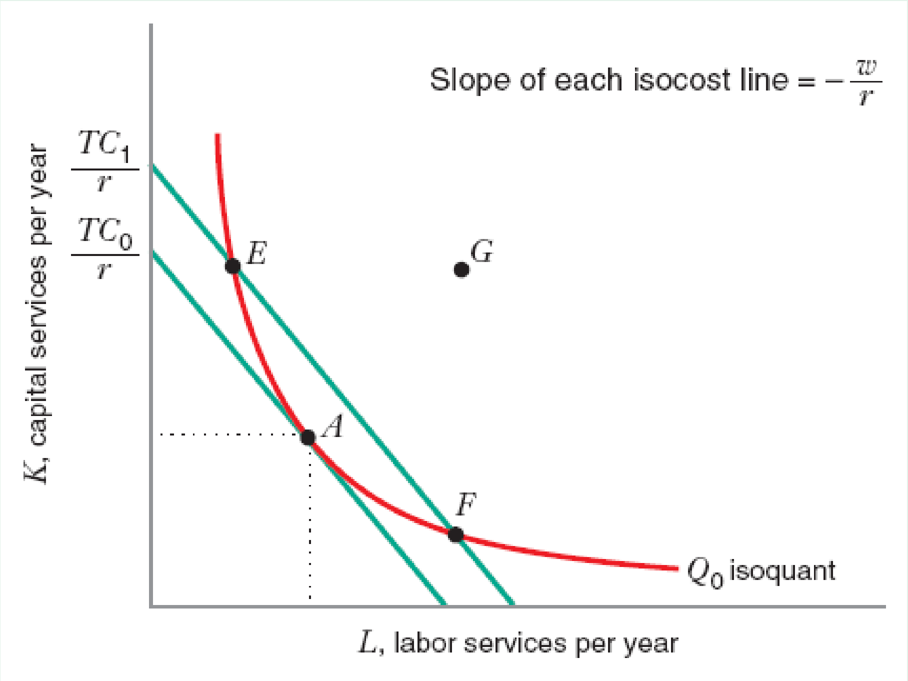
One $ on L gives greater output than one $ on K.

The firm can save more by employing **more L and less K.**

Long-Run Cost Minimization

- At point F, Isoquant is **flatter** than Isocost:

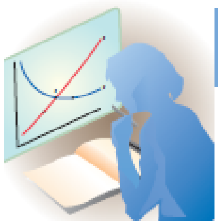
$$\frac{MP_L}{MP_K} < \frac{w}{r} \quad (or) \frac{MP_L}{w} < \frac{MP_K}{r}$$



One $ on K gives greater output than one $ on L.

The firm can save more by employing **more K and less L.**

Interior Solution



LEARNING-BY-DOING EXERCISE 7.2

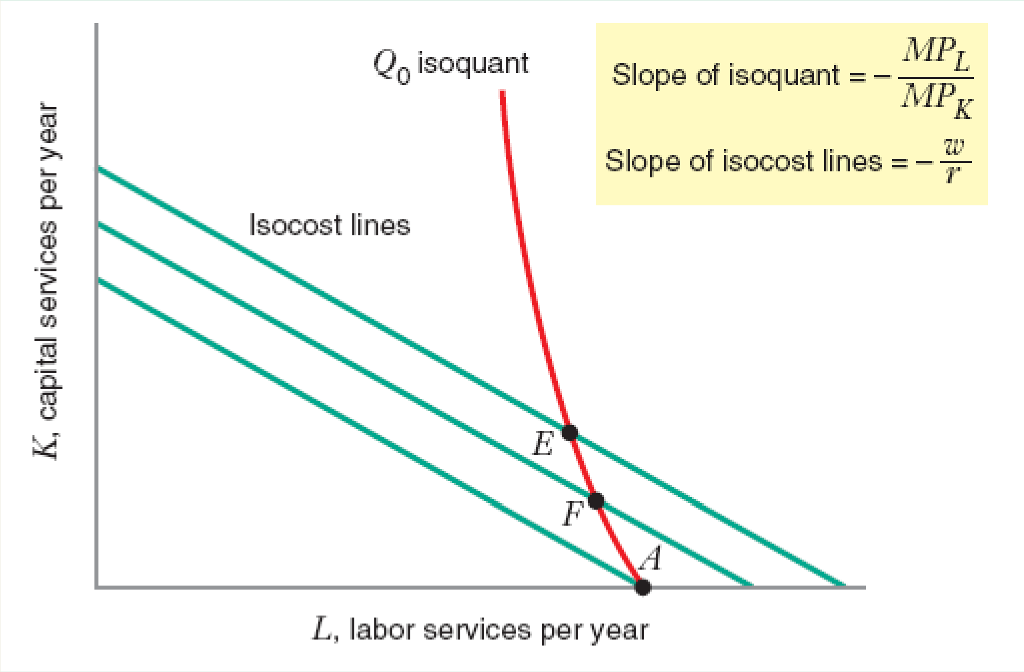
Finding an Interior Cost-Minimization Optimum

Problem The optimal input combination satisfies equation (7.1) [or, equivalently, equation (7.2)]. But how would you calculate it? To see how, let's consider a specific example. Suppose that the firm's production function is of the form $Q = 50\sqrt{LK}$. For this production function, the equations of the marginal products of labor and capital are $MP_L = 25\sqrt{K/L}$ and $MP_K = 25\sqrt{L/K}$. Suppose, too, that the price of labor w is $5 per unit and the price of capital r is $20 per unit. What is the cost-minimizing input combination if the firm wants to produce 1,000 units per year?

Corner Solution

In some cases, using one input is cheaper than using both inputs, and doing so can still produce the required output.

In such cases, we will have a corner solution.



$$-\left(\frac{MP_L}{MP_K}\right) < -\left(\frac{w}{r}\right)$$

$$\Rightarrow \quad \frac{MP_L}{w} > \frac{MP_K}{r}$$

Corner Solution

LEARNING-BY-DOING EXERCISE 7.3

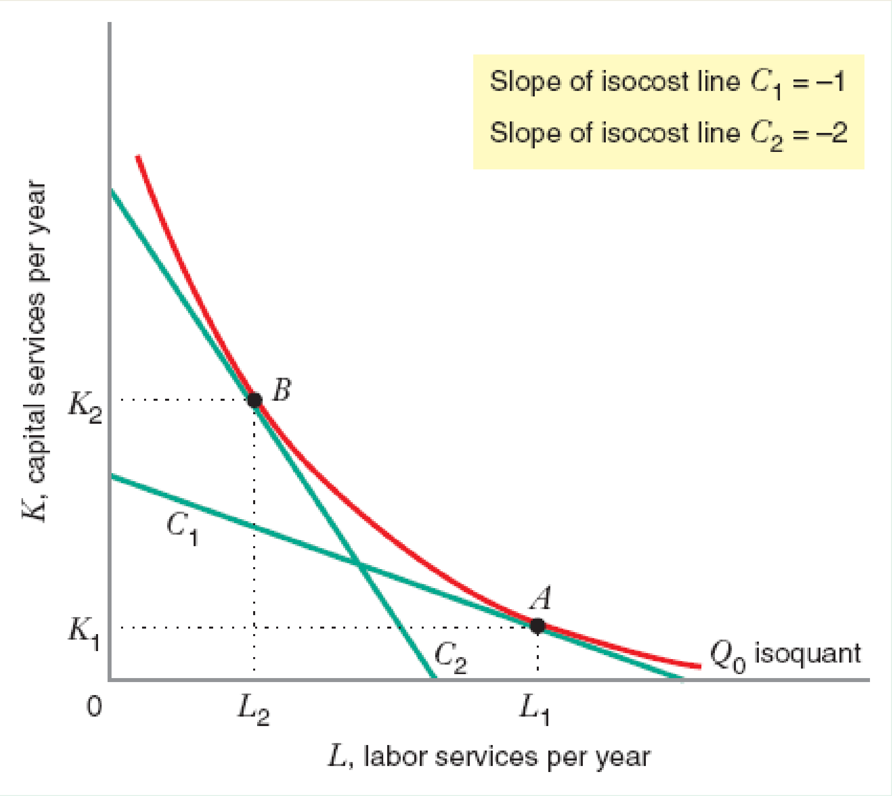
Finding a Corner Point Solution with Perfect Substitutes

Problem In Chapter 6 we saw that a linear production function implies that the inputs are perfect substitutes. Suppose that we have the linear production function $Q = 10L + 2K$. For this production function $MP_L = 10$ and $MP_K = 2$. Suppose, too, that the price of labor w is \$5 per unit and that the price of capital services r is \$2 per unit. Find the optimal input combination given that the firm wishes to produce 200 units of output.

ANALYSIS: Change in Relative Prices of Inputs

1. **A change in the relative price of inputs changes the slope of the isocost line.**
2. Given a diminishing $MRTS_{L,K}$, an increase in the price of one input will cause the cost-minimizing quantity of that input to go down.
3. Given that both inputs are perfect complements, an increase in the price of one input will not change the cost-minimizing quantity of that input.

ANALYSIS: Change in Relative Prices of Inputs



- Price of capital r = 1.
- Quantity of output Q_0 is constant.
- When price of labor w = 1, the isocost line is C1. >>> optimal point A.
- When price of labor w = 2, isocost line is C2. >>> optimal point B.

ANALYSIS: Change in Relative Prices of Inputs

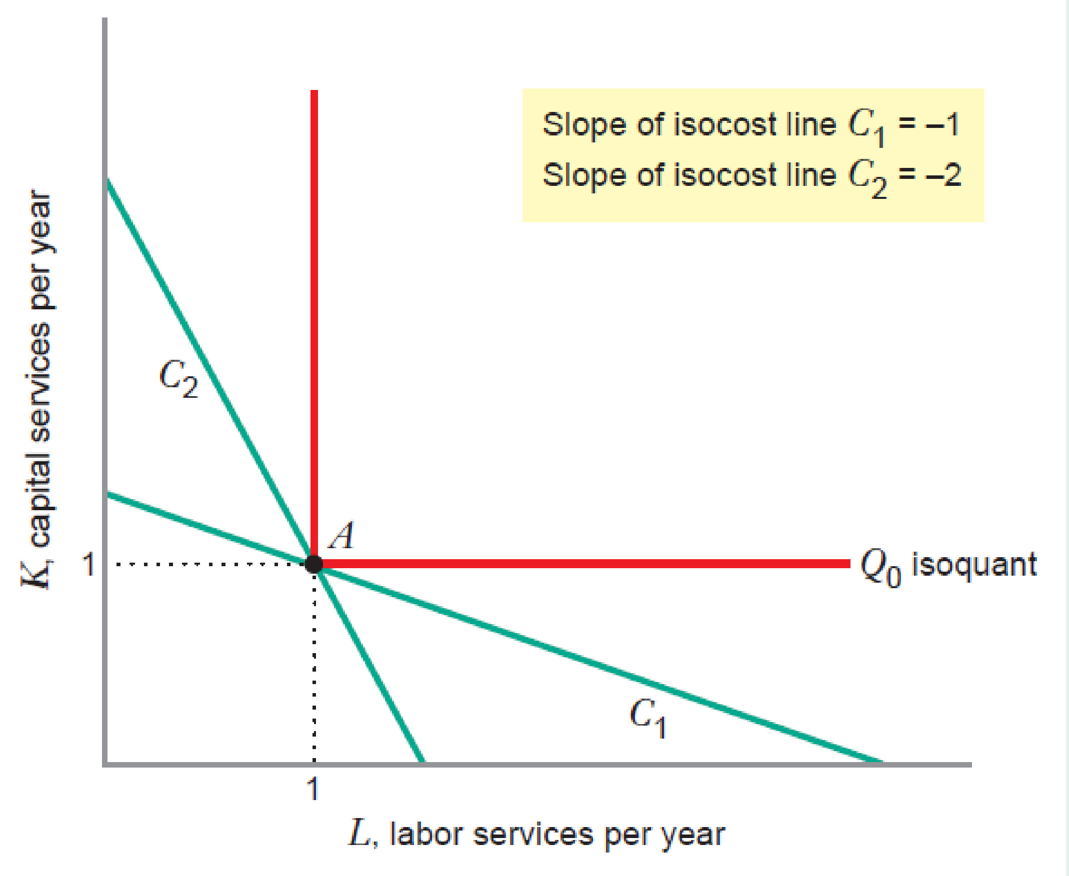


FIGURE 7.6 Comparative Statics Analysis of the Cost-Minimization Problem with Respect to the Price of Labor for a Fixed-Proportions Production Function
The price of capital $r = 1$, and the quantity of output Q_0 are held constant. When the price of labor $w = 1$, the isocost line is C_1 and the ideal input combination is at point A ($L = 1$, $K = 1$). When the price of labor $w = 2$, the isocost line is C_2 and the ideal input combination is still at point A. Increasing the price of labor does not cause the firm to substitute capital for labor.

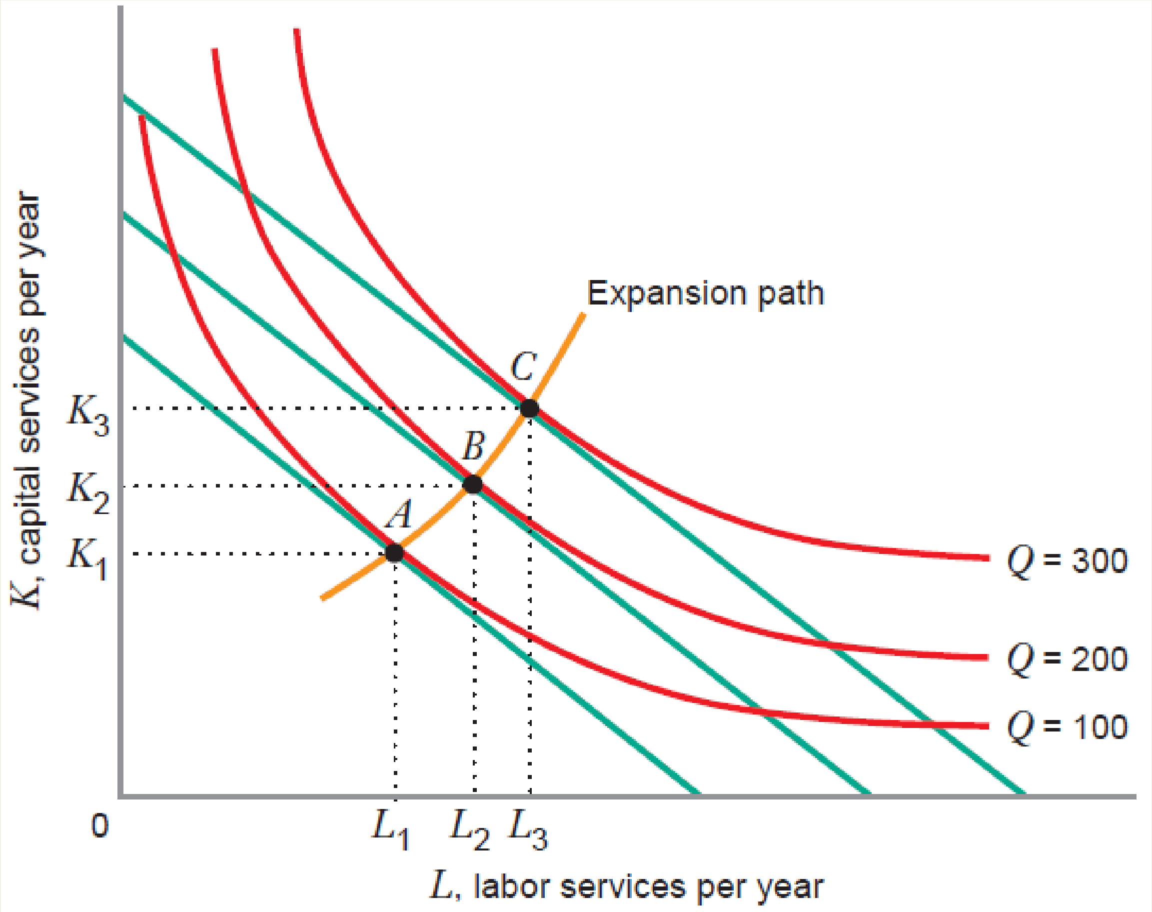
ANALYSIS: Change in Output

An increase in Q_0 moves the isoquant Northeast.

- **Expansion Path:** A line that connects the cost-minimizing input combinations as the quantity of output, Q, varies, holding input prices constant.
- **Normal Inputs:** An input whose cost-minimizing quantity increases as the firm produces more output.
- **Inferior Input:** An input whose cost-minimizing quantity decreases as the firm produces more output.

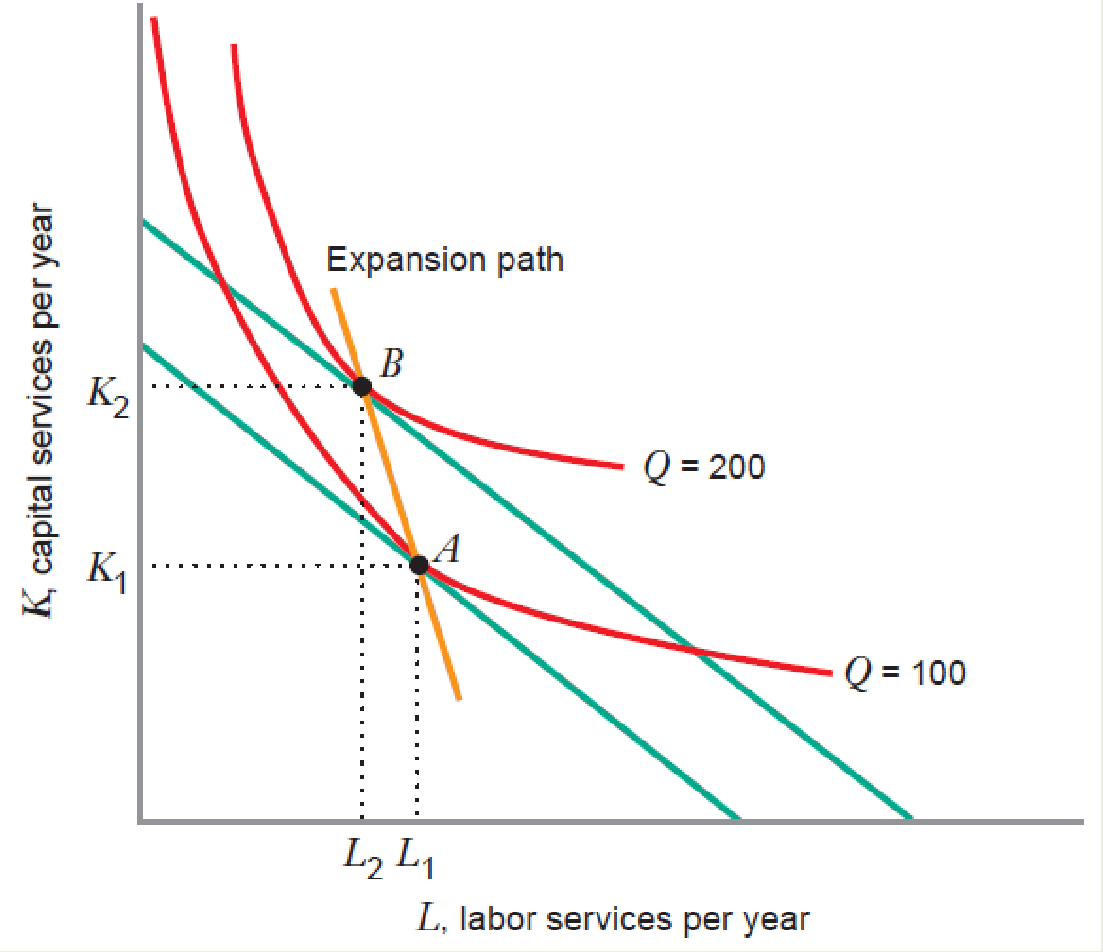
ANALYSIS: Change in Output

As output increases, the cost minimization path moves from point A to B to C when **both inputs are normal**.



ANALYSIS: Change in Output

As output increases, the cost minimization path moves from point A to B when **labor is an inferior input**.



Input Demand

Definition: A function that shows how the firm's cost-minimizing quantity of input varies with the price of that input.

Labor demand curve: Shows how the firm's cost-minimizing quantity of labor varies with the price of labor.

Capital demand curve: Shows how the firm's cost-minimizing quantity of capital varies with the price of capital.

Input Demand

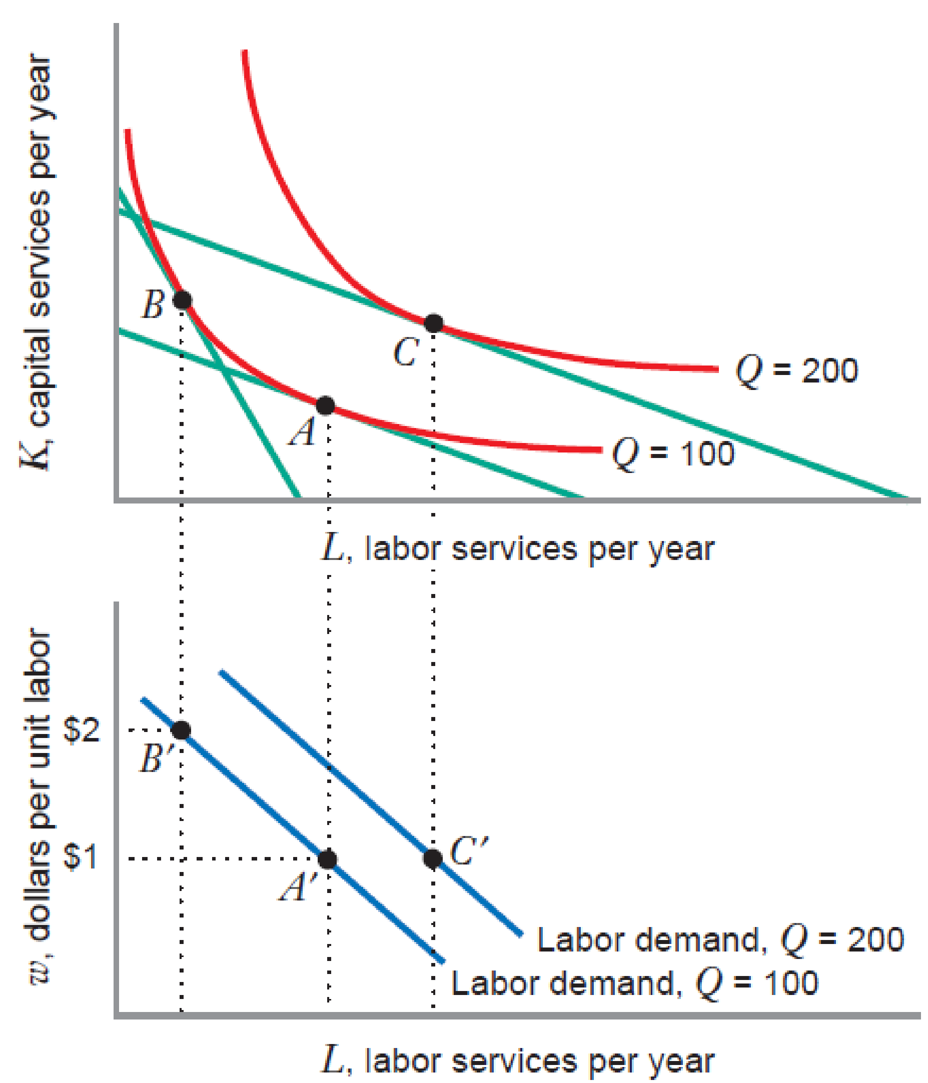
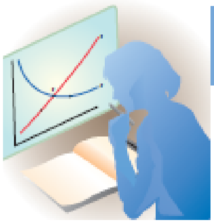


FIGURE 7.10

Comparative Statics Analysis and the Labor Demand Curve
The labor demand curve shows how the firm's cost-minimizing amount of labor varies as the price of labor varies. For a fixed output of 100 units, an increase in the price of labor from $1 to $2 per unit moves the firm along its labor demand curve from point *A′* to point *B′*. Holding the price of labor fixed at $1 per unit, an increase in output from 100 to 200 units per year shifts the labor demand curve rightward and moves the firm from point *A′* to point *C′*.

Input Demand



LEARNING-BY-DOING EXERCISE 7.4

Deriving the Input Demand Curves from a Production Function

Problem Suppose that a firm faces the production function $Q = 50\sqrt{LK}$. What are the demand curves for labor and capital?

Price Elasticity of Demand for Inputs

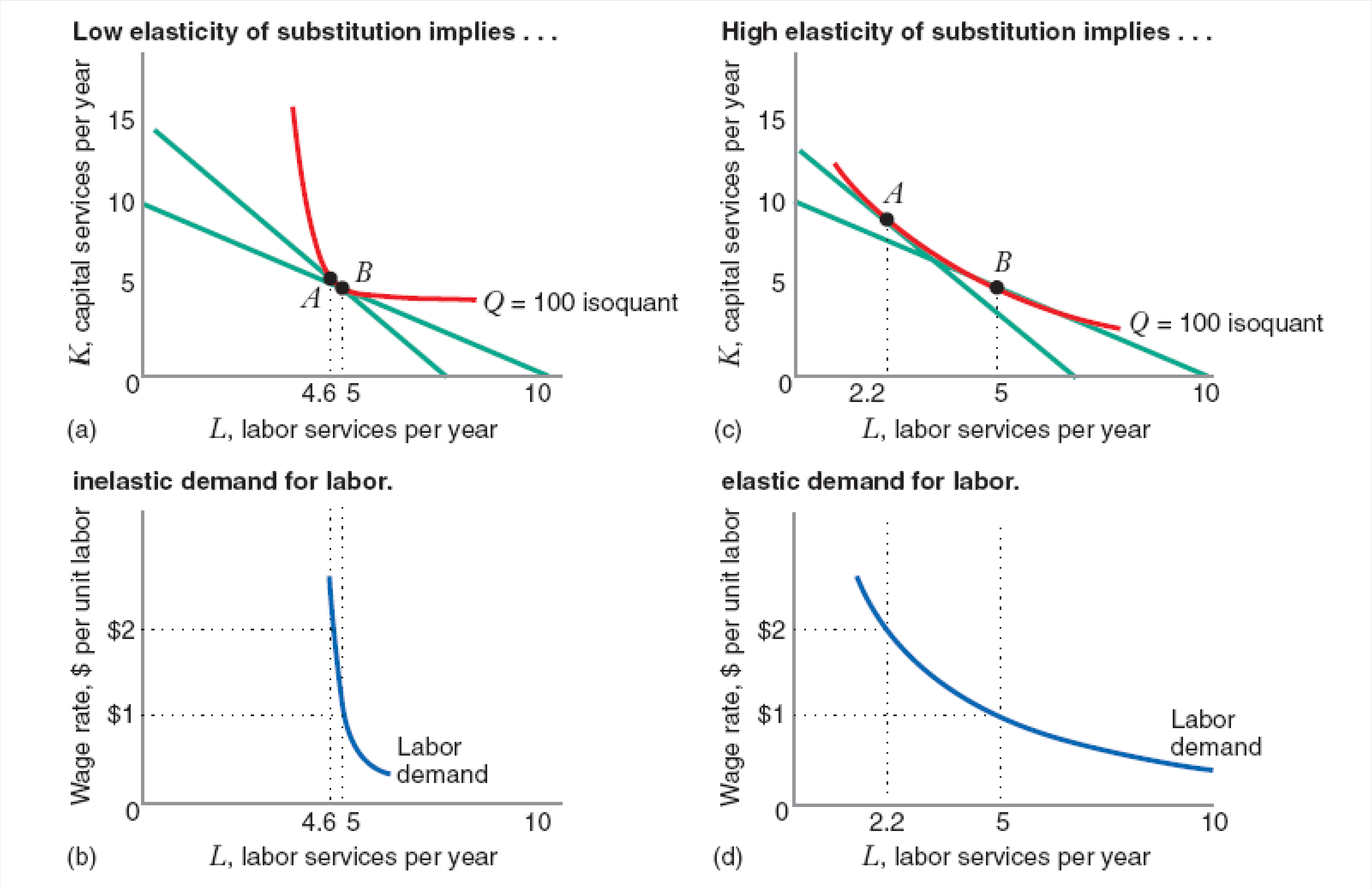
- Percentage change in the cost-minimizing quantity of labor with respect to a 1% change in the price of labor.

$$\varepsilon_{L,w} = \frac{\Delta L}{\Delta w}\frac{w}{L}$$

- Percentage change in the cost-minimizing quantity of capital with respect to a 1% change in the price of capital.

$$\varepsilon_{K,r} = \frac{\Delta K}{\Delta r}\frac{r}{K}$$

Price Elasticity of Demand for Inputs



Short-Run Cost Minimization

Total Variable Costs – the sum of total expenditures on variable inputs, such as labor and materials, at the short-run cost-minimizing input combination

Total Fixed Costs – the cost of fixed inputs; it does not vary with output

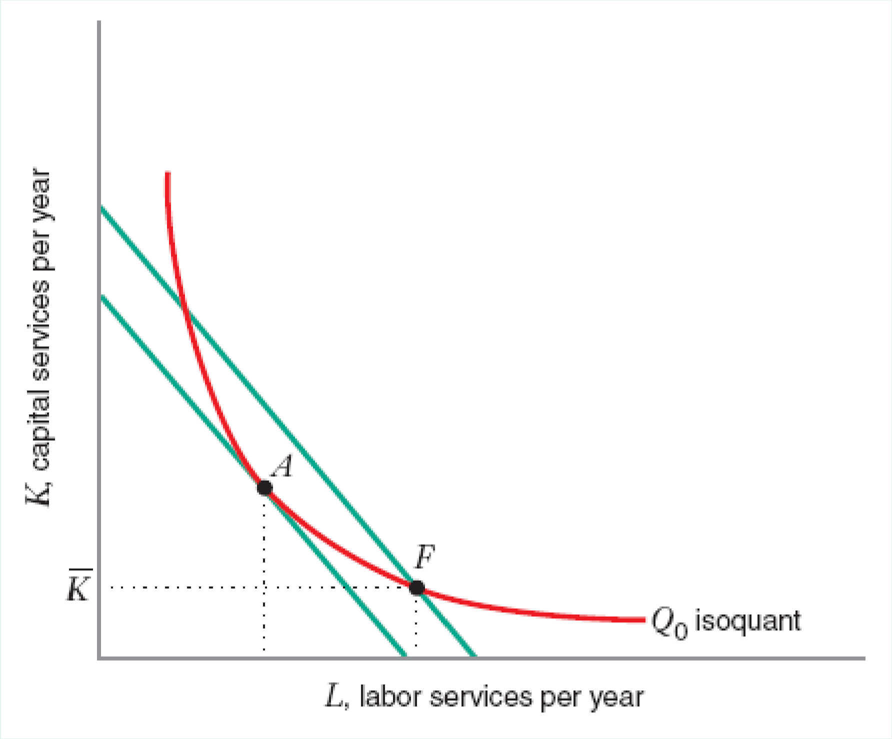
Sunk vs Non-Sunk – avoidable or not when Q = 0

Short-Run Costs can be classified as:

- Variable and non-sunk (labor and materials)
- Fixed and non-sunk (heating and lighting)
- Fixed and sunk (factory)

Short-Run Cost Minimization

In SR, capital is fixed at $\bar{K}$.



- Short-run combination is at Point F.
- If the firm were free to adjust all of its inputs, the cost-minimizing combination is at Point A.

Short-Run Cost Minimization – Input Demand

- In short run, one input is fixed, e.g. capital $\overline{K}$.
- Firm can vary the other input, labor.
- **The demand for labor will be independent of the price of labor or wage rate.**
- **Short-run demand for labor will depend on quantity produced.**
- As quantity increased, labor used increases holding capital fixed.

Short-Run Cost Minimization – Input Demand

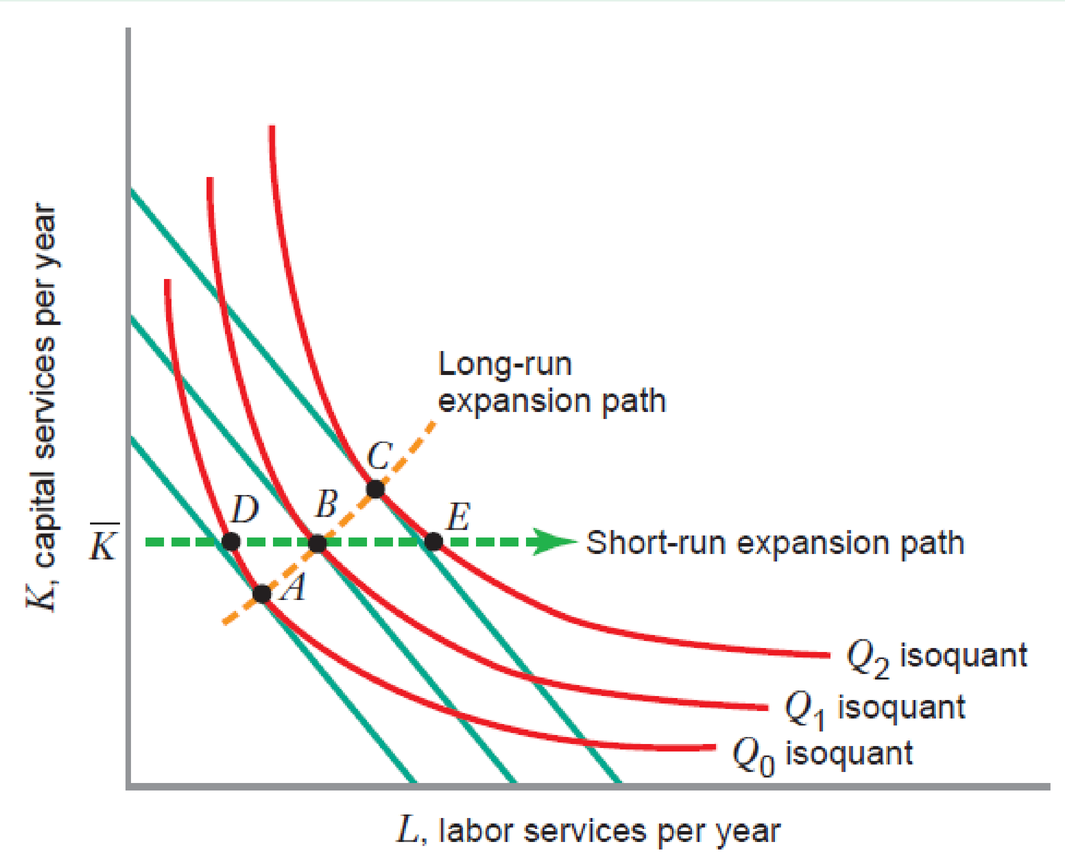


FIGURE 7.14 **Short-Run Input Demand versus Long-Run Input Demand**
In the long run, as the firm's output changes, its cost-minimizing quantity of labor varies along the long-run expansion path. In the short run, as the firm's output changes, its cost-minimizing quantity of labor varies along the short-run expansion path. These expansion paths cross at point *B*, where the input combination is cost-minimizing in both the long run and the short run.

Short-Run Cost Minimization – Input Demand

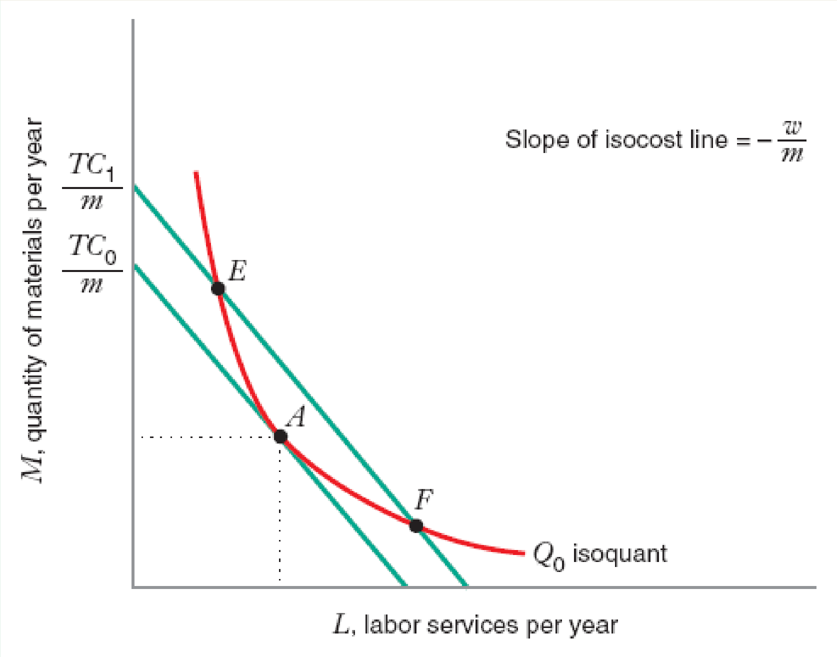
LEARNING-BY-DOING EXERCISE 7.5

Short-Run Cost Minimization with One Fixed Input

Problem Suppose that the firm's production function is given by the production function in Learning-By-Doing Exercises 7.2 and 7.4: $Q = 50\sqrt{LK}$. The firm's capital is fixed at $\overline{K}$. What amount of labor will the firm hire to minimize cost in the short run?

Short-Run Cost Minimization – 3 Inputs

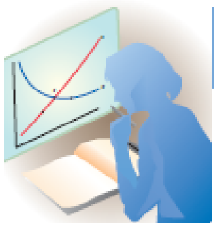
- When the short-run production requires 3 inputs – labor (L), fixed capital ($\overline{K}$), raw materials (M) – the analysis will be similar to the long-run cost minimization.



$$-MRTS_{L,M} = \frac{w}{m}$$

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$

Short-Run Cost Minimization – 3 Inputs



LEARNING-BY-DOING EXERCISE 7.6

Short-Run Cost Minimization with Two Variable Inputs

Suppose that a firm's production function is given by $Q = \sqrt{L} + \sqrt{K} + \sqrt{M}$. For this production function, the marginal products of labor, capital, and materials are $MP_L = 1/(2\sqrt{L})$, $MP_K = 1/(2\sqrt{K})$, and $MP_M = 1/(2\sqrt{M})$. The input prices of labor, capital, and materials are $w = 1$, $r = 1$, and $m = 1$, respectively.

Problem

(a) Given that the firm wants to produce 12 units of output, what is the solution to the firm's long-run cost-minimization problem?

(b) Given that the firm wants to produce 12 units of output, what is the solution to the firm's short-run cost-minimization problem when $K = 4$?

(c) Given that the firm wants to produce 12 units of output, what is the solution to the firm's short-run cost-minimization problem when $K = 4$ and $L = 9$?