Chapter 4: Understanding Interest Rates

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This chapter begins the second part of the course, which focuses on the behavior of interest rates.

The analysis starts on familiar ground, by reviewing some of the types of credit market instruments that we discussed earlier, in our overview of the financial system. In particular, it usefully classifies those credit market instruments into four types: simple loans, fixed-payment loans, coupon bonds, and discount bonds.

Next, it deals with the question: how can we come up with a unified way of measuring interest rates on all of these different types of credit market instruments?

It turns out that the concept of present value gives us the key to answering this question. So the chapter introduces us to this concept.

Then, the chapter shows us how we can use the concept of present value to define the yield of maturity, which is the most widely-applicable and most accurate measure of interest rates on credit market instruments. The chapter shows us how we can calculate the yield to maturity on each of the four main types of credit market instruments.

Although the yield to maturity is the most accurate measure of interest rates, there are two other measures of interest rates that are also widely-used and therefore of some importance. Hence, this chapter also includes a discussion of these alternative measures: the current yield and the yield on a discount basis.

Finally, the chapter concludes by highlighting an important distinction for investors in bonds: the distinction between the interest rate and the rate of return on a bond.

Perhaps the most important lesson that we learn from this chapter is that bond prices and interest rates are negatively related. That is, when interest rates rise, bond prices fall; and when interest rates fall, bond prices rise.

At first, this negative relationship between bond prices and interest rates might seem counterintuitive. But once we see how the various measures of interest rates are defined, the negative relationship actually becomes quite easy to see.

1 Types of Credit Market Instruments

In our overview of the financial system, we learned about a wide variety of credit market instruments. For the purposes of thinking about interest rates, it is useful to classify those credit market instruments into four types.

Credit market instruments = debt instruments.

Debt instrument = a contractual agreement by the issuer of the instrument (the borrower) to pay the holder of the instrument (the lender) fixed dollar amounts (interest and principal payments) at regular intervals until a specified date (maturity date) when a final payment is made.

1.1 Simple Loan

Provides the borrower with an amount of funds (principal) that must be repaid to the lender at maturity along with an additional amount (interest).

Example: Borrow \$100 today; repay \$100 principal plus \$10 interest in one year.

Many commercial loans are of this type.

1.2 Fixed-Payment Loan

Sometimes called a fully amortized loan.

Provides the borrower with an amount of funds that is to be repaid with interest by making fixed regular payments until maturity.

Example: Borrow \$1,000 today, repay \$126 per year for 25 years (as we'll see below, this loan has a 12% interest rate).

Auto loans and mortgages are of this type.

1.3 Coupon Bond

Pays the owner a fixed interest payment (coupon payment) each year until maturity, when a specified amount (face value or par value) is repaid.

The name "coupon bond" comes from the fact that years ago, before the time of computer record-keeping, such bonds came with coupons attached, which the holder would mail to the issue to request payment.

Example: Coupon bond with face value \$1,000, \$100 coupon payment, and ten-year maturity pays \$100 per year for ten years and \$1,000 at the end of ten years.

Coupon rate = Coupon payment as a percentage of face value.

Example:

$$\text{Coupon Rate } = \frac{\text{Coupon Payment}}{\text{Face Value}} = \frac{\$100}{\$1000} = 0.10 = 10\%$$

Many corporate bonds and US Treasury notes and bonds are of this type.

1.4 Discount Bond

Also called a "zero-coupon bond."

Bought at a price below face value (bought at a discount); makes no interest payments, but returns face value at maturity.

Example: Discount bond with $1,000 face value and one-year maturity sells for $900 today and returns $1000 in one year. The $100 difference between the purchase price and face value represents the interest payment.

US Treasury bills are of this type.

2 Present Value

The various types of credit market instruments all require payments at different times: simple loans and discount bonds make payments only at maturity, while fixed-payment loans and coupon bonds make regular payments until maturity.

How can we find a unified approach to measuring interest rates on these various types of instruments?

The key to answering this question lies with the concept of present value.

Present value captures the idea that a dollar received in the future is less valuable than a dollar received today.

To see how the concept of present value works, let's consider a series of examples.

Example 1: A simple loan of $100 requires the borrower to repay $100 principal plus $10 interest one year from now. For this simple loan, the interest payment expressed as a percentage of the principal is a sensible way of measuring the interest rate. In fact, when we measure the interest rate in this way, we are computing the simple interest rate:

$$\text{Simple Interest Rate } = i = \frac{\text{Interest}}{\text{Principal}} = \frac{\$10}{\$100} = 0.10 = 10\%$$

Example 2: If you make a simple loan of $100 at the simple interest rate $i = 0.10$ for one year, you get

$$\$100 + \$100 \times i = \$100 \times (1 + i) = \$100 \times (1.10) = \$110$$

at the end of the year. If you lend this \$110 out again for another year at the same simple interest rate $i = 0.10$, you get

$$\$110 + \$110 \times i = \$110 \times (1 + i) = \$110 \times (1.10) = \$121$$

at the end of the second year. Equivalently, we can write

$$\$100 \times (1 + i) \times (1 + i) = \$100 \times (1 + i)^2 = \$100 \times (1.10)^2 = \$121.$$

If you lend the \$121 out for a third year at the simple interest rate $i = 0.10$, you get

$$\$121 \times (1 + i) = \$100 \times (1 + i)^3 = \$100 \times (1.10)^3 = \$133.10$$

at the end of the third year.

Example 3: Starting with \$100, if the simple interest rate on one-year loans is i, and if you make these loans for n consecutive years, you get

$$\$100 \times (1 + i)^n$$

at the end of those n years.

Example 4: Working backwards, if the simple interest rate is $i = 0.10$, then:

$\$110 = \$100 \times (1 + i)$ received next year is worth $\$100 = \$110/(1 + i)$ today.

$\$121 = \$100 \times (1 + i)^2$ received two years from now is worth $\$100 = \$121/(1 + i)^2$ today.

$\$133.10 = \$100 \times (1+i)^3$ received three years from now is with $\$100 = \$133.10/(1+i)^3$ today.

This process of working backwards is called "discounting the future."

Example 5: If the simple interest rate is i, then the present value of \$1 received n years from now is defined as

$$\text{Present value of \$1 received } n \text{ years from now} = \frac{\$1}{(1 + i)^n} < \$1.$$

Thus, the idea of present value captures the face that \$1 received in the future is worth less than \$1 received today. More generally,

$$\text{Present value of \$}X \text{ received } n \text{ years from now} = \frac{\$X}{(1 + i)^n}.$$

3 Yield to Maturity

Yield to Maturity = the simple interest rate that equates the present value of payments received from a debt instrument to the price or value of that debt instrument today.

Also sometimes called the "internal rate of return."

This is the most accurate and widely-applicable measure of interest rates.

In fact, we can use this definition to compute the yield to maturity on each of our four types of credit market instruments.

3.1 Simple Loan

Consider a simple loan of \$100 that repays \$100 principal plus \$10 interest, for a total of \$110, in one year.

The yield to maturity i must satisfy

$$\text{Value today} = \$100 = \text{Present value of future payments} = \frac{\$110}{1+i}.$$

Let's solve for i:

$$\begin{aligned} \$100 &= \frac{\$110}{1+i} \\ \$100 \times (1+i) &= \$110 \\ \$100 + \$100 \times i &= \$110 \\ \$100 \times i &= \$110 - \$100 \\ i = \frac{\$110 - \$100}{\$100} &= \frac{\$10}{\$100} = 0.10 = 10\%. \end{aligned}$$

Thus, for simple loans, the yield to maturity equals the simple interest rate.

3.2 Fixed-Payment Loan

Consider a fixed payment loan of \$1,000 that requires payments of \$126 per year for 25 years.

The yield to maturity i must satisfy

$$\begin{aligned} \text{Value today} &= \$1000 \\ &= \text{Present value of future payments} \\ &= \frac{\$126}{1+i} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + ... + \frac{\$126}{(1+i)^{25}}. \end{aligned}$$

A financial calculator or a computer will generally be needed to solve for i. In this example, the yield to maturity turns out to be $i = 0.1183$, or approximately 12%.

More generally, for any fixed-payment loan, if

$$LV = \text{loan value (amount) today}$$

$$FP = \text{fixed annual payment}$$

$$n = \text{years to maturity}$$

$$i = \text{yield to maturity}$$

then

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + ... + \frac{FP}{(1+i)^n}.$$

3.3 Coupon Bond

Consider a coupon bond with \$1,000 face value, \$100 coupon payment (10% coupon rate), and ten-year maturity that sells at price P today.

The yield to maturity i must satisfy

$$P = \frac{\$100}{1+i} + \frac{\$100}{(1+i)^2} + ... + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}.$$

Note that since both the final \$100 coupon payment and the \$1,000 payment of face value are received at the end of the tenth year, both must be included in the present value calculation on the right-hand side.

Once again, a financial calculator or a computer is needed to solve for i. A range of results, depending on the purchase price P, are displayed in Mishkin's Table 1 (p.66):

Price of Bond P	**Yield to Maturity** i
\$1,200	7.13%
\$1,110	8.48%
\$1,000	10.00%
\$900	11.75%
\$800	13.81%

From this table, we can ascertain three important facts about the yield to maturity on a coupon bond:

1. When the bond price P equals the face value of \$1,000, the yield to maturity i equals the coupon rate.
2. The bond price P and the yield to maturity i are negatively related: when the bond price falls, the yield to maturity rises; and when the bond price rises, the yield to maturity falls.
3. The yield to maturity is below the coupon rate when the bond price is above face value and the yield to maturity is above the coupon rate when the bond price is below face value.

More generally, for any coupon bond, if

$$P = \text{today's bond price}$$

$$C = \text{annual coupon payment}$$

$$F = \text{face value}$$

$$n = \text{years to maturity}$$

$$i = \text{yield to maturity}$$

then

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}.$$

Once again, because the last coupon payment C and face value F are both received at the end of the nth year, both must be included in the present value calculation on the right-hand side.

Notice that in this general formula, we can see the inverse relationship between the bond price P and the yield to maturity i:

If i rises, then each term on the right-hand side gets smaller. Hence, P must fall.

If i falls, then each term on the right-hand side gets bigger. Hence, P must rise.

3.4 Discount Bond

Consider a US Treasury bill with one year maturity and face value \$1,000 that sells for \$900 today.

The yield to maturity i must satisfy

$$\$900 = \frac{\$1000}{1+i}$$

$$\begin{aligned}
\$900 \times (1+i) &= \$1000 \\
\$900 + \$900 \times i &= \$1000 \\
\$900 \times i &= \$1000 - \$900 \\
i = \frac{\$1000 - \$900}{\$900} &= \frac{\$100}{\$900} = 0.111 = 11.1\%
\end{aligned}$$

More generally, for any discount bond, if

$$\begin{aligned}
P &= \text{today's bond price} \\
F &= \text{face value} \\
n &= \text{years to maturity} \\
i &= \text{yield to maturity}
\end{aligned}$$

then

$$P = \frac{F}{(1+i)^n}$$

From this general formula, we can see that for a discount bond, too, the bond price and the yield to maturity are negatively related: when the yield to maturity falls, the bond price rises; and when the yield to maturity rises, the bond price falls.

In the special case where $n = 1$:

$$\begin{aligned}
P &= \frac{F}{(1+i)} \\
P \times (1+i) &= F \\
P + P \times i &= F \\
P \times i &= F - P \\
i &= \frac{F-P}{P}.
\end{aligned}$$

Suppose we rewrite this last formula as

$$\begin{aligned}
i &= \frac{F-P}{P} \\
&= \frac{F}{P} - \frac{P}{P} \\
&= \frac{F}{P} - 1.
\end{aligned}$$

Once again, the bond price and the yield to maturity are negatively related: when P falls, i rises; and when P rises, i falls.

4 Other Measures of Interest Rates

Although the yield to maturity is the most accurate and widely-applicable measure of interest rates, it is often difficult to calculate.

For this reason, a couple of other measures of interest rates are frequently used: the current yield and the yield on a discount basis.

4.1 Current Yield

Applies only to coupon bonds.

Recall that the yield to maturity on a coupon bond usually cannot be found without the help of a computer.

As a result, the yield to maturity on a coupon bond is often approximated by the current yield.

To calculate the current yield, let

$$P = \text{today's bond price}$$

$$C = \text{annual coupon payment}$$

$$i_c = \text{current yield}$$

then the current yield i_c is defined as

$$i_c = \frac{C}{P}.$$

Four facts about the current yield:

1. i_c better approximates i when today's bond price P is closer to the face value F and when the maturity of the bond is longer.
2. In fact:

   If $P = F$, then $i_c = i$.
   If $P > F$, then $i_c > i$.
   If $P <: F$, then $i_c < i$.
3. i_c and i always move in the same direction: i_c rises when i rises, and i_c falls when i falls.
4. The bond price P and the current yield i_c are negatively related: when P rises, i_c falls; and when P falls, i_c rises.

4.2 Yield on a Discount Basis

Applies only to discount bonds.

Although it is fairly easy to calculate the yield to maturity on a one-year discount bond, it is more difficult to calculate the yield to maturity on a discount bond with maturity less than or greater than one year.

For this reason, interest rates on discount bonds, including US Treasury bills, are often quoted in terms on the yield on a discount basis.

To calculate the yield on the discount basis for a discount bond, let

$$P = \text{today's bond price}$$

$$F = \text{face value}$$

$$i_{db} = \text{yield on a discount basis}$$

then the yield on a discount basis i_{db} is defined as

$$i_{db} = \frac{F - P}{F} \times \frac{360}{\text{days to maturity}}.$$

Recall that for a discount bond with one-year maturity

$$\text{yield to maturity} = i = \frac{F - P}{P}.$$

Thus, the yield on a discount basis calculation has two peculiarities:

1. It uses the percentage gain on face value, $(P - F)/F$, rather than the percentage gain on the purchase price $(F - P)/P$.
2. It considers the year to be 360 days long instead of 365.

As a result of both of these peculiarities, the yield on a discount basis always understates the yield to maturity.

Example: Discount bond with \$1,000 face value and one-year maturity sells for \$900 today:

$$P = \$900$$

$$F = \$1000$$

$$n = 1$$

and hence

$$i_{db} = \frac{F-P}{F} \times \frac{360}{\text{days to maturity}} = \frac{\$1000 - \$900}{\$1000} \times \frac{360}{365} = 0.099 = 9.9\%$$

but

$$i = \frac{F-P}{P} = \frac{\$1000 - \$900}{\$900} = 0.111 = 11.1\%.$$

Nevertheless, it is still true that the bond price and its yield on a discount basis are negatively related.

To see this, return to the definition

$$i_{db} = \frac{F-P}{F} \times \frac{360}{\text{days to maturity}}.$$

Since the P on the right-hand side has a minus side out in front of it, i_{db} falls when P rises, and i_{db} rises when P falls.

In addition, the yield on a discount basis and the yield to maturity always move in the same direction: i_{db} rises when i rises, and i_{db} falls when i falls.

5 Interest Rates and Returns

For investors, the distinction between interest rates and returns on bonds can be quite important.

The rate of return on a bond takes into account not only any interest payments made by the bond, but also any changes in the price of the bond itself.

Example: Consider a coupon bond with $1,000 face value and $100 coupon payment (10% coupon rate) that is bought today for $1,000.

If interest rates fall over the next year, the bond price will rise.

Suppose, to continue the example, that the investor is able to sell the bond one year from now at the price of $1,200.

Then the investors total earnings from holding the bond include the $100 coupon payment plus the capital gain of $1,200-$1,000 = $200.

Expressing these earnings as a percentage of the original purchase price gives us a total return on the investment of

$$\frac{\$100 + \$200}{\$1000} = \frac{\$300}{\$1000} = 30\%.$$

Of course, this process can work in the opposite direction as well.

Suppose instead that interest rates rise over the next year, so that the bond price falls to $800.

If the investor sells the bond after one year, the $100 coupon payment is completely offset by a capital loss of $1000-$800 = $200.

In this case, the total return would be

$$\frac{\$100 - \$200}{\$1000} = -\frac{\$100}{\$1000} = -10\%.$$

These examples reveal that although bonds are often considered to be very conservative investments, they can in fact be quite risky for those who do not plan to hold them until maturity.

6 Conclusion

This chapter has shown us that for any type of credit market instrument, the yield to maturity serves as the most accurate measure of the interest rate.

The yield to maturity is defined as the simple interest rate that equates the present value of payments received from a debt instrument to the price or value of that debt instrument today.

Two other measures of the interest rate, the current yield and the yield on a discount basis, are less accurate but are often easier to calculate.

Nevertheless, for any type of bond and any measure of the interest rate, bond prices and interest rates are negatively related: bond prices fall when interest rates rise, and bond prices rise when interest rates fall.