Production and Costs

Part 4

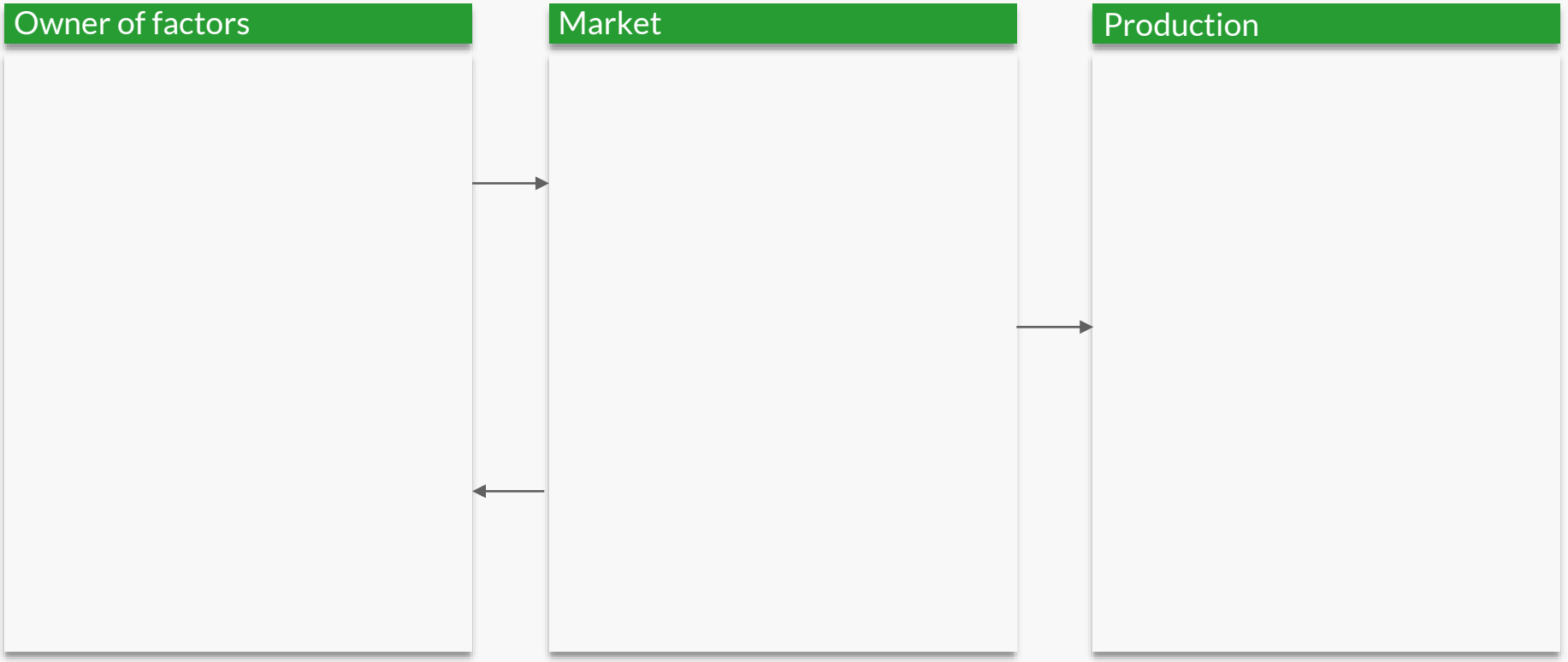
EE211

Principles of Microeconomics

Revision Aug 2020

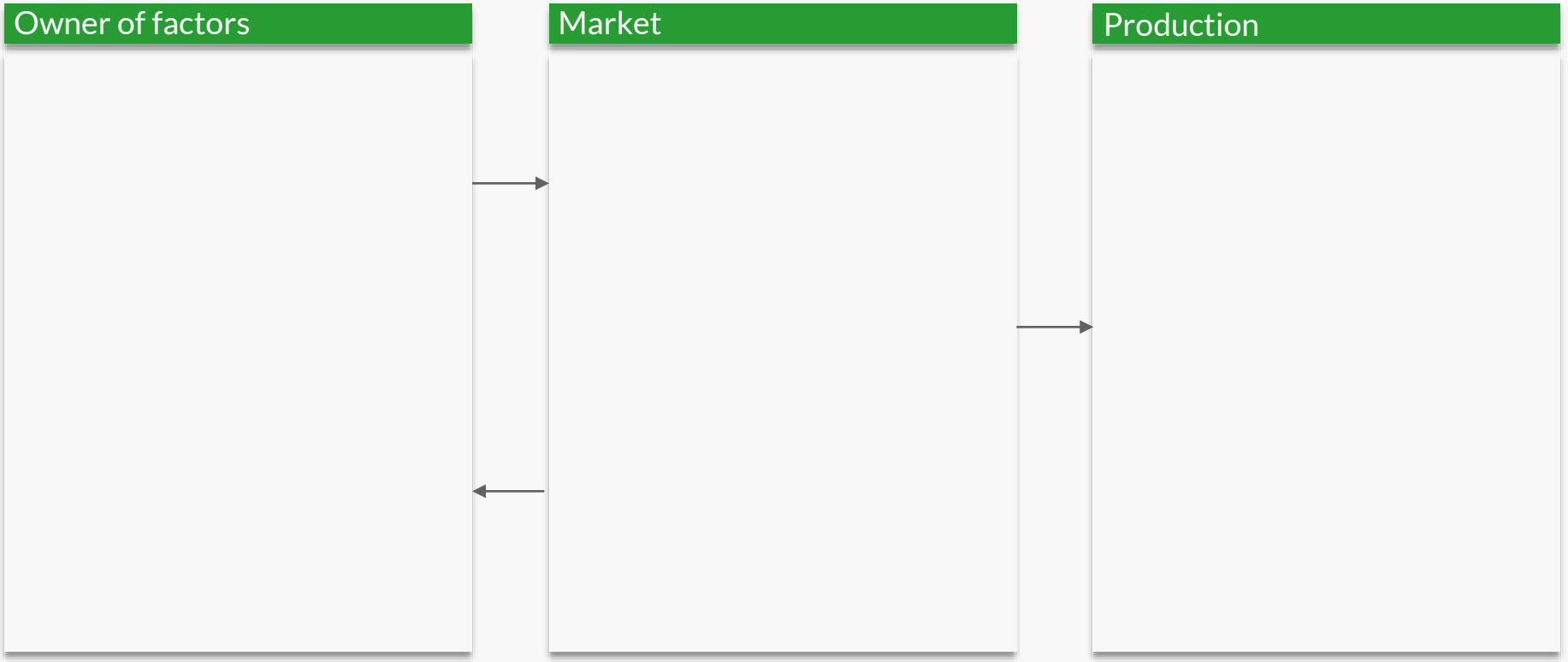
Understanding production

Let's depict a clear-cut production when owner of factors is separated.



Understanding production

Now let's turn to a little bit more complicated one when the owner of factors maybe the same person/entity.



Understanding production

The production process is simplified into a production function, which is defined as follows.

Definition 4.1

Production function *is a mathematical function which transforms resources into goods or services. It indicates the highest output q that a firm can produce for every specified combination of inputs. For simplicity, we assume that*

$$q = f(K, L)$$

Before we move on, some assumptions must be posed.

- Firm is assumed to be rational with its aim to maximize profit.
- Every unit of factors of production is considered equally in quality.
- Each factor of production can be indefinitely separable. (Continuous)
- In the long-run, if budget or cost is limited, the analysis will consider only the part where the satiation of production has not been reached.
- Ceteris paribus.

(1) Short-run production

Short and long-run production is not categorized by time frame of production. Instead, short run of production means that there is at least one **fixed factor** utilized, a factor which is assumed to be constant in the short-run.

Therefore, we usually assume that capital is a fixed factor while labor is a **variable factor**, a factor that firm can adjust its amount. The production function becomes

- $q = f(\bar{K}, L)$

Also note that return or compensation for capital is interest (r), while for labor is wage (w).

Since capital is assumed to be fixed in the short-run, the focus here is on the relation between **labor hired and number of output** ($L \Rightarrow q$). We further define types of output in the following manners on the right-hand side.

Each type of output is related in the following table.

Definition 4.2

Total product *is total output produced from a production process and a set of input, denoted by* ***TP*** *or* ***q****.*

Definition 4.3

Average product *is the average output per variable factor, denoted by* ***AP****.*

$$AP = \frac{TP}{L}$$

Definition 4.4

Marginal product *is the additional output from adding 1 more unit of variable factor into the production, denoted by* ***MP****.*

$$MP = \frac{\Delta TP}{\Delta L} = \frac{\partial TP}{\partial L} = TP_n - TP_{n-1}$$

(1)Short-run production

Table 4.1: Variable factor (L) vs. output (q)

Labor (L)	Total product (TP,q)	Average product (AP)	Marginal product (MP)
0	0	-	-
1	3		
2	8		
3	12		
4	14		
5	14		
6	12		

From the chart, we can see that when a firm decides to add more variable factor, marginal product increases at first and then decreases later on. This nature of short-run production is governed by the law of diminishing marginal product.

Definition 4.5

***Law of diminishing marginal product** states that in short-run, when firm increases variable factor, marginal product will increase at first then decrease until it becomes 0 and negative. The reason of diminishing marginal product is because of disproportionated use of factors of production due to the constant number of fixed factor.*

Let's look at an example of a coffee shop where espresso machine is considered as fixed factor while baristas are variable factor, illustrate this scenario in the box.

(1) Short-run production

A coffee shop scenario

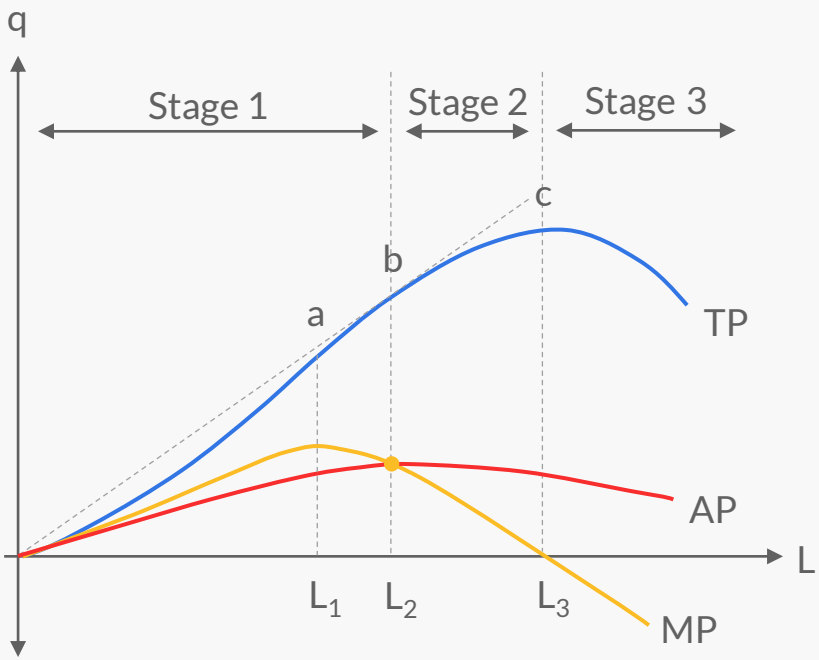
Let's think of another example when we fix a capital while having a variable factor.

Another scenario

Now if we graph *Table 4.1*, each type of measuring product, against number of product q, we have the following graph, which can be divided in to 3 stages.

(1) Short-run production

Figure 4.1: Graphing L and q



What should we notice in each stage?

(2) Short-run costs

Even though we learned that marginal product is diminishing in the short-run, seeking to maximize output is not firm's goal. (**MAXIMIZING PROFIT**)

Therefore, we need to further cost structure in the short-run. Cost is a value that one gives up to get another thing in return. For the production concept, production cost can be interpreted as **compensation or opportunity cost** for those factors since they are used for production and not otherwise.

Since we have fixed and variable cost, both are separated. We are going to consider these costs from three aspects, or term, as

(1) Total term: Total fixed cost (TFC), Total variable cost (TVC), and total cost (TC or STC)

(2) Average term: Average fixed cost (AFC), Average variable cost (AVC), and Average total cost (ATC or SAC)

(3) Marginal term: Marginal cost (MC or SMC)

Studying cost, on the other hand, focuses on relation between output and cost. (**q => c**) Now let's look at the definition.

Definition 4.6

- ***Total fixed cost*** *is the total cost for fixed factor, denoted by* ***TFC.***
- ***Total variable cost*** *is the total cost for variable factor, denoted by* ***TVC.***
- ***Total cost*** *is both fixed and variable cost combined, denoted by TC or STC.*

$$TC = TFC + TVC$$

Definition 4.7

- ***Average fixed cost*** *is the cost of fixed factor per one unit of output, denoted by* ***AFC.***

$$AFC = \frac{TFC}{q}$$

- ***Average variable cost*** *is the cost of variable factor per one unit of output, denoted by* ***AVC.***

$$AVC = \frac{TVC}{q}$$

- ***Average cost*** *is the total cost per one unit of output,* ***denoted by AC or SAC.***

$$AC = \frac{TC}{q} = AFC + AVC$$

(2) Short-run costs

Definition 4.8

Marginal cost is the additional cost when one more unit of output is produced, *denoted by* ***MC or SMC.***

$$MC = TC_n - TC_{n-1} = \frac{\Delta TC}{\Delta q} = \frac{dTC}{dq}$$

Prove that $MC = \frac{dTVC}{dq}$

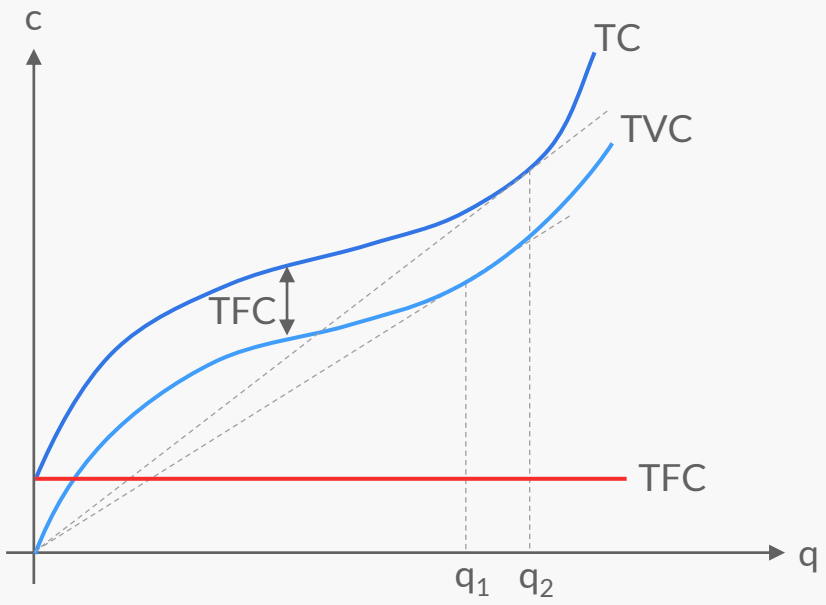
Now let's consider each type of cost and characterize them.

Table 4.2*: Output (q) and costs (c)*

(1) q	(2) TFC	(3) TVC	(4) TC	(5) AFC	(6) AVC	(7) AC	(8) MC
Output	Total fixed cost	Total variable cost	Total cost	Average fixed cost	Average variable cost	Average cost	Marginal cost
0	25	0					
1		12					
2		18					
3		20					
4		24					
5		35					

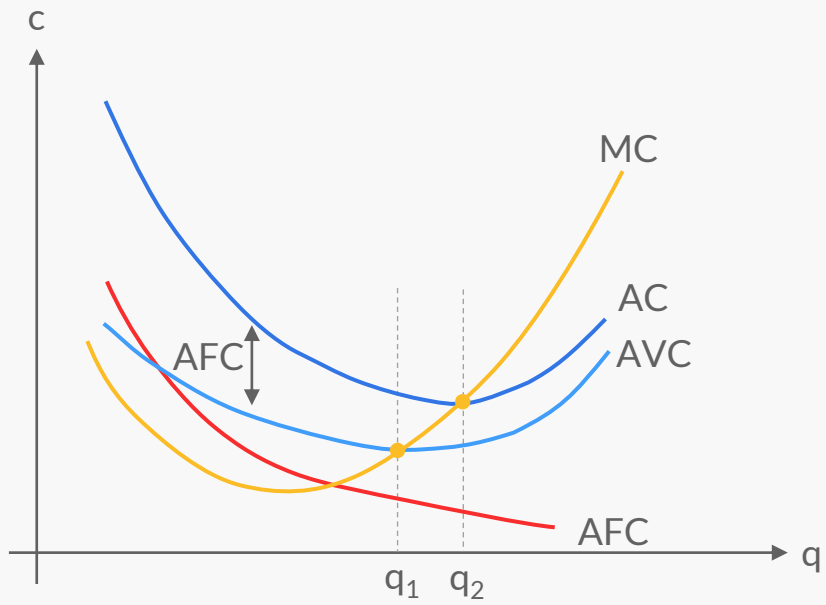
(2) Short-run costs

Figure 4.2: Cost in total terms



What should we notice from this graph?

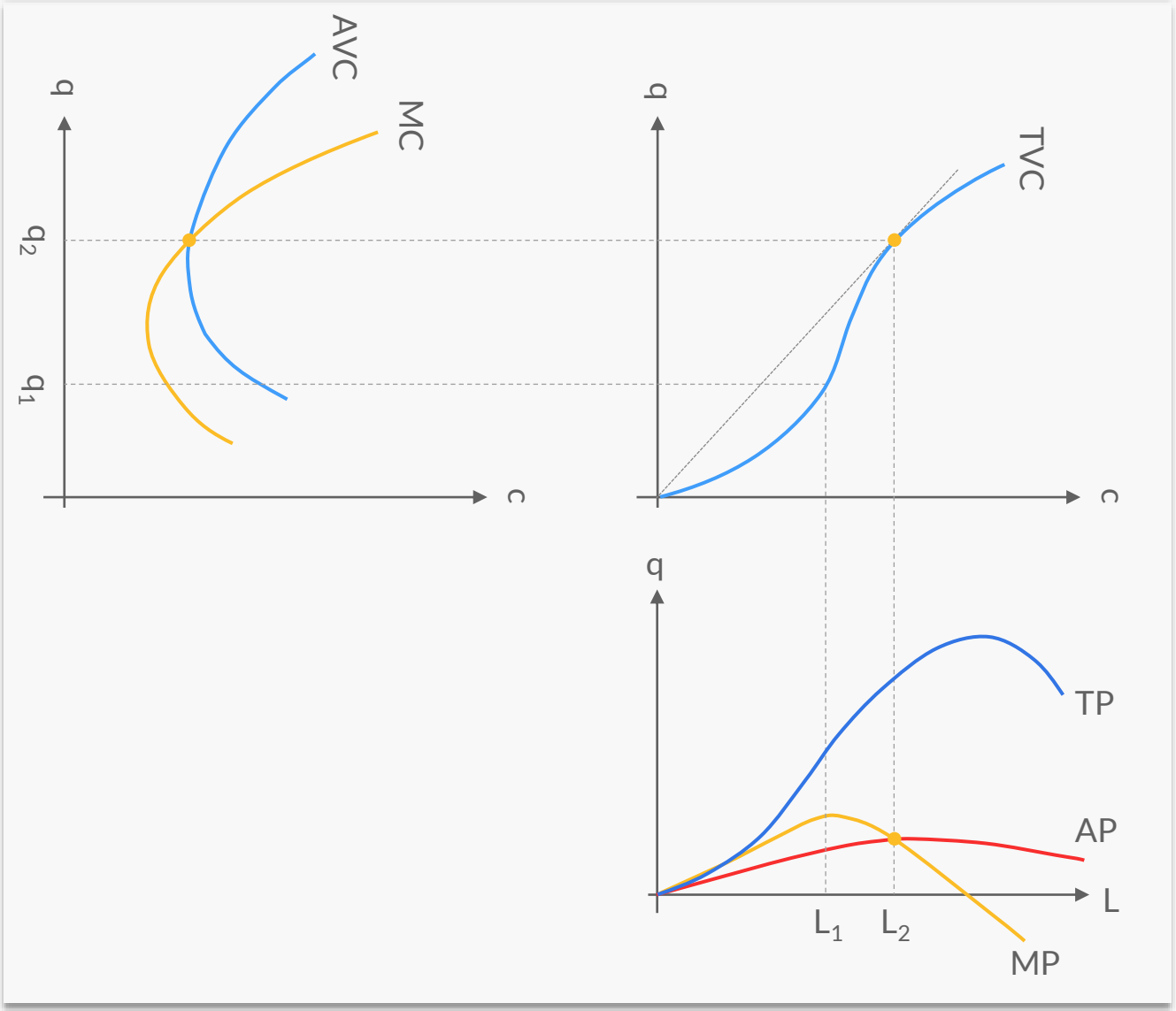
Figure 4.3: Cost in average and marginal terms



What should we notice from this graph?

(2) Short-run costs

Figure 4.4: relations of production and costs



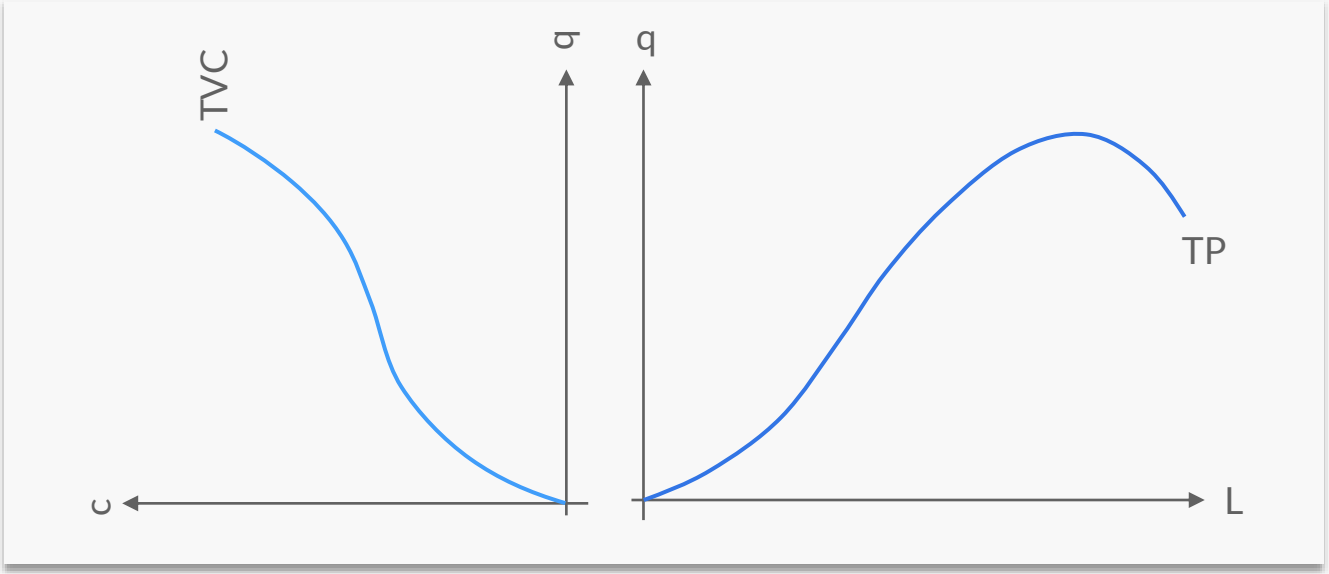
Putting $c = f(q(L))$ altogether. Note that

- Diminishing MP causes TP to be increasing in decreasing rate.
- Increasing MC and TVC is caused by diminishing MP.

(2) Short-run costs

Let's assume some numbers here. Given that a unit of labor costs 500 baht to hire, consider when TP increases both at increasing and decreasing rate.

Figure 4.5: TP and TVC as marginal cost is rising



(2)Short-run costs

Summary of short-run production

(1) Short-run production ($L \Rightarrow q$) means at least a fixed factor is utilized.

(2) Increasing variable factor into the production causes marginal product to diminish due to disproportionated combination of fixed and variable factor.

(3) Diminishing marginal product makes it costlier
($q \Rightarrow c$) to produce one more output when firm keeps adding more variable factor.

(4) Diminishing marginal product causes marginal cost to rise, also total variable cost, total cost ,average variable cost, and average cost to later rise.

Again, firm's objective is not to minimize cost. We then need to study firm's revenue in order to complete producer part. However, firm's revenue varies by market structure (or competitiveness). Hence, we will study firm's revenue and profit in the market section.

(1) Long-run production

On the other hand, long-run production occurs when a firm can adjust the amount of all factors of production. Production function is then represented in this form

- $q = f(K, L)$

Since firm can freely select an amount of all factors of production, the topic then shifts to how can firm select optimal combination for each level of output?

Supposed that only capital (K) and labor (L) are utilized in a production, a firm considers to produce $q = 10$ or $q = 20$, the question is how many units of capital or labor should firm utilize at each q or what is (K^*, L^*).

The table on the right-hand side illustrates many bundles that can lead to the same result.

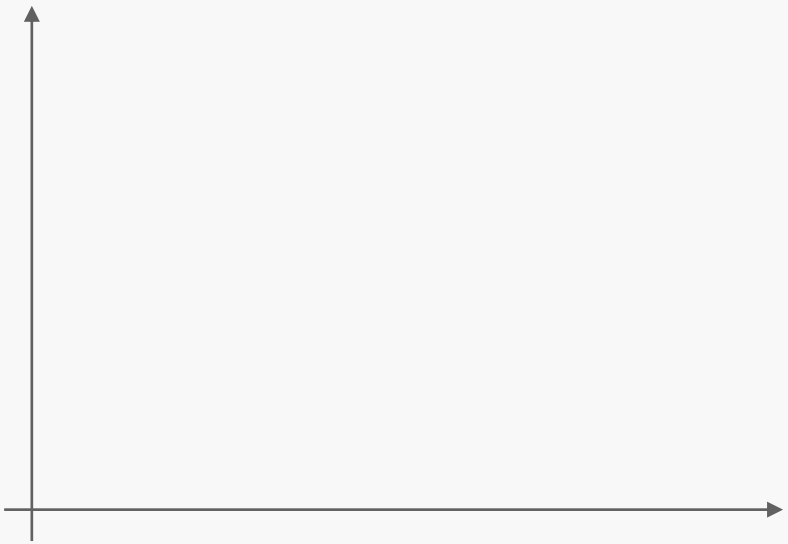
The quick answer would be (K^*, L^*) that **minimize cost** for each particular level of q. So we need to consider **capital and labor price** (r and w). The tools that we are going to use to analyze is called **Isoquant** and **Isocost**.

***Table 4.3**: Variable factors (K, L) and output (q)*

(1) Output	(2) Capital	(3) Labor
10	1	5
10	3	3
10	5	1
20	2	7
20	4	4
20	6	2

(1) Long-run production

Isoquant



Properties of Isoquant

- Factors bundles.
- Level of output yielded.
- Isoquant cannot intersect.
- Isoquant has negative slope and convex to the origin.

On an isoquant, there are many combinations of factors (K,L) that yield the same quantity of product, similarly to an IC curve. The isoquant curve is convex to origin in which on each point substitution rate of K and L is different.

Definition 4.9

Marginal rate of substitution is a ratio of substitution between two types of factor (K,L) on a point of an isoquant that yield the same level of output.

$$MRTS_{LK} = \frac{\Delta K}{\Delta L}$$

(1)Long-run production

Marginal rate of technical substitution

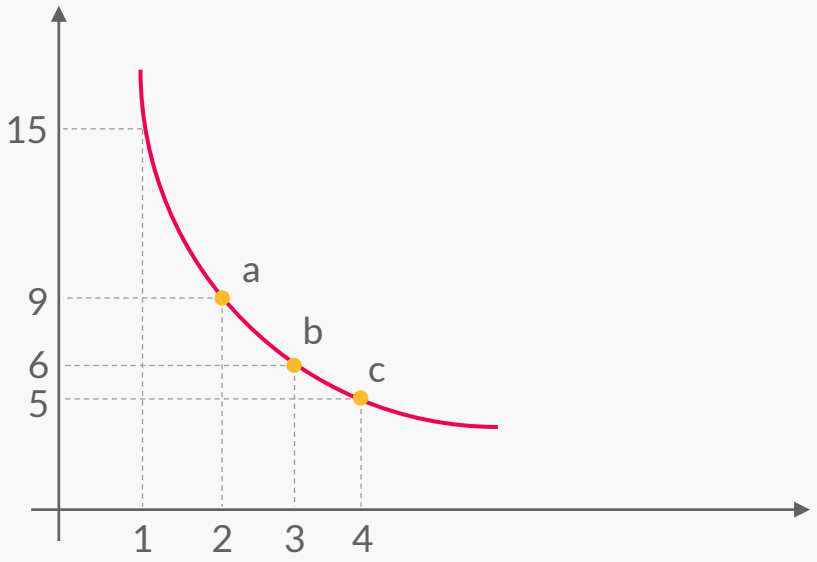
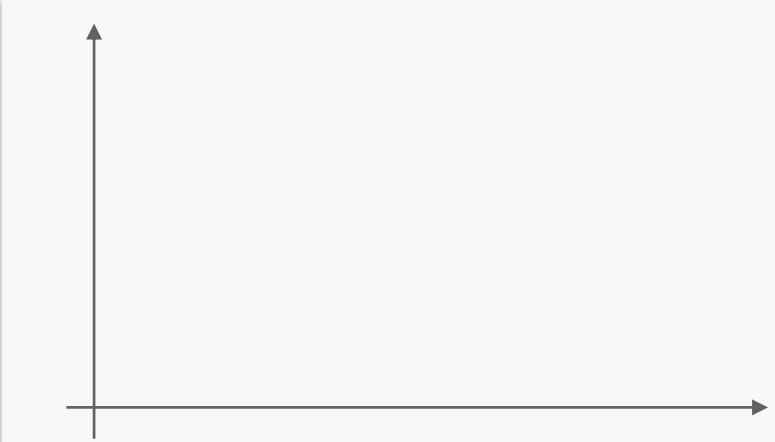


Figure out these $MRTS_{LK}$

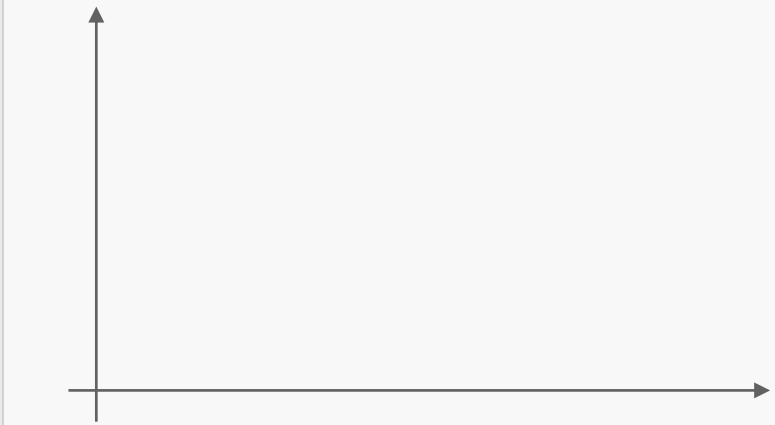
- $MRTS_{LK}(\text{a}) =$
- $MRTS_{LK}(\text{b}) =$
- $MRTS_{LK}(\text{c}) =$

MRTS reflects how two factors can be substituted. Let's consider two factors that are **perfectly substitutable** and **perfectly complementary**.

Perfectly substitutable

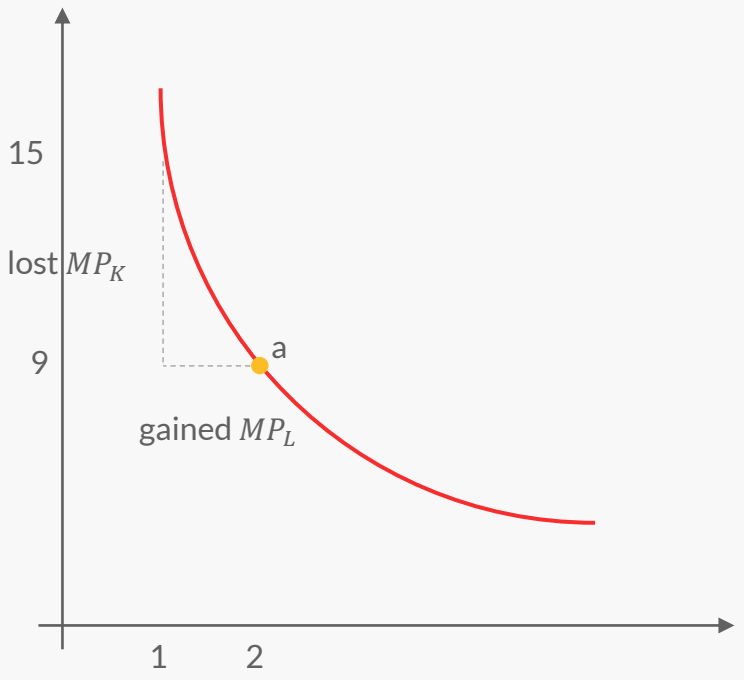


Perfectly complementary



(1) Long-run production

Marginal rate of technical substitution



As we learned that, in case of two substitutable factors, MRTS is the ratio of substituting two factors, resulting in the same amount of output, it also means that.

- $|MRTS_{LK}| = \left|\frac{\Delta K}{\Delta L}\right| = \frac{MP_L}{MP_K}$

Consider an example here.

- Supposed that the MRTS at one point is -6, it means that when giving up 6 of K while adding 1 unit of L into the production result in yielding the same amount of product. (or it can be 12 of K and 2 of L which is the same ratio)

- When a firm gives up 6 of K, this firm loses an amount of marginal product of capital (MP_K). We do not know how many.

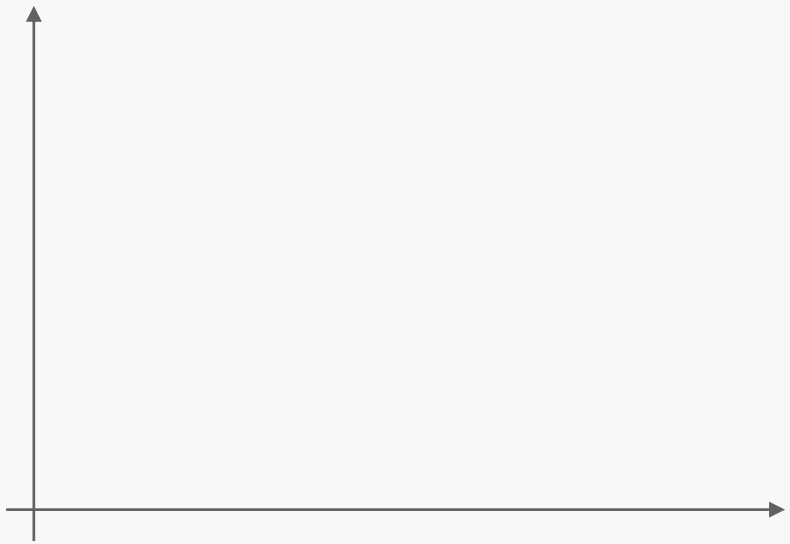
- When the firm adds 1 more unit of L, this firm gains an amount of product that was lost from MP_K. Again we do not know how many but it is the marginal product of labor (MP_L) from that 1 L.

- Those MP_K and MP_L must be equal since changing factors bundle on the same isoquant results in the same amount of output.

- Therefore, it means that 1 unit of L can produce equal MP of 6 units of K. That makes MP_L 6 times larger than MP_K.

(1) Long-run production

Isocost



Isocost is, once again, very similar to the budget line

Example: Given that a labor costs 25 baht a day and a unit of capital costs 50 baht a day, a firm can bear 200 baht cost per day. Draw the isocost.

Isocost shows combinations of capital and labor that cost equally. Assumed that production function consists of capital and labor, total cost (TC) equals interest multiply by number of capital plus with wage multiply by number of labor or $TC = rK + wL$

Isocost has a fixed slope if, during the analysis, price of labor and capital do not change. The slope can be defined as follows.

Definition 4.10

Marginal rate of market substitution is the substitution rate of two factors given by their price or the slope of isocost.

$$|MRMS_{LK}| = \left|\frac{\Delta K}{\Delta L}\right| = \frac{w}{r}$$

$MRMS_{LK}$ reflects relative factor price. Given the same example,

- Find the $MRMS_{LK} =$

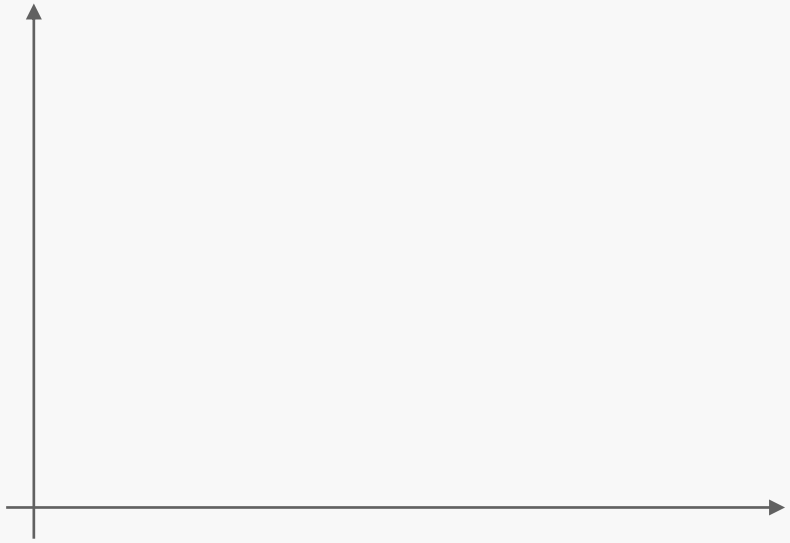
- If this firm can bear more cost up to 300 baht a day, how would isocost alter?

(1) Long-run production

Example: Given that a labor costs 25 baht a day and a unit of capital costs 50 baht a day, a firm can bear 200 baht cost per day at first. If wage increases to 25 baht a day and rent decreases to 10 baht a day.

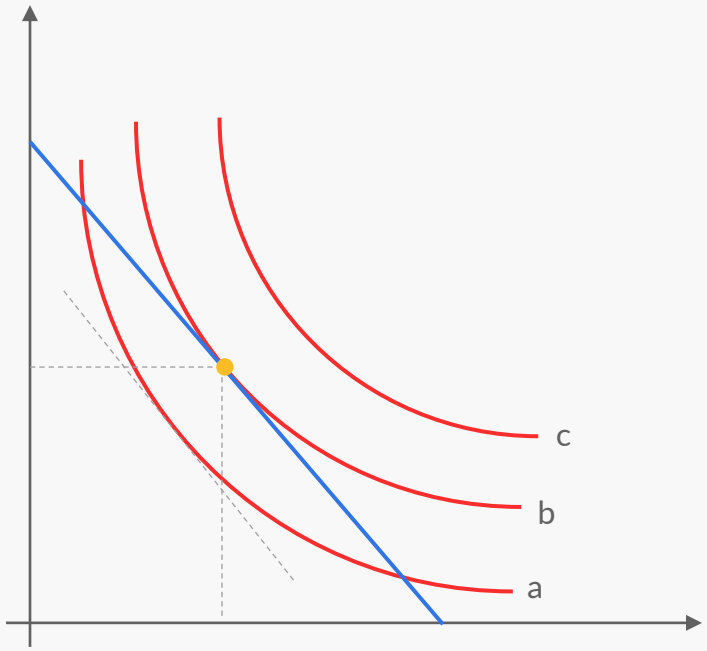
- Find the new $MRMS_{LK} =$
- Draw how isocost tilt.

Change in isocost



For each level of output on each isoquant, a b or c, there is a specific combination between K and L that will minimize cost (K^*, L^*).

Cost minimization

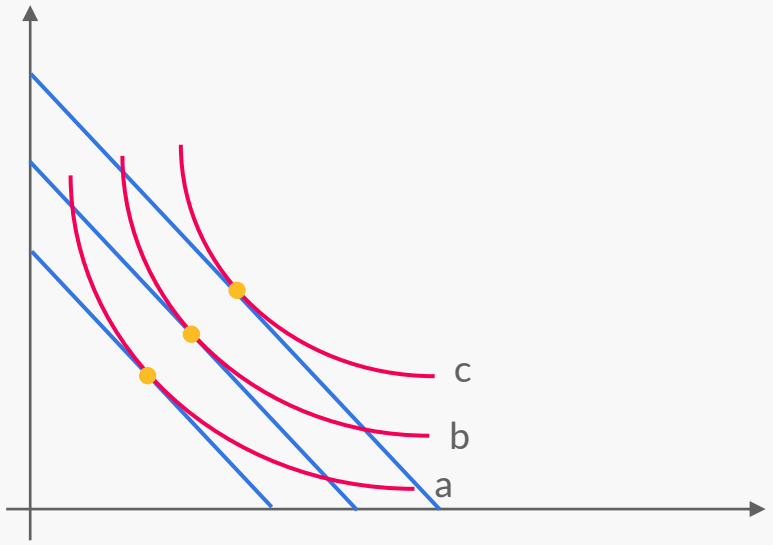


(1) Long-run production

When firm can bear more cost, the number of factors of production that minimize cost can be increased to the point where new isocost tangent with new isoquant. If we draw a line connecting between each equilibrium, we will get an **expansion path**.

This expansion path is the set of optimal combinations. Each combination indicates a pair of c and q, which this c is minimized with respect to given q. We can then plot relation between c and q in the long-run production.

Expansion path



From the expansion path, we can further study multiple topics. The law of returns to scale is one of them to understand proportional return of output to input.

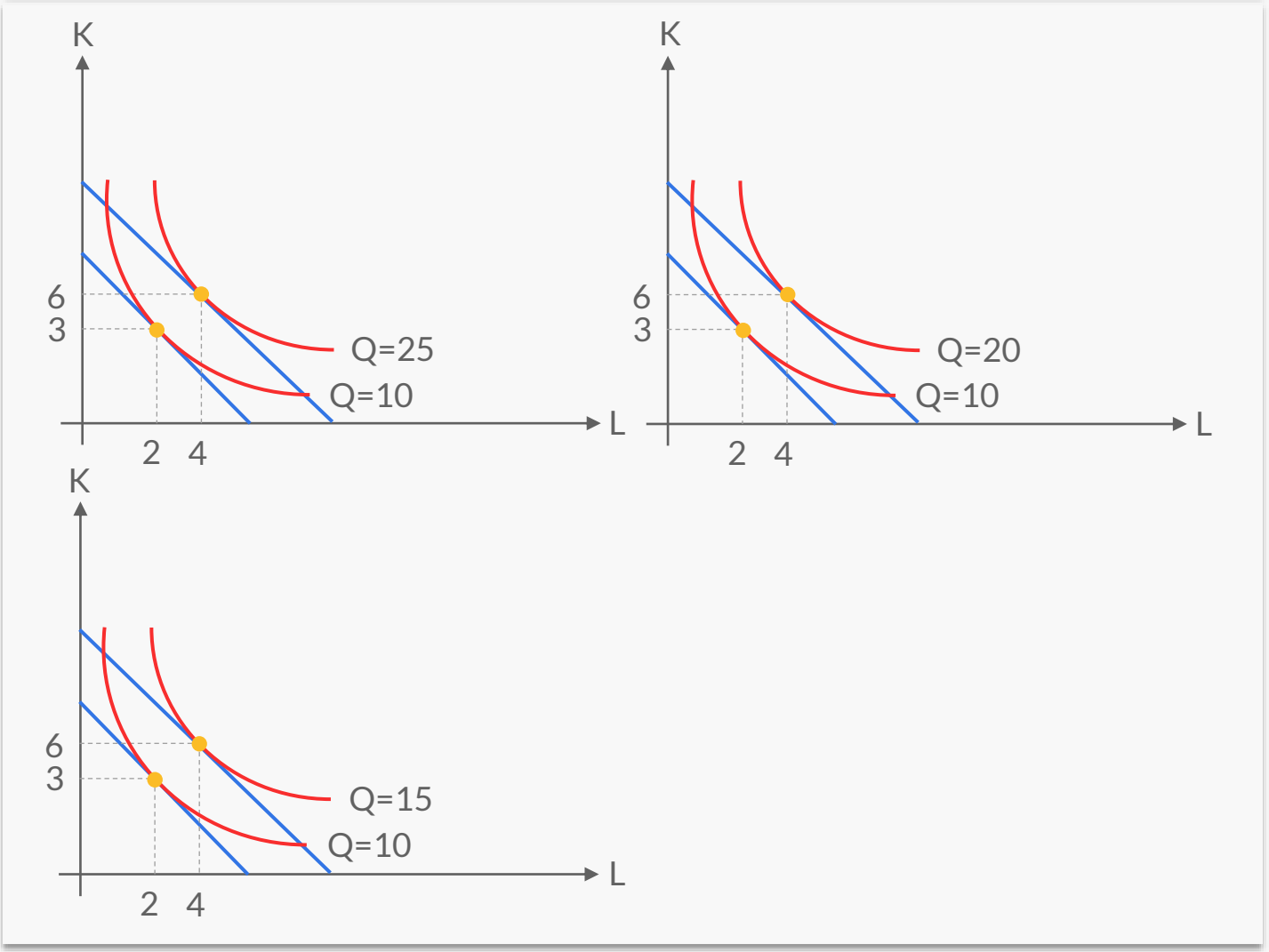
Definition 4.11

Law of returns to scale states that when firm increases labor and capital proportionally, the return will be one or another as stated below (we cannot consider the case that each factor increase unequally)

- Increasing Returns to Scale (IRS) is the case that proportion of output increases more than proportion of production factors.
- Constant Returns to Scale (CRS) is the case that proportion of output increases equally to proportion of production factors.
- Decreasing Returns to Scale (DRS) is the case that proportion of output increases less than proportion of production factors.

(1) Long-run production

Figure 4.6: Different returns to scale



Returns to scale will also dictate each phase of long-run cost curve.

(2) Long-run costs

For every combination of c and q that minimize cost for a firm, we can derive long-run costs from the expansion path.

Definition 4.12

- ***Long-run total cost*** *is the cost of all variable costs combined, denoted by* ***LTC.***
- ***Long-run average cost*** *is the average of total cost per one output, denoted by* ***LAC.***

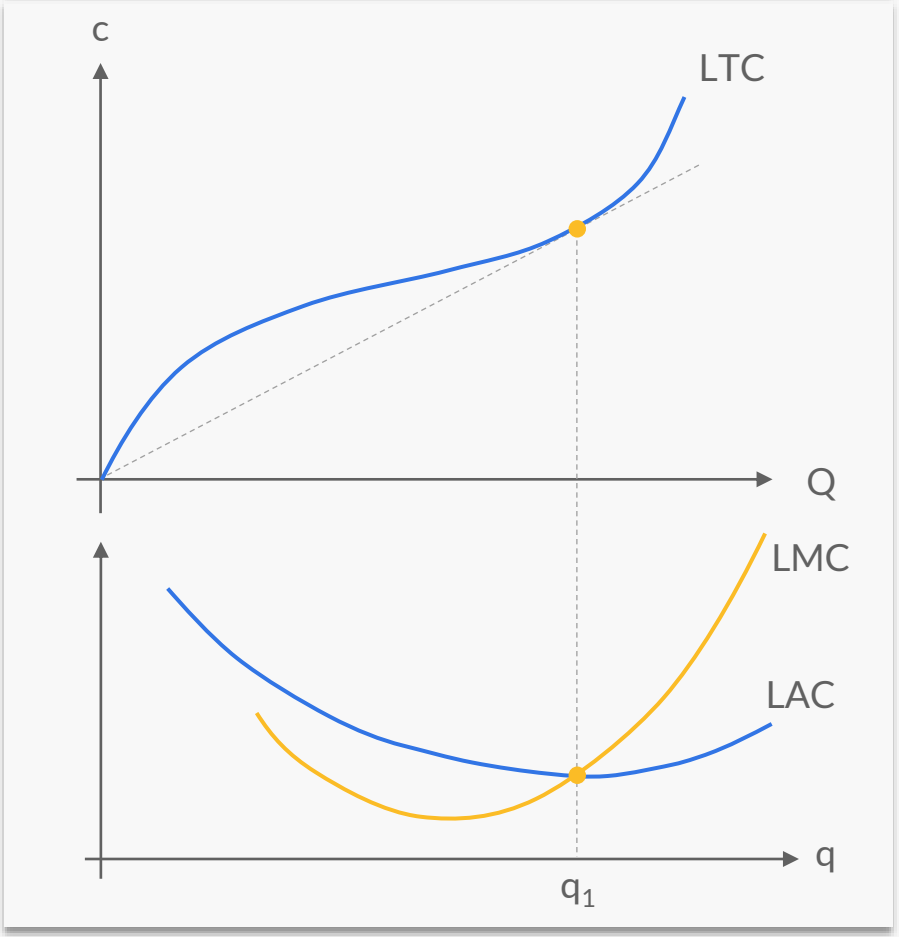
$$LAC = \frac{LTC}{Q}$$

- ***Long-run marginal cost*** *is the additional cost when firm produces one more unit of output, denoted by* ***LMC****.*

$$LMC = \frac{\Delta LTC}{\Delta Q} = \frac{dLTC}{dQ}$$

Graphing the long-run costs curve will result in the illustration on the right-hand side.

Figure 4.6: long-run costs



(2) Long-run costs

As we can see from the graph, even in the long-run, firms still encounter both decreasing and increasing marginal cost and average cost. To understand this, let's explore relationship between short and long-run first.

In the real-world scenario, firms mostly are in short-run production. Long-run production only occurs for a short period of time when firms decide to expand their production capability.

A metaphor for the situation is factory size. For example, for any industrial production, there must be some factors that cannot be adjusted immediately such as large and expensive machines or production building, setting up a new factory to expand production, or some made-by-order capital goods.

Therefore, the factory in the short-run production is when there is a capital good that cannot be adjusted, while in the long-run production is when the factory is actually expanding.

Short and long-run AC

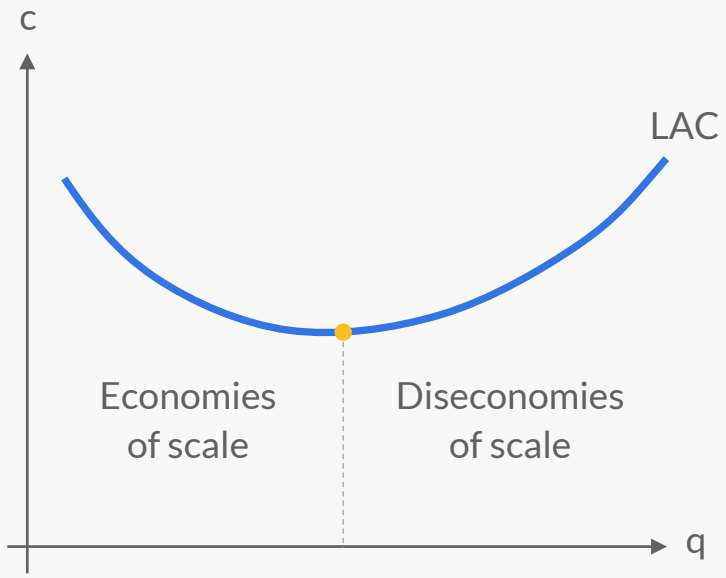
Both production stages are like the factory expansion as shown above. The short-run average cost of each line is the range that cannot be extended.

If the firm can choose unlimited factory sizes, when producing up to the lowest point of the short-run average cost, one will expand the new plant, causing the average cost to shift to another line which may result in lower the average cost level.

Therefore, the long-run average cost line is drawn from the lowest point of the average cost line.

(2) Long-run costs

Figure 4.7: Economies of scale



Expanding the factory cannot guarantee that the cost will be lower. The figure here shows that average cost in the long-run has the same U-shape as the short-run. Therefore, there are production ranges that expanding can both reduce the average cost (economies of scale) and raise the average cost (diseconomies of scale).

A firm gains cost advantage or economies of scale when the quantity produced is 'not too much' depending on each type of good or service, is a result from specialization, bulk order and other efficiency such as low search cost for material and specialized labor.

However, when firm produces 'too much', the opportunity cost can become higher such as search cost for labor and capital, cost of management, cost of monitoring, etc. This is the phase when a firm loses its advantage or diseconomies of scale.

(2) Long-run costs

Summary of long-run production

(1) Long-run production ($K, L \Rightarrow q$) means all factors are variable.

(2) To come up with long-run cost curves, it is based on a firm's decision to minimize cost for each level of q by choosing (K^*, L^*)

(3) Marginal and average long-run cost curves ($q \Rightarrow c$) still rise even in the long-run production due to economies of scale.

(4) Returns to scale and economies of scale are related.