

1. In Table 1,  $X_i$  is total econometrics exam point (total points are 100) and  $Y_i$  is GPA of each BE student.

Table 1

Student	$Y_i$	$X_i$	$X_i Y_i$
1	2.8	63	176.4
2	3.4	72	244.8
3	3.0	78	234
4	3.5	81	283.5
5	3.6	87	313.2
6	3.0	75	225
7	2.7	75	202.5
8	3.7	90	333

$$\sum X_i Y_i = 2012.4$$

$$\sum X_i = 621$$

$$\sum X_i^2 = (63)^2 + (72)^2 + (78)^2 + (81)^2 + (87)^2 + (75)^2 + (75)^2 + (90)^2 = 48717$$

$$\sum Y_i = 25.7$$

1.1 Now consider the two-variable model  $Y_i = \beta_1 + \beta_2 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of  $\beta_1$  and  $\beta_2$ . Interpret the regression.

1.2 Find  $\hat{Y}_i$  and  $\hat{u}_i$  and show that  $\sum_{i=1}^n \hat{u}_i \approx 0$

1.3 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_1)$ , and  $var(\hat{\beta}_2)$

1.1) Find  $\bar{X} = \frac{63+72+78+81+87+75+75+90}{8} = 77.625$

$$\bar{Y} = \frac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8} = 3.2125$$

Find  $\hat{\beta}_1$

$$\begin{aligned} \hat{\beta}_1 &= \bar{Y} - \beta_2 \bar{X} \\ &= 3.2125 - \beta_2 (77.625) \\ \hat{\beta}_1 &= 3.2125 - 77.625 (\beta_2) \quad \text{--- (1)} \end{aligned}$$

substitute (1) in (1)

$$\begin{aligned} \hat{\beta}_1 &= 3.2125 - 77.625 (0.0341) \\ &= 0.5655 \end{aligned}$$

Find  $\hat{\beta}_2$

$$\begin{aligned} \hat{\beta}_2 &= \frac{n \sum u_i Y_i - \sum u_i \sum Y_i}{n \sum u_i^2 - (\sum u_i)^2} \\ &= \frac{8(2012.4) - (621)(25.7)}{8(48717) - (621)^2} \\ &= \frac{16099.2 - 15959.7}{389736 - 385641} = 0.0341 \end{aligned}$$

$$\therefore \hat{\beta}_1 = 0.5655$$

$$\hat{\beta}_2 = 0.0341$$

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1.2)  $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + u_i$

From  $\hat{\beta}_1 = 0.5655$   
 $\hat{\beta}_2 = 0.0341$

$\hat{Y}_i = 0.44905 + 0.0356 X_i + u_i$

From  $\hat{Y}_i = \beta_1 + \beta_2 X_i$

Find  $Y_i$  :

$i=1, 0.5655 + 0.0341(63) = 2.7138$

$i=2, 0.5655 + 0.0341(72) = 3.0207$

$i=3, 0.5655 + 0.0341(78) = 3.2253$

$i=4, 0.5655 + 0.0341(81) = 3.3276$

$i=5, 0.5655 + 0.0341(87) = 3.5322$

$i=6, 0.5655 + 0.0341(75) = 3.123$

$i=7, 0.5655 + 0.0341(75) = 3.123$

$i=8, 0.5655 + 0.0341(90) = 3.6345$

Find  $\hat{u}_i$ , From  $Y_i = \hat{u}_i + \hat{Y}_i \rightarrow \hat{u}_i = Y_i - \hat{Y}_i$

$i=1, 2.8 - 2.7138 = 0.0862$

$i=2, 3.4 - 3.0207 = 0.3793$

$i=3, 3.0 - 3.2253 = -0.2253$

$i=4, 3.5 - 3.3276 = 0.1724$

$i=5, 3.6 - 3.5322 = 0.0678$

$i=6, 3.0 - 3.123 = -0.123$

$i=7, 2.7 - 3.123 = -0.423$

$i=8, 3.7 - 3.6345 = 0.0655$

$$\sum_{i=0}^n \hat{u}_i = 0.0862 + 0.3793 + (-0.2253) + (0.1724) + (0.0678) + (-0.123) + (-0.423) + 0.0655 = 0$$

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1.3 Find  $var(\hat{u}_i), var(\hat{\beta}_1),$  and  $var(\hat{\beta}_2)$

$$1.9) \quad var(\hat{u}_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4947}{8-2} = 0.0725$$

$$var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum X_i^2} \sigma^2 = \frac{621}{511.875} \cdot (0.0725) = 0.0880$$

$$var(\hat{\beta}_2) = \frac{\sigma^2}{\sum X_i^2} = \frac{0.0725}{511.875} = 0.0001416$$

2. Data is listed in the table

$X_i$	$Y_i$
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model  $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$   
Find estimators of  $\beta_1$  and  $\beta_2$  from the OLS method and interpret the meaning.

2.2 Find the value of  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass  $(\bar{X}, \bar{Y})$ ?

2.4 If  $X_i = 18$ , what is the predicted  $Y$ ?

2.5 Find  $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

$$2.1) \quad \bar{X} = (10+12+14+16+18+22+24+26+28+30) \div 10 = 20$$

$$\bar{Y} = (0+2+5+6+7+10+10+15+16+20) \div 10 = 9.1$$

$$\sum X_i^2 = (10)^2 + 12^2 + 14^2 + \dots + 30^2 = 4440$$

$$\sum X_i Y_i = 10(0) + 12(2) + 14(5) + 16(6) + 18(7) + 22(10) + 24(10) + 26(15) + 28(16) + 30(20) = 2214$$

$$\hat{\beta}_2 = \frac{\sum X_i Y_i - n \bar{X} \bar{Y}}{\sum X_i^2 - n \bar{X}^2} = \frac{2214 - 10(20)(9.1)}{4440 - 4000} = 0.8954$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - 0.895(20) = -8.808$$

$$2.2) \quad \hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i, \text{ Find } \hat{Y}_i$$

$$\text{When } i=1, \quad -8.808 + 0.8954(10) = 0.1455$$

$$i=2, \quad -8.808 + 0.8954(12) = 1.9364$$

$$i=3, \quad -8.808 + 0.8954(14) = 3.7273$$

$$i=4, \quad -8.808 + 0.8954(16) = 5.5182$$

$$i=5, \quad -8.808 + 0.8954(18) = 7.3091$$

$$i=6, \quad -8.808 + 0.8954(22) = 10.8904$$

$$i=7, \quad -8.808 + 0.8954(24) = 12.6813$$

$$i=8, \quad -8.808 + 0.8954(26) = 14.4722$$

$$i=9, \quad -8.808 + 0.8954(28) = 16.2636$$

$$i=10, \quad -8.808 + 0.8954(30) = 18.0549$$

Find  $\hat{u}_i$

$$\text{When } i=1, \quad \hat{u}_1 = -0.145$$

$$i=2, \quad \hat{u}_2 = 0.064$$

$$i=3, \quad \hat{u}_3 = 1.273$$

$$i=4, \quad \hat{u}_4 = 0.482$$

$$i=5, \quad \hat{u}_5 = -0.304$$

$$i=6, \quad \hat{u}_6 = -0.891$$

$$i=7, \quad \hat{u}_7 = -2.682$$

$$i=8, \quad \hat{u}_8 = 0.527$$

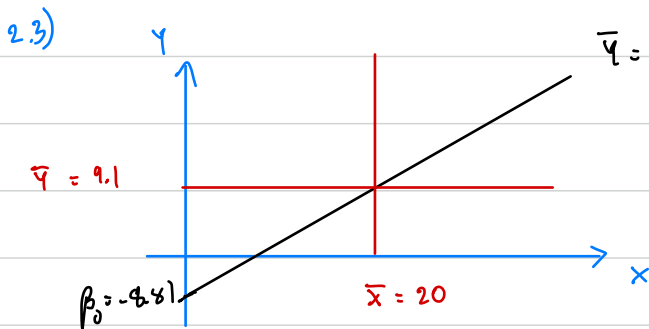
$$i=9, \quad \hat{u}_9 = -0.264$$

$$i=10, \quad \hat{u}_{10} = 1.945$$

$$\sum_{i=1}^{10} \hat{u}_i = (-0.145) + 0.064 + 1.273 + 0.482 + (-0.304) + (-2.682) + 0.527 - 0.264 + 1.945 = 0$$

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$$\bar{Y} = -8.81 + 0.895(20)$$

$$9.1 = 9.1$$

 $\therefore$  The line pass through  $(\bar{X}, \bar{Y})$ 

2.4)

$$Y_i = \beta_0 + \beta_1 X_i$$

$$= -8.81 + 0.895(16)$$

$$= 5.51$$

2.5)

$$Var(\hat{u}_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.091}{8} = 1.7614$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum (n_i - \bar{n})^2} = \frac{1.7614}{440} = 0.0040$$

$$Var(\hat{\beta}_2) = \frac{\sum n_i^2 (\sigma^2)}{n \sum (n_i - \bar{n})^2} = \frac{440(1.7614)}{10(440)} = 1.7774$$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim \text{NIID}(0, \sigma^2)$$

Find an OLS estimator of  $\beta_1$ . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$\rightarrow Y_i = \beta_1 + \beta_2 X_i + u_i$  are suit in CLRM assumption 1, linear regression model

Since  $\hat{\beta}_1$  is unbiased estimator,  $E(\hat{\beta}_1) = \beta_1$

$$\text{From } \hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} \quad \text{--- ①}$$

$$\hat{\beta}_2 = \frac{\sum x_i Y_i}{\sum x_i^2} \quad \text{Given } k = \frac{x_i}{\sum x_i^2}$$

Substitute  $k$  in ①

$$\hat{\beta}_1 = \bar{Y} - \sum k Y_i \bar{X} \quad \text{--- ②}$$

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\text{From ② } \hat{\beta}_1 = \frac{\sum Y_i}{n} - \sum k Y_i \bar{X} = \sum \left( \frac{1}{n} - k \bar{X} \right) Y_i$$

$$= \sum \left( \frac{1}{n} - k \bar{X} \right) (\beta_1 + \beta_2 X_i + u_i)$$

$$= \sum \left( \frac{\beta_1}{n} - \beta_1 k \bar{X} + \frac{\beta_2 X_i}{n} - \beta_2 k \bar{X} + \frac{u_i}{n} - u_i k \bar{X} \right)$$

$$= \sum \left( \frac{\beta_1}{n} + \frac{\beta_2 X_i}{n} + \frac{u_i}{n} - \beta_1 k \bar{X} - \beta_2 k \bar{X} - u_i k \bar{X} \right)$$

$$= \sum \frac{\beta_1}{n} + \beta_2 \frac{\sum x_i}{n} + \frac{\sum u_i}{n} - \bar{X} \beta_1 \sum k_i - \beta_2 \sum k_i \bar{X} - k \bar{X} \sum u_i$$

$$\hat{\beta}_1 = \beta_1 + \beta_2 \bar{X} - \frac{\sum u_i}{n} - \beta_2 \bar{X} - \bar{X} \sum k_i u_i$$

$$\hat{\beta}_1 = \beta_1 - \frac{\sum u_i}{n} - \bar{X} \sum k_i u_i \quad \text{--- ③}$$

Add expected value to the equation ③

$$E(\hat{\beta}_1) = E(\beta_1) + \bar{X} E(\sum k_i u_i)$$

$\rightarrow$  From Assumption 3, Zero mean value of disturbance  $u_i$ ; treat as given

$$E(u_i | X_i)$$

$$E(\hat{\beta}_1) = E(\beta_1) + \bar{X} \sum k_i E(u_i) \stackrel{=0}{}$$

$$E(\hat{\beta}_1) = \beta_1$$

Therefore  $\hat{\beta}_1$  is an unbiased estimator of  $\beta_1$

$$\text{From Proof: } \sum_{i=1}^n k_i = 0$$