Chapter 11 Consumption: Utility, Indifference Curve and Budget Line

Economics
- To do your best under some constrai

Consumer's Problem The consumer wants to maximize satisfaction (utility) by deciding what and how much to consume under a limitation of income

max satisfaction
under income limita

2 Types of Utility

1. Cardinal Utility—the consumer assigns a numerical value to denote his level of satisfaction. For example, a noodle for lunch gives the consumer a utility of 3 utils (units of utility).

This numerical value of 3 is entirely up to the consumer. He can assign value of 30, 300, or 17. However, if a pizza gives him twice the satisfaction he gets from noodle, then he will assign a number double that given to the noodle.

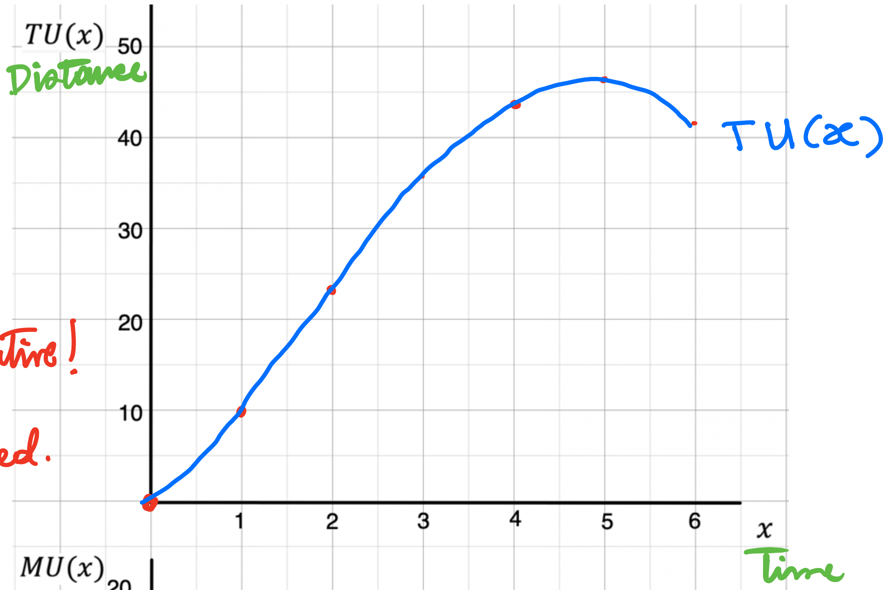
A total utility of consuming an increasing quantity of a product always increases at the beginning. Beyond a quantity, the total utility will decrease.

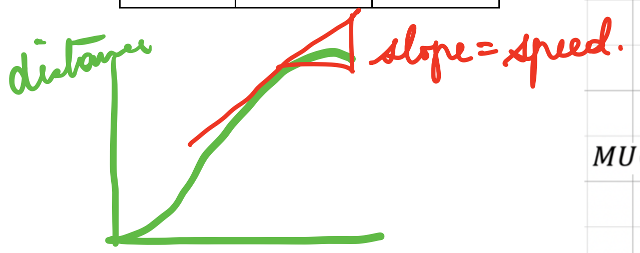
$$TU(x) = Total\ Utilty\ from\ consuming\ x\ units$$
$$MU(x) = Marginal\ Utilty\ from\ consuming\ x\ units$$
$$= the\ change\ of\ TU\ per\ 1\ unit\ change\ of\ x$$

If the consumer consumes 1, 2, 3, … units of x, we may have an example of $TU(x)$ and $MU(x)$ as given in the following table

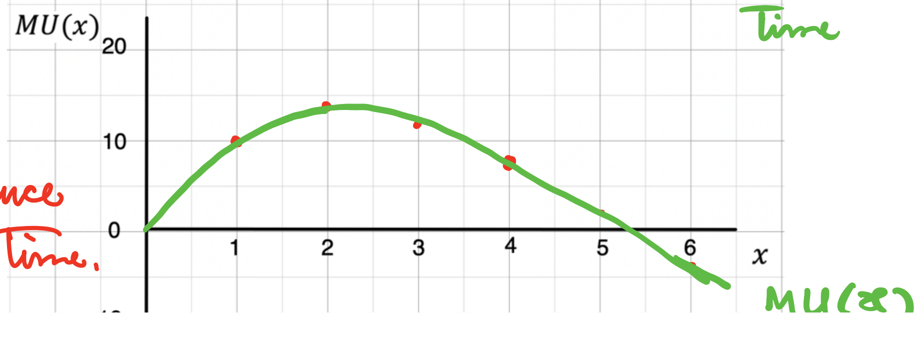
x	$TU(x)$	$MU(x)$
0	0	
1	10	$\boxed{10}$
2	24	14
3	36	12
4	44	8
5	46	2
6	42	-4

negative!





speed = rate of change of distance
per 1 unit change in time.

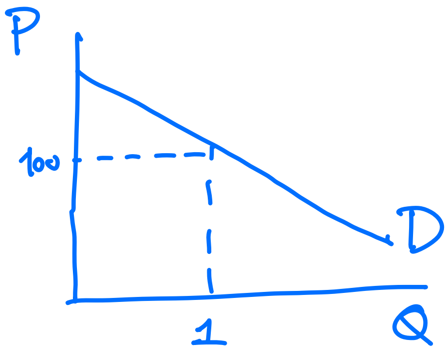


If we know speed. with Δt = change in time ΔD = change in distance $\approx$ speed $\cdot \Delta t$.

If the consumer is allowed to consume any decimal of x,

$$\begin{aligned} MU(x) &= marginal\ utilty\ from\ consuming\ x\ units \\ &= rate\ of\ change\ of\ TU(x)\ per\ unit\ of\ x \\ &= \frac{d}{dx}TU(x) \end{aligned}$$

After certain amount of consumption x, the Marginal Utility of x always is decreasing. This is one explanation why the Demand curve has a negative slope. To induce the consumer to buy the another unit, the price has to be lowered.



Given the value of $MU(x)$ is known, if we allow a change in $x = \Delta x$, then we can approximate the change in $TU(x)$ as follows:

$$\boxed{\Delta TU(x) \approx MU(x)\Delta x}$$

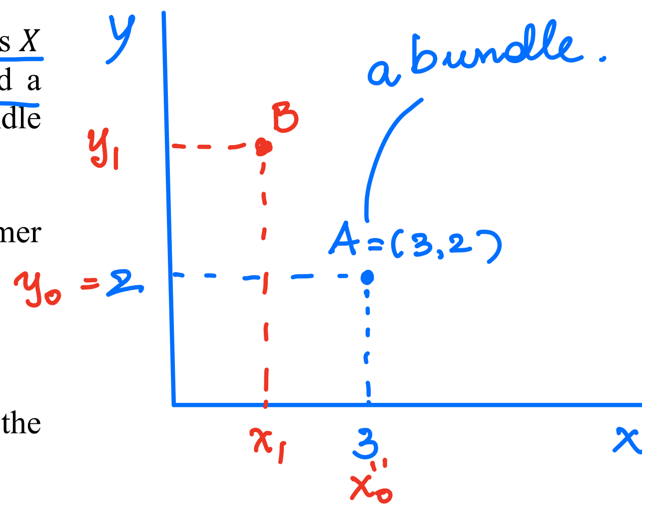
* — to be used later!

For example, when $x_0 = 10$, we have $MU(x) = 12$. If we increase x by $\Delta x = 0.1$, then TU will change approximately by

$$\begin{aligned} \Delta TU(x) &\approx MU(x)\Delta x \\ &= 12(0.1) = 1.2. \end{aligned}$$

- The eventual decline of Marginal Utility can explains the Law of Demand.

2. Ordinal Utility Assume the consumer has 2 products X and Y to consume. A ***bundle*** is a quantity of X and a quantity of Y to be consumed by a consumer. A bundle (x_0, y_0) is represented by a point in the XY plane.



Ordinal utility approach assumes that the consumer is 'rational' according to the following assumptions:

Assumptions:

1) Given two bundles $A = (x_0, y_0)$ and $B = (x_1, y_1)$, the consumer can say one of these:

$\succ$

1) $A \succ B$ (A is preferred to B)
2) $B \succ A$ (B is preferred to A)
3) $A \sim B$ (A and B are indifferent)(

Equivalently, we can say:

$\succsim$

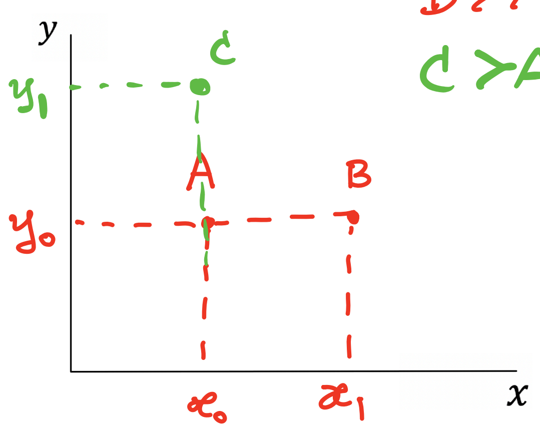
1) $A \succcurlyeq B$ (A is not less preferred to B)
2) $B \succcurlyeq A$ (B is not less preferred to A)

3) $A \sim B$ (A and B are indifferent)

2. More is always better

$B \succ A$. (or $A \prec B$)

$C \succ A$.

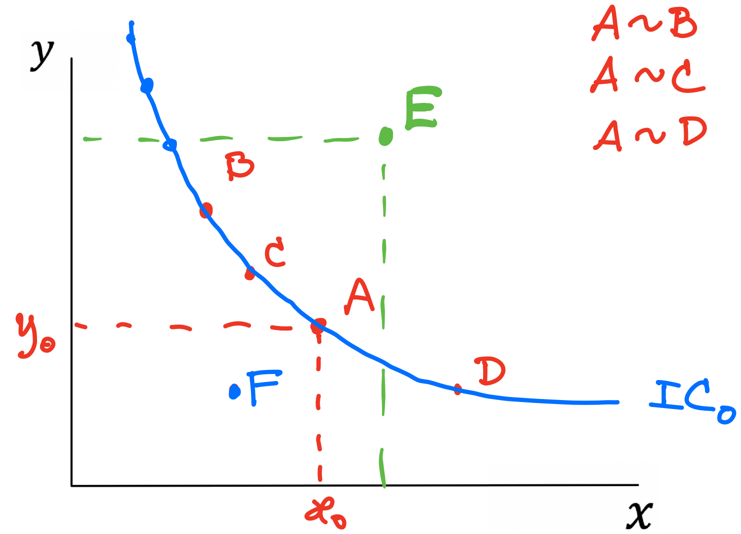


3. Transitivity: If $A \succ B$, and $B \succ C$, then $A \succ C$.

$A \succsim B$ $B \succsim C$ $A \succsim C$

4. Given a bundle A, the consumer can tell all other bundles that are equally preferred. This creates an Indifference Curve (IC) passing through point A.

$A \sim B$
$A \sim C$
$A \sim D$



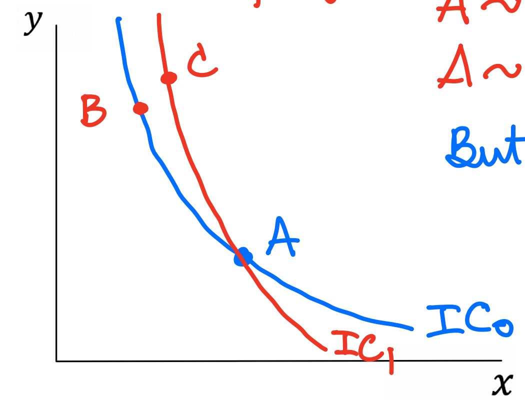
Indifference Curve (IC) is a curve whose every point gives the same satisfaction (utility) to the consumer.

- An IC separates the graph into 3 parts
  a) All the bundles ***on*** the IC
  b) All the bundles ***above*** the IC — $E \succ A$ $E \succ D$? — Yes! By transitivity!
  c) All the bundles ***below*** the IC $A \succ F$ → every point on IC_0 is preferred to F

Properties of Indifference Curves

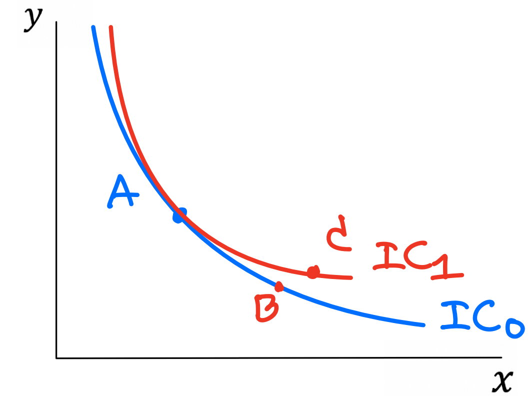
1. For any bundle $A = (x_0, y_0)$, there is exactly one IC passing through it. That is, no two IC's can intersect nor be tangent to each other.

If they intersect



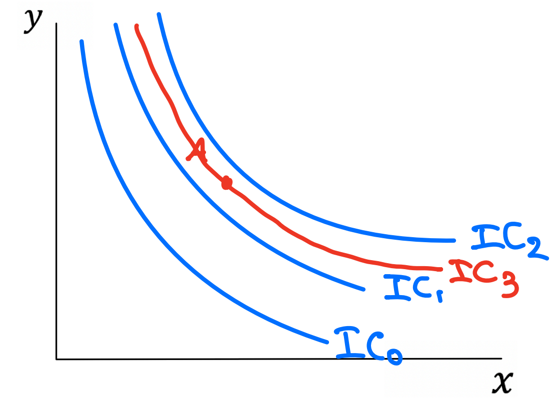
$A \sim B$ - on IC_0
$A \sim C$ - on IC_1
But $B \prec C$

} violate transitivity

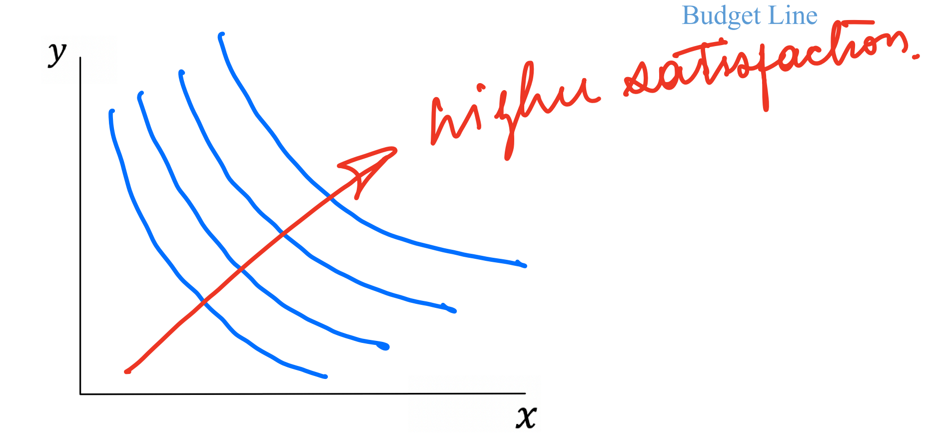


IC_0 and IC_1 are tangent and touch each other at A.

2. There are infinite number of IC's, each never intersect nor is tangent to another

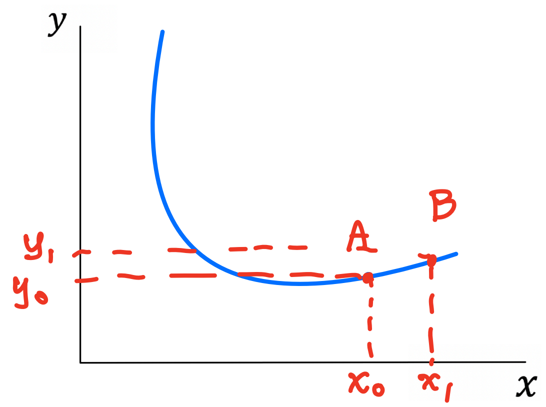


3. Higher IC means higher satisfaction. — each higher IC the consumer can climb up, $\Rightarrow$ higher satisfaction



4. Each IC always has negative slope

- What if there is a part of an IC with positive slope?

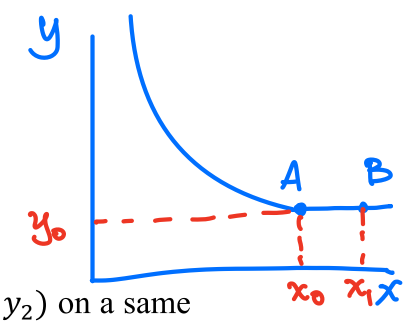


$B \succ A$

But B and A are on same IC
$B \sim A$

$\therefore$ contradiction!

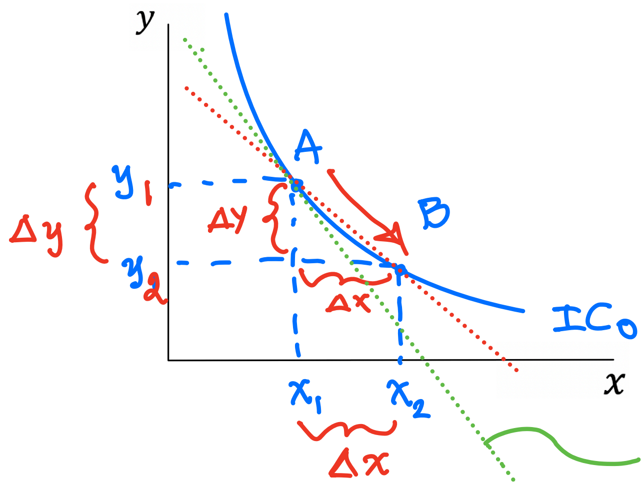
- Can an IC have zero slope?



No! because B must be on a higher IC as B has more of x

- If we move from $A = (x_1, y_1)$ to $B = (x_2, y_2)$ on a same IC, there is

- an increase in $x = x_2 - x_1 = \Delta x > 0$
- a decrease in $y = y_2 - y_1 = \Delta y < 0$



slope of IC_0 at B

$$= -\frac{MU_x(x_2, y_2)}{MU_y(x_2, y_2)}$$

slope at A is

$$\frac{dy}{dx} = -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}$$

The ratio $\frac{\Delta y}{\Delta x}$ is the exchange rate the between x and y with no change in the satisfaction level.

at A

Δx causes an increase in $TU \approx \; MU_x(x_1, y_1)\Delta x$ +
Δy causes a decrease in $TU \approx \; MU_y(x_1, y_1)\Delta y$ −

With total change being zero, we have

$$MU_x(x_1, y_1)\Delta x + \; MU_y(x_1, y_1)\Delta y \approx 0$$
$$MU_y(x_1, y_1)\Delta y \approx -MU_x(x_1, y_1)\Delta x$$
$$\frac{\Delta y}{\Delta x} \approx -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}$$

When we move from A to B that is very close together. So close that $\Delta x \to 0$, and we have the slope of IC at A as

$$\boxed{\frac{dy}{dx} = -\frac{MU_x(x_1, y_1)}{MU_y(x_1, y_1)}}$$

- At point A the slope can be found by drawing a line tangent to A. The slope of IC at A is equal to the slope of the tangent line.
- Slope at point $A = (x_1, y_1)$ is called the **Marginal Rate of Substitution** (MRS). It is the **instantaneous exchange rate** between x and y such that **in the mind of the consumer** his satisfaction (utility) is unchanged.

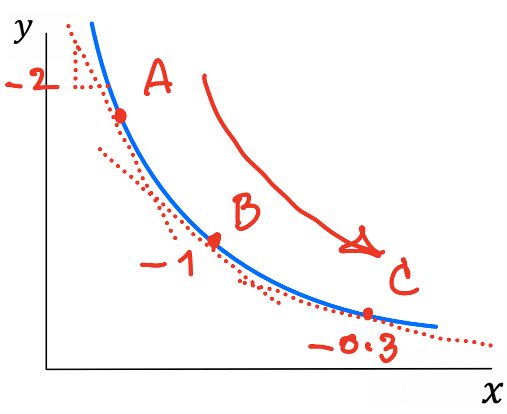
if MRS = −1 exchange rate in the mind of consumer is 1 to 1

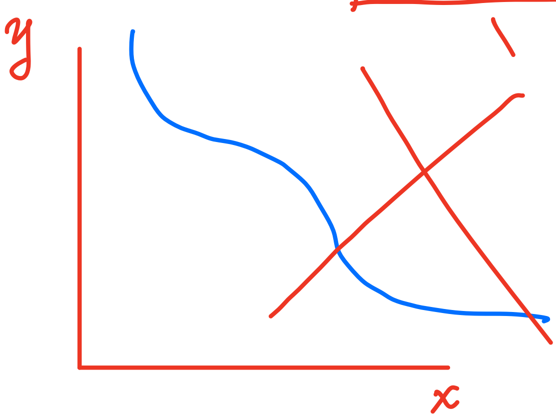
- $\boxed{MRS = -\frac{MU_x(x_1,y_1)}{MU_y(x_1,y_1)}}$ — negative.

if MRS = −2. → for 1 of x willing to sacrifice 2 of Y.

- This leads to the 5th property of IC (Assumption)

5. Diminishing MRS On a given IC, as the consumption of x increases, the value of $|MRS|$ decreases





Consumption Problem: maximizing satisfaction under a budget constraint
(income)

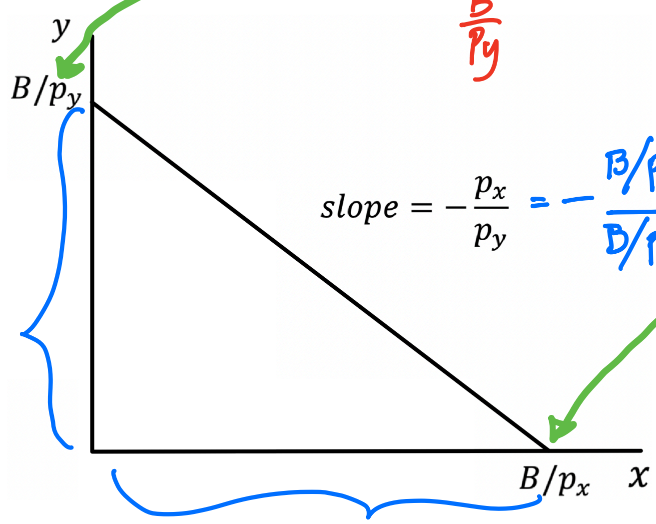
Budget Line A consumer is assumed to have a fixed income or budget B that he can buy x and y at fixed prices p_x and p_y, respectively. The equation for budget line is given by,

amt. of x bought

$$p_x x + p_y y = B$$

if $x = 0$, $y = \frac{B}{P_y}$

if $y = 0$, $x = \frac{B}{P_x}$



$$slope = -\frac{p_x}{p_y} = relative\ price\ of\ x\ interms\ of\ y$$
$$= exchange\ rate\ between\ x\ and\ y\ in\ the\ market$$

slope $= -\frac{P_x}{P_y} = -2 = -\frac{10}{5}$

the consumer can get 1 more of x by exchanging 2 unit of y.

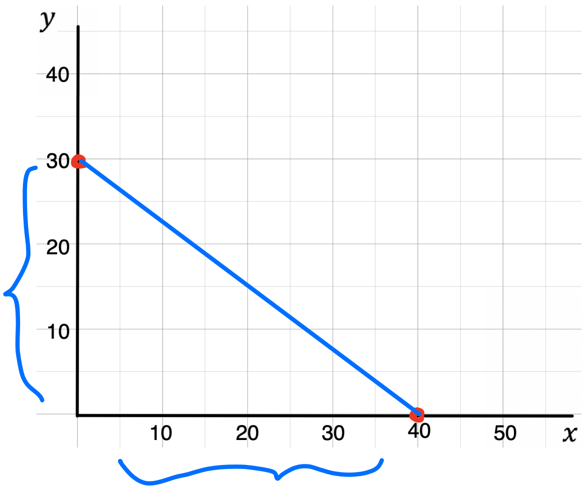
- Every point on the budget line is a bundle the consumer can afford.

Example: $p_x = 3, p_y = 4, B = 120$

$\frac{B}{P_y} = \frac{120}{4} = 30$

$\frac{B}{P_x} = \frac{120}{3} = 40$

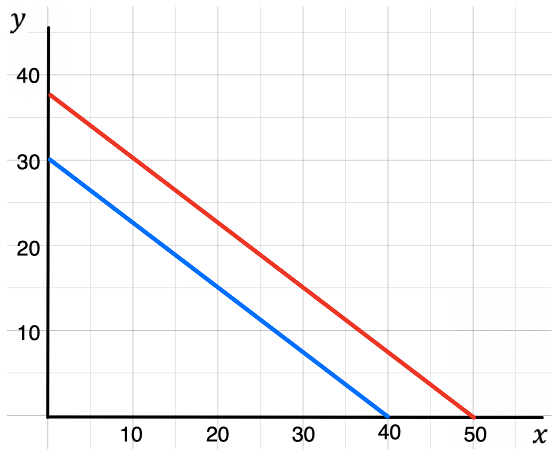
slope $= -\frac{30}{40} = \frac{-P_x}{P_y}$



$$slope = -\frac{p_x}{p_y} = -\frac{30}{40} = \frac{-P_x}{P_y} = -\frac{3}{4}$$

Changes of Budget Line

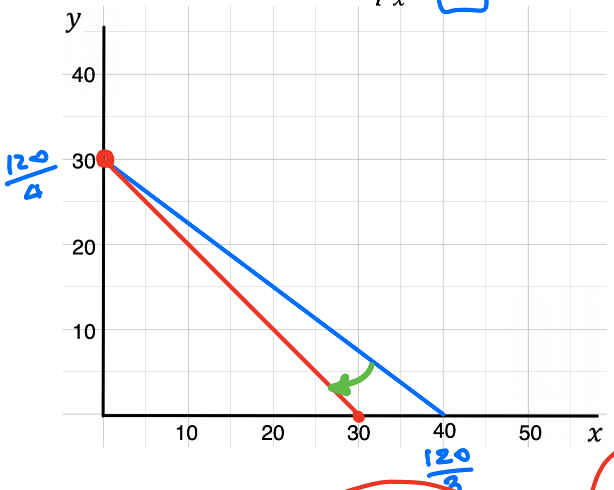
1. Income increases: Income increases from, $B = 120$ to $B' = 150$.



but slope remains unchanged.

$$slope = -\frac{P_x}{P_y}$$

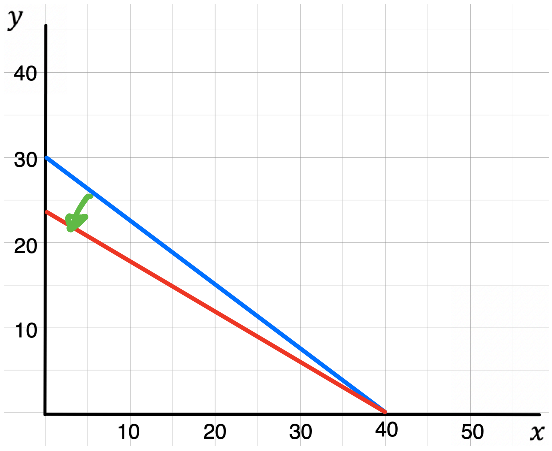
2. Price of x increases $p_x = \boxed{3}$ increases to $p_x = \boxed{4}$ → causing the budget line to rotate inward, with y-intercept unchanged.



new slope is $-\frac{P_x'}{P_y} = -\frac{4}{4} = -1$

– steeper

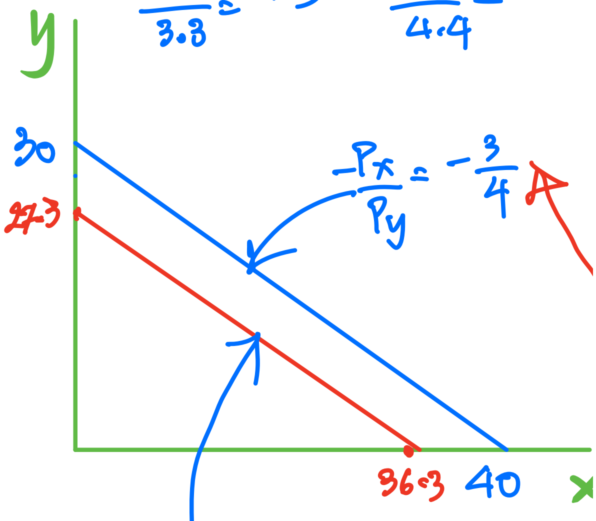
Price of y decreases $p_y = 4$ increases to ~~$p_x = 3$~~ $P_y = 5$



$P_x = 3, \quad P_y = 4.$

$P_x' = 3.3, \quad P_y' = 4.4$

$\frac{120}{3.3} = 36.3 \qquad \frac{120}{4.4} = 27.3$



$-\frac{P_x}{P_y} = -\frac{3}{4}$

$-\frac{3.3}{4.4} = -\frac{3}{4}$ – same!

- In Economics, only one thing is allowed to change at a time. That is, either income or price of a product changes.
- If we have inflation so that the prices p_x and p_y increases at the same percentage of 10%, what is the effect on the budget line?
- If we have 10% inflation and at the same time income also increase by 10%, how will the budget line change?

$\cancel{1.1} P_x x + \cancel{1.1} P_y y = \cancel{1.1} B.$