



B.E. International Program

Faculty of Economics, Thammasat University



EE 320 Introductory Mathematical Economics

Semester 2/2013

Practice Problem 5 (Differential Calculus)

Suggested Answers

1. Exercise 7.1 questions 2 and 3(a, c, d) in Chiang and Wainwright (2005)

- Ans. Q.2 (a) $4x^{-5}$ (b) $3x^{-2/3}$ (c) $20w^3$
 (d) $2cx$ (e) abu^{b-1} (f) abu^{-b-1}
- Q.3 (a) $f'(x) = 18; f'(1)=f'(2)=18$.
 (c) $f'(x) = 10x^{-3}; f'(1)=10; f'(2) = 1/4$
 (d) $f(x) = x^{1/3}; f'(1) = 1; f'(2) = 2^{1/3}$

2. Exercise 7.2 questions 1, 2, 4, 7, and 10 in Chiang and Wainwright (2005)

Ans. Q.1 $VC = Q^3 - 5Q^2 + 12Q; \quad \frac{dVC}{dQ} = MC = 3Q^2 - 10Q + 12$

Q.2 $C = AC \cdot Q = Q^3 - 4Q^2 + 174Q; \quad \frac{dC}{dQ} = MC = 3Q^2 - 8Q + 174$

This is a long-run cost function because the fixed cost is zero.

Q.4 (b) $= AR \cdot Q = 60Q - 3Q^2; MR = \frac{dR}{dQ} = 60 - 6Q$

(c) Yes.

(d) The MR curve is twice as steep as the AR curve.

Q.7 (a) $\frac{(x^2-3)}{x^2}$

(b) $-\frac{9}{x^2}$

(c) $\frac{30}{(x+5)^2}$

(d) $\frac{acx^2+2adx-bc}{(cx+d)^2}$

Q.10 (a) $MC = 6Q + 7$; $AC = 3Q + 7 + \frac{12}{Q}$

(b) $MR = 10 - 2Q$ $AR = 10 - Q$

(c) $MP = a + 2bL - cL^2$ $AP = a + bL - cL^2$

3. Exercise 7.3 questions 1, 3, and 4 in Chiang and Wainwright (2005)

Ans. Q.1 $\frac{dy}{dx} = -2x[3(5 - x^2)^2 + 2]$

Q.3 (a) $\frac{dy}{dx} = 18x(3x^2 - 13)^2$

(b) $\frac{dy}{dx} = 189x^2(7x^3 - 5)^8$

(c) $\frac{dy}{dx} = 5a(ax + b)^4$

Q. 4 Both methods yield the same answer $\frac{dy}{dx} = -32(16x + 3)^{-3}$

(Questions 4 – 5 are from Dowling, 2012)

4. Find the marginal and the average functions for each of the following total functions.

a) $TC = 3Q^2 + 7Q + 12$

Ans. $MC = 6Q + 7$; $AC = 3Q + 7 + \frac{12}{Q}$.

b) $\pi = Q^2 - 13Q + 78$

Ans. $\frac{d\pi}{dQ} = 2Q - 13$; $\frac{\pi}{Q} = Q - 13 + \frac{78}{Q}$

5. Given $C = 1200 + 0.8Y_d$, $Y_d = Y - T$, and $T = 100$, use the derivative find the marginal propensity to consume (MPC).

Ans. $MPC = 0.8$

6. Differentiate the following functions:

a) $y = \frac{x^2 - 9x + 20}{x - 5}$

Ans. $\frac{dy}{dx} = 1, x \neq 5$

b) $y = \sqrt{x + \sqrt{x}}$

Ans. $\frac{dy}{dx} = \frac{1}{2(x+\sqrt{x})} \left(1 + \frac{1}{2\sqrt{x}}\right) = \frac{\sqrt{x}+1}{2(x\sqrt{x}+x)}$

c) $y = \ln(e^x + x)$

Ans. $\frac{dy}{dx} = \frac{e^x+1}{e^x+x}$

d) $y = x^4 e^x$

Ans. $\frac{dy}{dx} = x^3 e^x (x + 4)$

7. Given the total-product function:

$$Q = 2L + 5L^2 - L^3$$

a) Find the average product (AP) function.

Ans. $AP(L) = 2 + 5L - L^2$

b) Find the marginal product (MP) function.

Ans. $MP(L) = 2 + 10L - 3L^2$

c) Determine the slopes of the AP and MP functions. What can you conclude about their relative slopes?

Ans. $\frac{d(AP)}{dL} = 5 - 2L$; $\frac{d(MP)}{dL} = 10 - 6L$

The MP curve is twice as steep as the AP curve.