

Quiz 2: Date: May 5, 2022 from 11.00-12.30

Question 1 (40 marks)

Score.....

Consider the Muliperiod model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$\max_{C_s, \omega_s, \forall t} E_t \left[\sum_{s=t}^{T-1} \delta^s \left(\frac{C_s^{(1-\gamma)}}{1-\gamma} \right) + \delta^T \left(\frac{W_T^{(1-\gamma)}}{1-\gamma} \right) \right]$$

Assume that there is no wage income ($y_t = 0 \forall t$) and a constant risk-free rate return asset , $R_{ft} = R_f$. Also assume that $n=1$ and the return of a single risky asset, R_{rt} , is independently and identically distributed over time. Denote the proportion of wealth invested in the risky asset at date t as ω_t .

Please read and answer the following questions carefully and completely.

Score.....

Question 1.1 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-1, C_{T-1}^* and ω_{T-1}^* , and give an explicit expression for C_{T-1}^*

Score.....

Question 1.2 (10 marks) Solve for the form of $J(W_{T-1}, T-1)$.

Score.....

Question 1.3 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-2, C_{T-2}^* and ω_{T-2}^* , and give an explicit expression for C_{T-2}^*

Score.....

Question 1.4 (10 marks) Solve for the form of $J(W_{T-2}, T-2)$. Based on the pattern for $T-1$ and $T-2$, provide expressions for the optimal consumption and portfolio weight at any date $T-t$, $t=1, 2, 3, \dots$