RISK AND TERM-STRUCTURE OF INTEREST RATE

EE431 Semester 1/2017

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Theories to account for the pattern and behavior of yield curve, rates that vary with TTM…

- **Risk Structure of Interest rate**

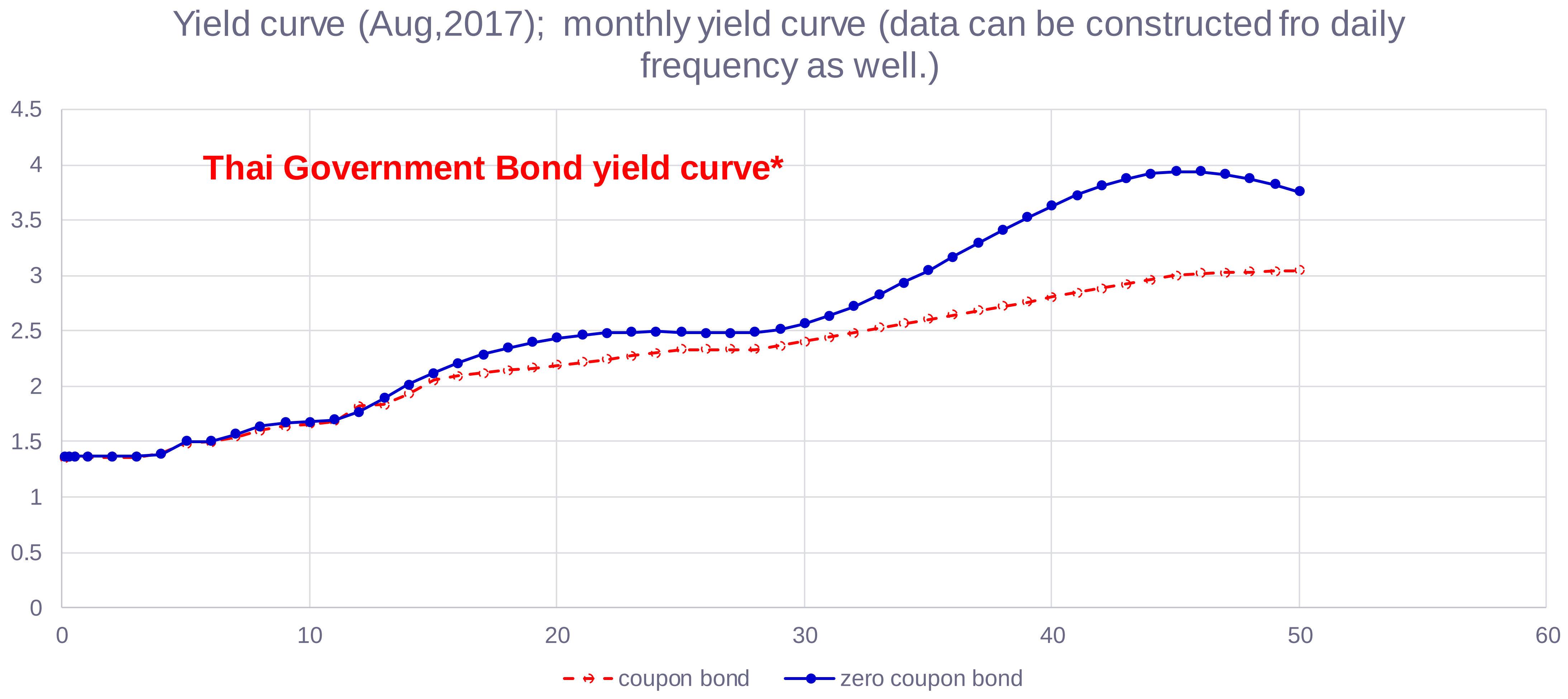
- **Term Structure of Interest rate**

Term Structure of Interest Rate

- Interest rates vary with *term/time to maturity, i.e. remaining periods to maturity date.*

- Explaining about the *dynamic of yield curve*.
  - How yield curve gets move overtime.

Observation 2: Yield across TTM (Yield curve)



Get to know the definition: Coupon yield *v.s.* Zero-coupon yield

- Two types of yield curve: *coupon yield* and *zero-coupon yield*.
  - The latter is called yield curve for spot rates.

- The type of bond that is mostly traded is the coupon bond.
  - Thus, yield of coupon bond, aka coupon yield (not the same as coupon rate!), can be simply constructed from the bond pricing formula.
  - That is, coupon yield is the implied discount rate that sets market price equal to the sum of present value of all remaining cash-flow of the bond with j-year remained before maturity.

Coupon yield v.s. Zero-coupon yield

- On the other hand, zero-coupon bond is normally observable in the short-term market, i.e. TTM less than 1 year.
- We do not in fact have data for Long-term yield of zero coupon bond rate.
  - No firm would borrow by issuing 15-year zero-coupon bond.
- Then, how do we construct the zero-coupon yield?
  - Market data of zero-coupon yield is then *constructed and statistically* ***inferred*** from the using theoretical restrictions imposed on the relationship among prices of coupon bonds, coupon yields and spot rates.

Zero coupon yield and Coupon-bond yield

- Consider a simple example with 1-year bond and 2-year bond, where i_1 and i_2 represent the yield of 1-year and 2-year coupon bond.

$$P_{t=0}^{1} = \frac{CF_1^1}{1 + i_1} \ and$$

$$P_{t=0}^{2} = \frac{CF_1^2}{1 + i_2} + \frac{CF_2^2}{(1 + i_2)^2}$$

Zero coupon yield and Coupon-bond yield

- Now, introduce $\{s_1, s_2\}$ as the corresponding zero-coupon yield.
  - s_j = Discount rate that is specifically used for calculating the present value of the cash flow that will be receiving at the period (year) j-th.
- 1-year bond = 1-year zero-coupon bond with its face value of CF_1^1.
- 2-year bond = combination of (i) 1-year zero-coupon bond CF_1^2 and (ii) 2-year zero-coupon bond with CF_2^2.
- By principle of asset valuation#2, we know that

$$P_{t=0}^1 = \frac{CF_1^1}{1+s_1} \; and \; P_{t=0}^2 = \frac{CF_1^2}{1+s_1} + \frac{CF_2^2}{(1+s_2)^2}$$

Term Structure of Interest Rate

- Interest rates vary with term/time to maturity
- Explaining about the dynamic of yield curve.
- What important features of yield curve do we try to explain?
  - Three stylized-facts for the yield curve

Three stylized-facts for yield curve

- **Fact 1:** Yields at different TTM co-move.

- **Fact 2:** If short-term yield is (exceptionally) low (high), yield curve tends to be *steep* upward (downward) sloping.
  - Upward sloping ➔ SR yield $<$ LR yield
  - Downward sloping ➔ SR yield $>$ LR yield

- **Fact 3:** ***Normally,*** yield curve is upward sloping.

How to Rationalize Stylized Facts?

- *Expectations Theory*
- *Segmented Markets Theory*
- *Liquidity Premium (Preferred Habitat) Theory*

Remark!

- Expectations Theory: 1&2, but not 3
- Segmented Markets: 3, but not the first two.
- *Preferred Habitat Theory: Expected theory + Segmented market → 1&2&3 can be explained.*

How to Rationalize Stylized Facts?

- ***Expectations Theory***
- *Segmented Markets Theory*
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Expectation Hypothesis: Assumption

- Investors are *risk neutral.*
- *Bonds can be perfectly substituted.*

What do they imply?

- *#1 → only maximize the expected pay-off (do not care about risk!)*
- *#2 → no biased decision.*

Expectation Hypothesis: the prediction under the simplest version

TTM = 1 year

$$i_{t=0,2} = \frac{i_{t=0,1} + i^e_{t=1,1}}{2}$$

Yield of 2-year bond = average of yield of 1-year bond over 2 years

TTM 2 year

Expected yield of 1-year bond at time “t”, (in the next year)

Expectation Hypothesis: Derivation/Intuition

- Suppose a representative risk-neutral investor consider an investment decision for 2 years.

- As the investor is risk-neutral, he/she cares for the *highest expected return* from the investment.

Two possible investment strategies

- **Option 1:** Roll-over strategy with one-year bond in each year. (short-term strategy)
  - Suppose that yield of 1-year bond is $i_{t=0,1}$.
  - In the next year, yield of 1-year bond is expected to be $i^e_{t=1,1}$

Expectation Hypothesis: expected return under option 1

Expected return from Option 1 =

$$\left(1 + i_{t=0,1}\right)\left(1 + i^{e}_{t=1,1}\right) - 1$$

On its approximation

$$i_{t=0,1} + i^{e}_{t=1,1}$$

An alternative investment strategy: Option 2

- **Option 2:** *Long-term investment* with 2-year bond.
  - Suppose that yield of the 2-year bond is $i_{t=0,2}$

Expected return from Option 2 → $(1 + i_{t=0,2})^2 - 1$

On its approximation → $2 \times i_{t=0,2}$

Equilibrium: No-arbitrage condition

$$\underbrace{\boldsymbol{RER^{1,e}}}_{i_{t=0,1} + i^e_{t=1,1}} \begin{Bmatrix} > \\ < \end{Bmatrix} \underbrace{RER^{2,e}}_{2 \times i_{t=0,2}} \Rightarrow 2i_{t=0,2} = i_{t=0,1} + i^e_{t=1,1}$$

Rationality and No-Arbitrage Condition: An equalization of expected return obtained under the two investment options.

$$i_{t=0,2} = \frac{i_{t=0,1} + i^e_{t=1,1}}{2}$$

Expectations Hypothesis: *Generalization*

- Long-term T-period investment v.s. Roll-over short-term investment for T periods.

$$i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i_{t=j,1}^{e}}{T}$$

Long-term yield should be equal to the average of expected short-term yield over the duration of the remaining times/periods to maturity of the long-term bond.

Expectation Hypothesis: Fact 1

- Why does short-term yield and long-term yield co-move?
  - Suppose that current short-term yield ($i_{t=0,1}$) increases, while given that $i^e_{t=1,1}$, $i^e_{t=2,1}$, …. are fixed.

$$i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i^e_{t=j,1}}{T}$$

- Mathematically implied by EH, the average of interest rate will increase, and hence $i_{t=0,T}\uparrow$.

How tight do they commove?: evidence from 2005M1-2017M9.

	R1M	R3M	R6M	R1YR
R1M	1	0.997995	0.990083	0.968111
R3M	0.997995	1	0.996559	0.979335
R6M	0.990083	0.996559	1	0.991306
R1YR	0.968111	0.979335	0.991306	1
R2YR	0.902792	0.919897	0.942647	0.97503
R3YR	0.832165	0.851757	0.879886	0.926238
R4YR	0.747176	0.767354	0.79788	0.853303
R5YR	0.706364	0.724983	0.754556	0.812287
R6YR	0.663297	0.681325	0.711142	0.772238
R7YR	0.629757	0.646797	0.676096	0.738146
R8YR	0.602851	0.618559	0.64687	0.709318
R9YR	0.580742	0.595844	0.623935	0.686988
R10YR	0.546415	0.561243	0.5894	0.653821
R11YR	0.527782	0.541557	0.568603	0.632919
R12YR	0.515876	0.529029	0.555341	0.619423
R13YR	0.510568	0.523043	0.548554	0.612347
R14YR	0.493402	0.504989	0.529659	0.59297
R15YR	0.490684	0.50125	0.524726	0.586896
R16YR	0.493704	0.503572	0.52582	0.586933
R17YR	0.492329	0.501395	0.522362	0.582328
R18YR	0.495001	0.503508	0.523495	0.582194
R19YR	0.500082	0.508064	0.527442	0.585437
R20YR	0.504188	0.511843	0.531026	0.588574

- In practice, correlation is not 1.
  - $i_{t=0,1} \uparrow \Rightarrow i^e_{t=1,1}, i^e_{t=2,1}, \ldots$ may be also adjusting in the way that offsets the directional change of the current short-tem yield, and hence causing an imperfect correlation.
  - This point is to be explained later when we discuss about the dynamic yield curve.

Expectation Hypothesis: Fact 2

“yield curves tend to have steep slope when short rates are low and downward slope when short rates are high”

How?

Expectation Hypothesis: Fact 2

- Yield curve is "downward sloping" if short-term rate is exceptionally high.
  - Fist, note that downward sloping = STR > LTR!
  - ($i_{t=0,1}$) is exceptionally high → $i^e_{t=1,1}$, $i^e_{t=2,1}$ is expected to drop because current short-term rate is "too" high.
  - Average would be lower than ($i_{t=0,1}$) .
  - Following the Expected hypothesis, long-term interest rate will be lower than the current short-term rate.

Expectation Hypothesis: Fact 3

- Normally, yield curve is (mostly) upward sloping.
- To explain fact#3, it requires that average of expected future short-term interest rate is higher than today.

$$i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i^{e}_{t=j,1}}{T}$$

- Reasonable? No, because it implies that we always need a rising trend of market interest rate to account for the regularity of upward sloping yield curve.

How to Rationalize Stylized Facts?

- *Expectations Theory*
- ***Segmented Markets Theory***
- *Liquidity Premium (Preferred Habitat) Theory*

Segmented Market Model (SMM)

- Shortcomings of the EH → an assumption is relaxed
- Bonds with different TTM are NOT substitutable.
- Investors and Borrowers have made choice of investment decision and fund raising with *biasedness*.
- SMM assumes a strong version of *biased preference*; no substitution at all, and hence implies that markets are *completely segmented.*
- Implication: The determination of yield at each TTM will be separately determined in each market.

Segmented Market Model: **Bond Holder**

- Biased preference implies that investors prefer some types of bods to others. (make sense!?!?)

- Biasedness over TTM of bonds: relatively high demand of short-term bond to long-term bond.

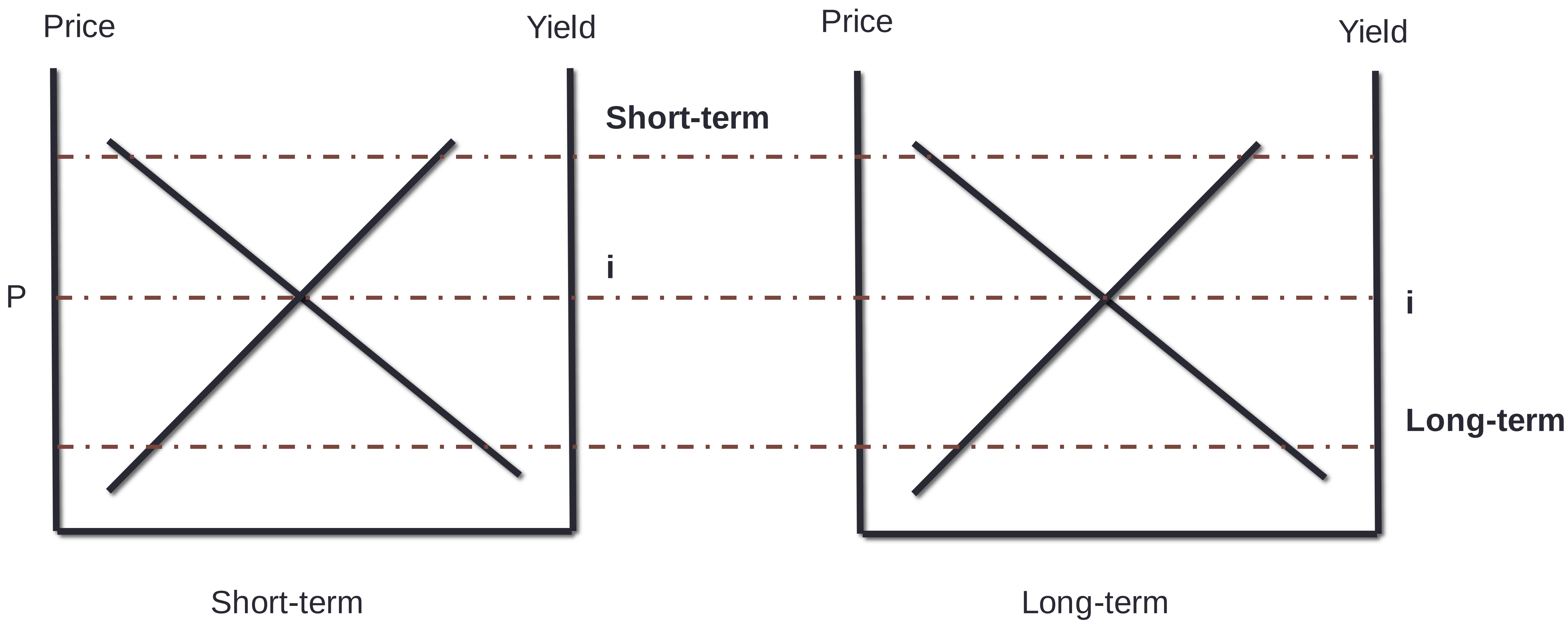
- Why? Uncertainty resolved sooner; the better.
  - Event Risk is lower.
  - Inflation Risk is lower.

Segmented Market Model: Bond Issuer

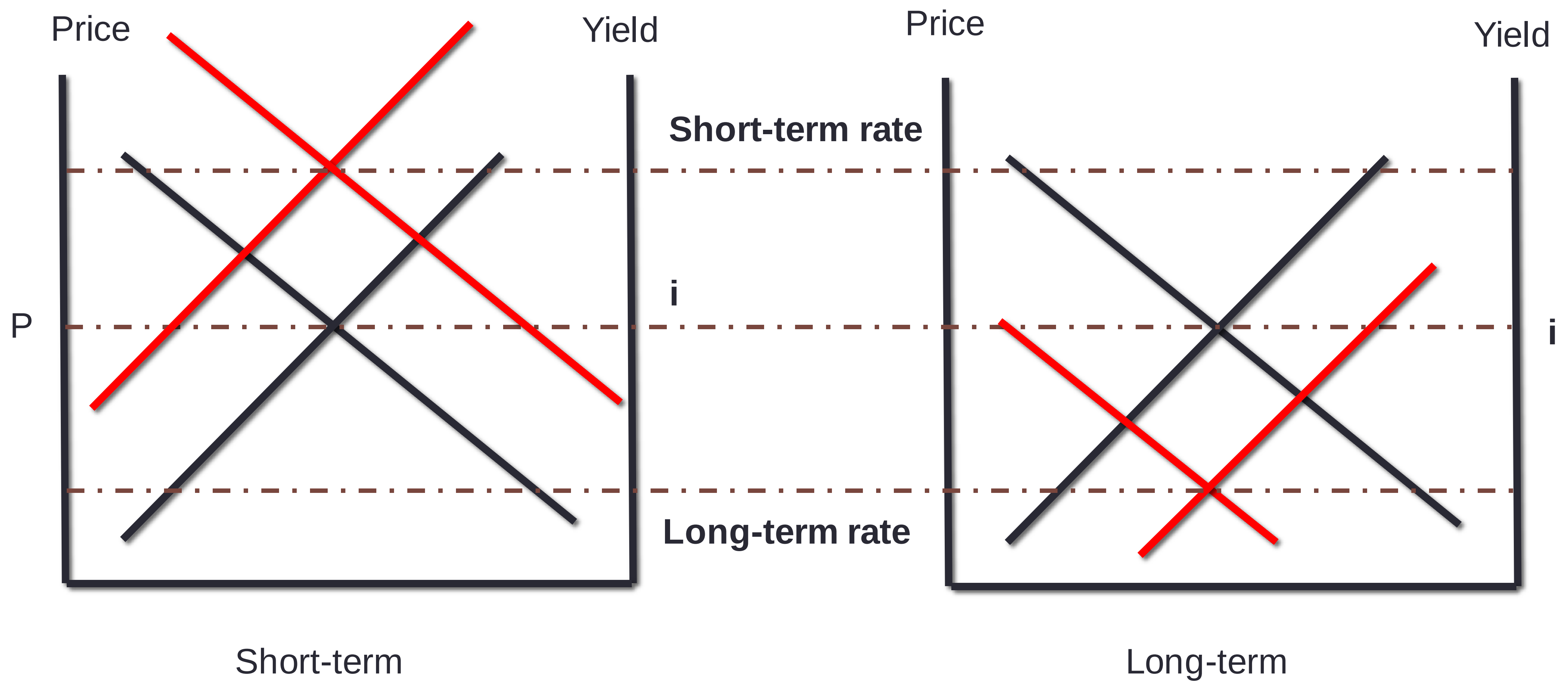
- Similarly, bond issuers usually prefer securing long-term funds, rather than short-term funds.

- There is a risk associated with the failure to refinance, i.e. roll-over risk.

Relative demand and supply for short-term and long-term bond: no biased / identical bonds



Relative demand and supply for short-term and long-term bond: with biased preference taken into account.



Problem with SMM

- No linkage between Short-term and long-term market.
- Changes in short-term market will not affect long-term market as we assume that each of them is separate on their own.
- So, our assumption is too strong.

Liquidity Premium or Preferred Habitat

- Risk aversion investor! ➔ Not only expected return still matters; but now risk also matters as well.
- What risk?
  - Event risk
  - Inflation risk
  - Interest rate risk
  - Liquidity risk
- All these risks vary with respect to time. How?
  - The longer, the bigger.

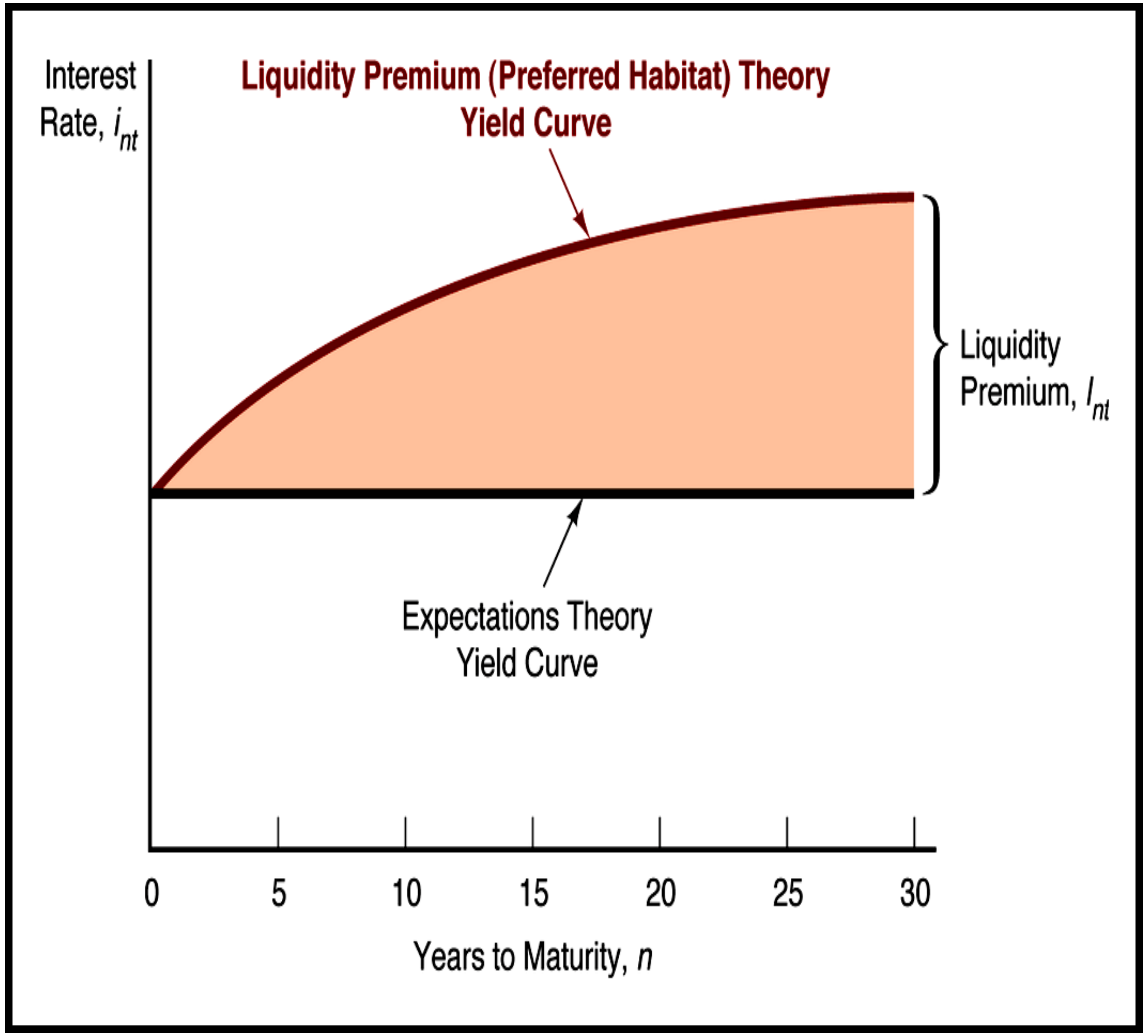
Liquidity Premium or Preferred Habitat

- Long-term bond is more riskier than short-term bond; you need to get some extra return on the top, i.e. the required **term premium**.
- Term premium $i(T)$ varies over TTM.
  - The longer TTM, the higher required premium.

$$\boldsymbol{i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i^e_{t=j,1}}{T} + i(T)}$$

$i(T)$ **: an increasing function with respect to "time to maturity"**

Liquidity Premium or Preferred Habitat



- Fact 3 can be explained under more general (reasonable) assumptions.
- Even if investors expected a constant path of future short-term rate, yield curve can still be upward sloping.