# INTRODUCTION TO GAME THEORY

- GAME = A strategic interaction among players.
  (=Mutual interdependence)

- COMPONENTS OF A GAME

  **Players** (P) : 2 or more than 2 players

  **Actions** (A) : Actions / Strategies / Moves available to each player.

  **Payoffs** (P) : outcomes based on your decision and another player's decision.

  Information (I) : Info. about
  - rule of the game (How game is conducted)
  - who moves first? Who moves second? or Both move simultaneously?
  - Is the game played only once or repeatedly?
  - etc.

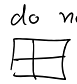
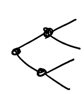
In short ⇒ P – A – P – I

- Types of games

  – One shot game Vs Repeated game
    - finitely repeated game
    - infinitely repeated game.

  – Simultaneous move game Vs. Sequential move game.
    ex: • car racing • penalty kick rock – paper – scissor | ex: chess, O–X

  – Cooperative game Vs Non-cooperative game.
    ex: • coalition game • Cartel | (game that players do not cooperate.

- Game Representation
  1. Payoff matrix 
  2. Game tree 

Payoff Matrix

Consider 2 players: 1 & 2. Each player has 2 actions.

Player 2 (column player) (unit: million baht)

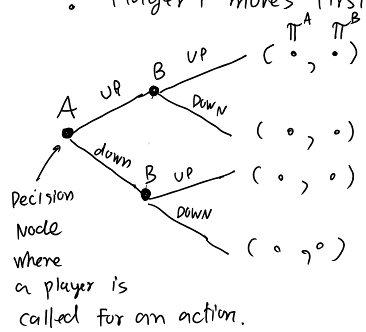
| Player 1 (Row player) | LEFT | RIGHT |
|---|---|---|
| UP | (10, 2) | (8, 3) |
| DOWN | (12, 4) | (10, 1) |

(First number, Second number)
First number → Player 1's Payoff
Second number → Player 2's Payoff

| Player 1 (Row player) | | | Player 1's Payoff | Player 2's Payoff |
|---|---|---|---|---|
| DOWN | (12, 4) | (10, 1) | | |

## Game tree

- Two players: A and B
- Two actions by each player
- Player 1 moves first and player 2 moves second.




# Solving a game

Consider an advertising game.

- Two players: Firm A and Firm B
- Two actions: Advertise, Do not advertise
- Simultaneous move game: Both choose an action simultenasly.

| | | Firm B ADV | Firm B DO NOT ADV |
|---|---|---|---|
| Firm A | ADV | (10, 5) | (15, 0) |
| Firm A | DO NOT ADV | (6, 8) | (10, 2) |

Q: What is the outcome of the game?

Fact#1 Regardless of what Firm B chooses or plays, Firm A always chooses "ADV."
So "ADV" is Firm A's dominant strategy

Fact#2 Firm B has a dominant strategy which is "ADV"

Fact#3 As both players have a dominant strategy and of course they will play the dominant strategy, then outcome of the game is (ADV, ADV)

The process used to find the outcome here is so called
"Elimination of dominated strategies."

# Solving a game by using "Nash Equilibrium Concept."

John F. Nash → Mathematician
→ Ph.D. (Princeton)
→ Nobel Prize winner (1994) w/ Selten and Harsanyi

Watch → "A beautiful Mind" in Netflix

| Firm A \ Firm B | LEFT | RIGHT |
|---|---|---|
| UP | $(10^*, 5^D)$ | $(15^*, 0)$ |
| DOWN | $(6, 8^D)$ | $(10, 2)$ |

<u>Consider Firm A</u> IF B plays "LEFT", A's best choice/response is to play "UP" (since $10 > 6$)

IF B plays "RIGHT", A's best response is to play "UP" (since $15 > 10$)

<u>Consider Firm B</u> IF A plays "UP", B's best response is to play "LEFT" (since $5 > 0$)

IF A plays "DOWN", B's best response is to play "LEFT" (since $8 > 2$)

There fore, $(s_A^*, s_B^*) = (UP, LEFT)$ is a Nash equilibrium

$$(\pi_A(s_A^*, s_B^*), \pi_B(s_A^*, s_B^*)) = (10, 5).$$

| | |
|---|---|
| <u>Nash equilibrium</u> = | Set of actions/strategies in which each player does its best given its rival's action/strategy. |

In formal language: " in a Nash equilibrium I am doing the best I can given what you are doing & you are doing the best you can given what I am doing "

· In a Nash equilibrium: no one would have incentive to deviate/or to alter his/her action, given an action chosen by his/her rival.

## Prisoner's Dilemma

Consider 2 suspects : Mr. A & Mr. B

Pareto superior outcome

| | | B: QUIET | CONFESS |
|---|---|---|---|
| A | QUIET | (−1, −1) | (−12, 0) D |
| | CONFESS | (0, −12) * | (−8, −8) * D |

# number of years in jail.

Pareto-inferior outcome

Consider A: if B plays "QUIET", A's BR is "CONFESS" $(0 > -1)$
if B plays "CONFESS", A's BR is "CONFESS" $(-8 > -12)$

Consider B: if A plays "QUIET", B's BR is "CONFESS" $(0 > -1)$
if A plays "CONFESS", B's BR is "CONFESS" $(-8 > -12)$

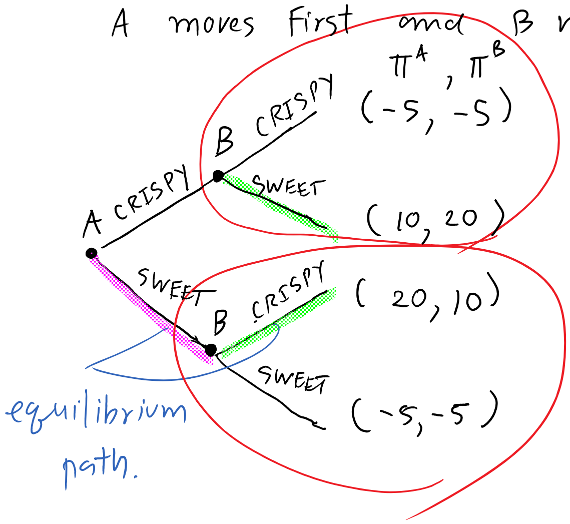
Therefore, A Nash equilibrium is ( CONFESS, CONFESS )

$$[\Pi_A(\text{CONFESS}, \text{CONFESS}), \Pi_B(\text{CONFESS}, \text{CONFESS})] = [-8, -8]$$

# Sequential Move game

Consider 2 Firms: A & B

A moves first and B moves second.




unit: million baht

Backward Induction : looking ahead and reasoning backward.

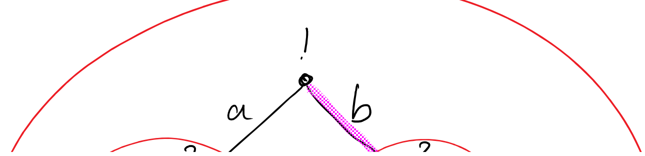
$$(s_A^*, s_B^*) = (\text{SWEET}, \text{CRISPY})$$

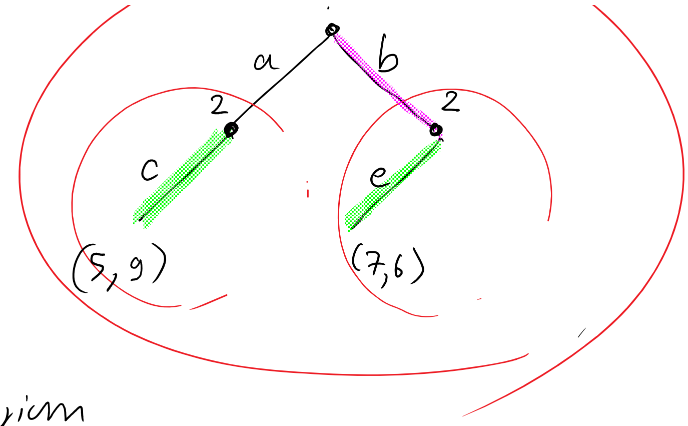
$$[\Pi_A(s_A^*, \ ^*), \Pi_B(s_A^*, s_B^*)] = (20, 10)$$

"First mover advantage" : $\Pi_A > \Pi_B$ in this outcome.
(20) (10)

By moving first, it leaves B little choice but to choose "CRISPY" to avoid crash in the same market.

Exercise






In equilibrium,

# Battle of Sexes

| | | WIFE | |
|---|---|---|---|
| | | BULL FIGHT | SHOPPING |
| HUSBAND | BULL FIGHT | (10*, 5 D) | (0, 0) |
| | SHOPPING | (0, 0) | (5*, 10 D) |

- Two Nash equilibria

  (BULL FIGHT, BULL FIGHT)

  &

  (SHOPPING, SHOPPING)

- which one will be played depends on players' bargaining power.

- Each player has NO dominant strategy:
  - The best response by H depends on what W chooses
  - The best response by W depends on what H chooses.

# Matching pennies

| | | ② | |
|---|---|---|---|
| | | H | T |
| ① | H | (1, -1) | (-1, 1) |
| | T | (-1, 1) | (1, -1) |

- No Nash equilibrium
- zero-sum game